

LESSON 8

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- LEVEL INDICATORS

- Superscripts
- Baseline Indicator
- Subscripts
- More about Superscripts and Subscripts

- GROUPING SIGNS, cont.

FORMAT

DISPLAYED MATHEMATICAL EXPRESSIONS

LEVEL INDICATORS

8.1 Definition: A mathematical expression can contain symbols placed above or below the baseline level (the normal line of type in print). Superscripts appear above the baseline; subscripts appear below the baseline. Here are some expressions which contain superscripts and subscripts. The baseline is marked for orientation.

__baseline__ 10^5 H_2O -4 $^\circ\text{C}$ πr^2 $^{235}_{92}\text{U}$ __baseline__

Note: The font size is increased for the isolated examples in this lesson to help in determining the levels.

Superscripts

8.2 Superscript Level Indicator: In braille, indicators are used to identify the level of a superscript or subscript. The superscript level indicator is used to show that the symbols immediately following it appear on the first level above the baseline of writing.

Superscript Indicator	⠠
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➤ 10^7 ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠

➤ $y = [g(x)]^n$ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠

The characters in the superscript are spaced according to the rules of the Nemeth Code.

➤ $3^{-0.05\text{T}}$ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠

Remember, a numeral does not need a numeric indicator when it follows and is unspaced from a mathematical character or indicator. A letter, a shortform letter combination, or a Roman numeral does not need an English letter indicator unless it is preceded by a space or by one or more punctuation marks and followed by a space or by one or more punctuation marks.

➤ $x x^3$ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠ ⠠

8.3 Returning to the Baseline Level: When a superscript is the last item in the expression, the following circumstances bring the reader back to the baseline level: a space, a comma followed by a space, or a punctuation indicator. "A space" includes the space before a Nemeth Code termination indicator and/or transition to a new braille line.

- A space returns the reader to the baseline.

Example 8.3-1 10^2 10^3 10^4

Example 8.3-2 10^2 and 10^3

Example 8.3-3 Which term of $(a + b)^{12}$ has the factor b^5 ?

Example 8.3-4 5.3×10^{-3} 5.3×10^{-1} 5.3×10^0 5.3×10^3

- A comma followed by a space returns the reader to the baseline.

Example 8.3-5 Which is the largest area? 3 cm^2 , 7 m^2 , 9 ft^2

Example 8.3-6 Several ways to notate the number nine: $5 + 4$, 3^2 , $17 - 8$, -9×-1 .

Reminder: The multipurpose indicator (dot 5) is placed between the symbols \times and $-$ to show that they are printed side-by-side.

PRACTICE 8B

1. Tell what number each of the following names: 6^2 , 5^{-3} , and $(3.15)^4$.
2. Which expression denotes the area of a circle?
 - a. 360°
 - b. πr^2
 - c. πd
3. The symbol for "feet squared" is ft^2 . $1 \text{ ft} \times 1 \text{ ft} = 1 \text{ ft}^2$
4. 6.02×10^{23} "is an approximation of "Avogadro's number".

Introduction to the Baseline Indicator

8.5 Function of the Baseline Indicator: In an unspaced expression, a return to the baseline level is brought about by the use of the baseline indicator.

Example 8.5-1 Draw a 30°-60°-90° triangle.

Notice that the baseline indicator is the same symbol as the multipurpose indicator—dot 5. The indicator's function is determined in context.


Example 8.5-2 Adding cubic meters: $2 \text{ m}^3 + 2 \text{ m}^3 = 4 \text{ m}^3$

Reminder: The superscript indicator is unspaced from the abbreviation.

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8.6.1 Combinations: Each level indicator is read *as it relates to the baseline level*. The effect of one level indicator is terminated by another level indicator. Keep this in mind as you read the following examples.

Prime Sign ' ∙

$\gg A^{2''}$

The prime signs belong with the number 2 in the superscript.

 $\gg A^{m+m+m'} s$
[illegible]

1. Use a calculator to find 9^{9^9} .
2. Find the r^{th} term of $(x + y)^n$.
3. Label the x^2 's and x^3 's.
4. What is the meaning of x''^3 ?
5. Simplify: $(x^3 - y^3)^2 - (x^3 + y^3)^2$.
6. x^{y^nz} or x^{y^2z}

Subscripts

8.8 Subscript Level Indicators: *Except* as stated in **8.10** below, the subscript level indicator is used to show that the symbols immediately following it appear on the first level below the baseline of writing.

Subscript Indicator 

$$\gg f_n \quad \begin{array}{ccc} \bullet\bullet & \cdot\cdot & \bullet\bullet \\ \cdot\cdot & \bullet & \bullet\bullet \\ \cdot\cdot & \bullet & \bullet\bullet \end{array}$$
$$\gg a_{(k+1)} \quad \begin{array}{ccccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

Subscripts may carry subscripts of their own. In such cases, the subscript level indicator is doubled, tripled, etc. to indicate subscripts on the second, third, or lower levels.

Subscript with Subscript $\ddot{\bullet} \ddot{\bullet}$ (two levels below the baseline)

Subscript with Subscript with Subscript $\ddot{\text{c}}\ddot{\text{c}}\ddot{\text{c}}$ (three levels below the baseline)


$$\gg n_{x_y} \quad \begin{array}{cccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$
[illegible]

Note that the subscript indicator $\substack{\cdot}{\cdot}$ is the same symbol as the English letter indicator. The indicator's function is determined in context.

Recall that an English letter indicator is not used in an unspaced mathematical expression unless the letter is printed in a mathematically-significant typeform. Compare these transcriptions of the letter "i" in regular type and bold type:

$\gg 3i$ 
Spoken: three i

≫ **3i** ⠼⠨⠦⠨⠦⠨⠦⠨⠦⠨⠦
Spoken: three bold i

$\gg 3_i$ 
Spoken: three sub i

$\gg 3_i$ ⠼⠨⠦⠢⠨⠶⠨⠧⠨⠥⠨⠩⠨⠎
Spoken: three sub bold i

PRACTICE 8E

$$\begin{array}{cccc}
 3_c & 10_8 & ?_3 & x_{2+k} \\
 \delta_{ij} & 10_7 & \text{Ca}(\text{OH})_2 & y_{-2} \\
 y_{n_k} & 3_{\text{five}} & P_{3n} & a_{m1}
 \end{array}$$

8.9 Returning to the Baseline Level: The same circumstances discussed with superscripts bring the reader back to the baseline level from a subscript: a space, a comma followed by a space, a punctuation indicator, or a baseline indicator. "A space" includes the space before a Nemeth Code termination indicator and/or transition to a new braille line.

Example 8.9-1 Add in base 12: 27TE_{12} and $\text{E}5\text{T}_{12}$; $4\text{E}9_{12}$, $8\text{T}2_{12}$, and $\text{T}0\text{E}_{12}$.

$$\begin{array}{ccccccc}
 2 & 7 & \text{T} & \text{E} & 12 & + & \text{E} & 5 & \text{T} & 12 \\
 4 & \text{E} & 9 & + & 8 & \text{T} & 2 & + & \text{T} & 0 & \text{E} \\
 12 & & & & & & & & & & 12
 \end{array}$$

Example 8.9-2 $27\text{TE}_{12} + \text{E}5\text{T}_{12} = \text{---}_{12}$

$$\begin{array}{ccccccc}
 2 & 7 & \text{T} & \text{E} & 12 & + & \text{E} & 5 & \text{T} & 12 & = & \text{---} & 12
 \end{array}$$

Note that the level indicator is unspaced from the long dash to which it applies.

Example 8.9-3 $(R_H T_H) + (R_S T_S)$

$$(R_H T_H) + (R_S T_S)$$

⠠ is the subscript indicator throughout this expression. Letters H and S are subscripts.

PRACTICE 8F

1. Name the numeral in base ten equal to:
a. 47_8 b. 34_6 c. 1101_2
2. $C_{\text{hex}} = 12_{\text{dec}} = 14_{\text{oct}}$
3. What do we know if $P_{n_r} = (x_{n_k}, y_{n_k})$?
4. $7_8 - 4_8$

- 0-10** **C** **G** **N** **S** **T** **E** **R** **I** **N** **G** **A** **L** **E** **X** **A** **M** **P** **L** **E**

- the subscript is *numeric*;
- the numeral is a *right subscript to a letter*;
- the subscript is on the *first* level below the baseline of writing.

Figure 1 consists of 10 small diagrams arranged horizontally, each showing a 10x10 grid of dots. The dots represent particles in a lattice. The diagrams show the progression of a process over time, with the number of particles increasing from 1 to 10. The diagrams are labeled 1 through 10.

When a subscript does not require a subscript indicator, a return to the baseline is implied following the numeric subscript. A baseline indicator is not needed.

Letters H and O are on the baseline of writing. Numeral 2 is a subscript.

All numerals are subscripts. A baseline indicator is not needed to show that the letters and the plus signs are on the baseline level because no subscript indicators are used.

The subscripts are numerals 3 and 2. Everything else is on the baseline of writing, including the closing parenthesis. The second subscript requires a level indicator because it is not a right subscript to a letter.

8.10.1 Further Conditions: The definition of "numeric" and "letter" include the following:

- The numeric subscript may contain a *segmenting comma* or a *decimal point*.

» x_{1,000} ⠨⠭⠏⠒⠐⠔⠎ (The numeral 1,000 is a right subscript to the letter x.)

➤ X_{5.3} ⠠⠭⠨⠠⠼⠠⠅⠠⠗⠠⠅ (The numeral 5.3 is a right subscript to the letter x.)

- The letter may carry one or more *primes*.

$\gg X'_2$ $\begin{smallmatrix} \bullet\bullet \\ \bullet\bullet \end{smallmatrix} \begin{smallmatrix} \bullet\bullet \\ \bullet\bullet \end{smallmatrix} \begin{smallmatrix} \bullet\bullet \\ \bullet\bullet \end{smallmatrix}$ (The numeral 2 is a right subscript to X' .)

- The letter may be taken from *any alphabet* in *any typeform*.

[illegible]

i₁ ⠠⠊⠨ (bold English letter i, subscript one.)

- The letter may also carry a *superscript* (See **8.16**, *Simultaneous Superscripts and Subscripts*)

- The letter may be part of a *two-letter chemical abbreviation*.*

 (Two-letter chemical abbreviation for chlorine, subscript four.)

*A one- or two-letter "abbreviation" for a chemical compound follows spacing rules for letters. For example, Fe_2O_3 is brailled without spaces:

8.10.2 Restrictions: Just because a numeral is a right subscript to a letter does not mean that the special rule can be applied. The subscript level indicator must be used if any of the following conditions apply:

- If the letter is functioning as a numeral in a nondecimal numeration system a subscript indicator is required. (This example shows base 12 numeration.)

 TE₁₂

- If the letter is part of a word or abbreviation a subscript indicator is required.

» five₃ ⠠⠋⠗⠑⠨⠠⠋⠗⠑⠨⠠⠋⠗⠑⠨⠠⠋⠗⠑⠨⠠⠋⠗⠑⠨

- If the subscript contains any symbol other than a numeral with its comma or decimal point a subscript indicator is required.

$$\gg x_{2k} \quad \begin{array}{cccc} \bullet & \bullet & \cdot & \cdot & \cdot & \bullet & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \bullet & \bullet & \cdot & \cdot & \cdot & \bullet & \cdot \end{array}$$
$$\gg x_{2'} \quad \begin{array}{cccc} \bullet & \bullet & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \bullet & \bullet & \bullet & \bullet \end{array}$$
$$\gg x_{-2} \quad \begin{array}{cccc} \bullet\bullet & \cdot & \cdot & \cdot \\ \cdot & \bullet & \cdot & \bullet \\ \bullet\bullet & \bullet & \bullet\bullet & \bullet \end{array}$$


— If the subscript carries a superscript or subscript of its own a subscript indicator is required.

$$\gg x_{2n} \quad \begin{array}{cccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

- Numeric subscripts on the second or lower levels always require their appropriate subscript level indicators.

$$\gg P_{n_1} \quad \begin{array}{ccccccc} & \bullet & & \bullet & & & \\ \cdot & & \cdot & & \cdot & & \cdot \\ \cdot & \bullet & \cdot & \bullet & \cdot & \cdot & \cdot \\ \cdot & & \cdot & & \cdot & & \cdot \\ & \bullet & & \bullet & & & \end{array}$$

8.10.3 Rewind: Lesson 7 looked at unspaced number/letter combinations. 7.16 illustrated that, when a number appears to the right of a letter on the baseline level, a multipurpose indicator is necessary.

➤ R12 

Without the multipurpose indicator, the number will read as a subscript.

But if the expression is a subscript, no indicator is needed because the level in effect continues.

$$\gg t_{R12} \quad \begin{array}{cccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

8.10.4 Summary: The rules regarding use/nonuse of the subscript indicator are summarized as follows.

A subscript level indicator is not used before a numeric subscript on the first level below the baseline of writing when the numeral is a right subscript to a letter. The numeric subscript may contain a segmenting comma or a decimal point. The letter may carry one or more primes or a superscript, may be taken from any alphabet in any typeform, and may be part of a two-letter chemical abbreviation. However, the letter must not be used as a numeral in a nondecimal numeration system or as part of a word or an abbreviation.

The subscript level indicator must be used with a numeral on the first level below the baseline if the subscript contains any symbol other than a numeral with its comma or decimal point, or if the subscript carries a superscript or subscript of its own. Subscripts on the second or lower levels always require their appropriate subscript level indicators.

PRACTICE 8G

- 1) These expressions need subscript indicators in braille: y_{-2} , x_{2+k} , a_{m1} , x_{3n} , x_{y_2} .
 - 2) These expressions do not use a subscript indicator in braille: x_1 , ax_2 , CO_2 , $z_{.7}$, β_2 .
 - 3) Decide whether these expressions require a subscript indicator and braille them correctly: shape_4 , Q'_2 , $\text{C}_6\text{Fe}_2\text{O}_{12}$, n_k , x_{2k} , $P_{r_{st}}$, D_{56} , $G_{9,999}$, $2\text{E}6\text{B}_{16}$.
 - 4) Use $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ to denote the direction vectors v_1 and v_2 .
 - 5) *Chemistry*: While Na_2ZnCl_4 could be cooled in the normal way, $\text{Na}_2[\text{CoCl}_4]$ had to be quenched in the liquid N_2 .
 - 6) $f_1(x) = g(x) \cdot q_2(x) + f_2(x)$
-

8.11 Spaces Within Superscripts and Subscripts: A space usually returns the reader to the baseline. Various strategies are used to retain the level in effect when a space occurs within a superscript or a subscript.

8.11.1 Commas: As noted above, the effect of a level indicator is terminated by a comma unless the comma occurs in a long numeral (a "segmenting" comma). *Recall:* A segmenting comma maintains the level in effect,

$\gg x_{1,000}$

The numeral 1,000 is a right subscript to the letter x.

but a comma followed by a space re-establishes the baseline. The return to the baseline starts at the comma. No baseline indicator is needed.

$\gg x^2, y^2, z^2$

$\gg (a_1, a_{1i}, a_{2i})$

Example 8.11-1 Add: 2_{five} , 3_{five} , and 4_{five} .

8.11.3 Comparison Signs: The space before a comparison sign returns the reader to the base-illustrated in the next example.

$$\gg S_u = a$$

Example 8.11-4 Fill in the blank with a simplified superscript: $10^{7+3} = 10^{-}$

[illegible]
$$\gg P_{n_1, n_2, \dots}$$

8-16

The level extends through the required spaces before and after an ellipsis or a long dash.

$$\gg x^{1+3+5+\dots+(2n-1)}$$

The superscript is $1 + 3 + 5 + \dots + (2n - 1)$. The superscript level extends through the spaces before and after the ellipsis.

Exception: If an ellipsis or long dash in a superscript or subscript is followed by a sign of comparison or by literary text, reading returns to the baseline level without the need for a baseline indicator.

$$\gg 10^{7+} = 10^{21}$$

The first superscript is $7+$ ____. The superscript level extends through the required space before the long dash. The sign of comparison after the dash is assumed to be at the baseline level.

$$\gg 10^{7+} \text{ equals } 10^{21}$$

The first superscript is $7+$ ____. The superscript level extends through the required space before the long dash. The word after the dash is assumed to be at the baseline level.

8.11.5 Ellipsis or Long Dash on the Baseline of Writing: Because the space before an ellipsis or long dash maintains the level in effect, an indicator is required to return to the baseline when the symbol is printed there. The baseline indicator takes the place of the required space.

$$\gg a^1 b^2 c^3 d^4 \dots z^n$$

There is no need to indicate a return to the baseline after a numeric subscript that does not require a subscript indicator.

$$\gg r_1 \dots r_n$$

The ellipsis is printed on the baseline. The subscripts are 1 and n.

8.11.6 Segmented Numbers: The effect of a level indicator extends through the space inserted in a numeral for the purpose of dividing it into short regular groups of digits.

$$\gg e^{3.14159\ 26535}$$

The superscript is 3.14159 26535

PRACTICE 8H

Figure 1 shows a 3x10 grid of 30 small 3x3 dot patterns. Each pattern contains 8 dots arranged in various configurations. The patterns are numbered 1 through 30 in a single row below the grid.

(a) (b) (c) (d) (e)

The figure shows 12 stages of a pattern's growth on a 4x4 grid. The dots are represented by black squares on a white background. The stages are numbered 1 through 12.

- Stage 1: (1,1)
- Stage 2: (1,1), (1,2)
- Stage 3: (1,1), (1,2), (2,1)
- Stage 4: (1,1), (1,2), (2,1), (2,2)
- Stage 5: (1,1), (1,2), (2,1), (2,2), (3,1)
- Stage 6: (1,1), (1,2), (2,1), (2,2), (3,1), (3,2)
- Stage 7: (1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1)
- Stage 8: (1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2)
- Stage 9: (1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2), (1,3)
- Stage 10: (1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2), (1,3), (1,4)
- Stage 11: (1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2), (1,3), (1,4), (2,3)
- Stage 12: (1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2), (1,3), (1,4), (2,3), (2,4)

The figure consists of 12 sub-diagrams arranged in a single row, each showing a 5x5 grid of dots. Black dots represent the presence of a certain state at a specific location and time. The sequence shows a pattern that starts as a small cluster of dots on the left and grows and moves towards the right over time.

More about Superscripts and Subscripts

8.12 Superscript and Subscript Combinations: Combinations of subscripts to superscripts or of superscripts to subscripts require level indicators composed of two or more braille symbols. Keeping in mind that level indicators relate to the baseline, transcribing the following types of expressions is an exercise in logical thinking.

Superscripts with Subscripts

Superscript With Subscript

[illegible]

n_1 (n, subscript one) is in the superscript position. "1" is a super-subscript.

$$\gg 2^{Y_0} = Y_1 \quad \begin{array}{ccccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

Y_0 (Y, subscript zero) is in the superscript position. "0" is a super-subscript.

Reminder: The subscript indicator is omitted for a numeric subscript to a letter only for subscripts that are located on the *first level below the baseline of writing*. The super/sub indicator is needed to show a numeric subscript in the superscript position.

Subscripts with Superscripts

Subscript With Superscript


$$\gg x_{n^2} \quad \begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \end{array} \quad \begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \end{array} \quad \begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \end{array} \quad \begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \end{array} \quad \begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \end{array}$$

n^2 (n, superscript two) is in the subscript position. "2" is a sub-superscript.

$$\gg P_{3^n} \quad \begin{array}{cccccc} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

3^n (3, superscript n) is in the subscript position. "n" is a sub-superscript.

Reminder: The subscript indicator is required when a numeric subscript to a letter carries a superscript or subscript of its own.

PRACTICE 8I

Instructions: Descriptions are included in a righthand column for your benefit—do not braille the descriptions. Analyze the levels as you braille and as you proofread.

Superscripts with Subscripts

$$2^{\aleph_0} = \aleph_1$$

\aleph_0 is in the superscript position. (The aleph is not bold.)

$$a = 2^{k_1} \text{ and } b = 2^{k_2}$$

k_1 and k_2 are in the superscript position.

$$(ab)^x = 2^{k_1x} \cdot 2^{k_2x}$$

x , k_1x , and k_2x are superscripts.

$e^{i\theta_1}$ times $e^{i\theta_2}$ equals $e^{i(\theta_1+\theta_2)}$

$i\theta_1, i\theta_2$, and $i(\theta_1 + \theta_2)$ are superscripts.

Subscripts with Superscripts

Z_{5^n}

5^n is in the subscript position.

$7t_s^4$

S^4 is in the subscript position.

8.13 Left Subscripts and Superscripts: The appropriate level indicator is brailled before a subscript or superscript printed to the left of its related sign.

$$\gg 3X_1 \quad \begin{array}{ccccc} & & & \bullet\bullet & \\ \bullet & \bullet\bullet & \bullet & \bullet\bullet & \bullet \\ \bullet & & & \bullet\bullet & \end{array}$$

$$\gg \quad {}_{48}C_9 \times {}_4C_4$$

Reminder: The subscript indicator is not used when a numeral is a right subscript to a letter on the baseline.

$$\gg \quad {}^{14}\text{C} \quad \begin{array}{cccccc} \bullet & & & & & \bullet \bullet \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

8.13.1 Raised Negative Sign: In some texts, negative numbers are shown with a raised negative sign. The raised position of the negative sign must be shown in braille.

$$\gg -4 \quad \begin{array}{cc} \bullet & \bullet \\ \vdots & \vdots \\ \bullet & \bullet \end{array} \quad \begin{array}{cc} \bullet & \bullet \\ \vdots & \vdots \\ \bullet & \bullet \end{array} \quad \begin{array}{cc} \bullet & \bullet \\ \vdots & \vdots \\ \bullet & \bullet \end{array} \quad \begin{array}{cc} \bullet & \bullet \\ \vdots & \vdots \\ \bullet & \bullet \end{array}$$

Example 8.13-1 Explain why $4 + ^{-}4 = 0$.

The figure consists of 10 sub-diagrams arranged in a single row, each showing a pattern of black dots on a grid. The patterns evolve from left to right. The first pattern is a small cluster of 5 dots. The second pattern is a 2x2 square of 4 dots. The third pattern is a 3x2 rectangle of 6 dots. The fourth pattern is a 4x2 rectangle of 8 dots. The fifth pattern is a 5x2 rectangle of 10 dots. The sixth pattern is a 6x2 rectangle of 12 dots. The seventh pattern is a 7x2 rectangle of 14 dots. The eighth pattern is an 8x2 rectangle of 16 dots. The ninth pattern is a 9x2 rectangle of 18 dots. The tenth pattern is a 10x2 rectangle of 20 dots.

Punctuation mode is literary.

Example 8.13-4 Express average normal body temperature in $^{\circ}\text{C}$; in $^{\circ}\text{F}$.

The "C" is followed by a semicolon. No punctuation indicator is used because abbreviations are punctuated in literary mode.

8.14 Further Combinations: The Nemeth codebook illustrates additional combinations of superscripts and subscripts. Due to the obscurity of such complex combinations, only a few examples are shown in this lesson book. Proper interpretation of these characters will require reading the surrounding text in order to apply the correct indicators.

$\gg_{yx}n$ n is on the baseline of writing and has a left subscript.

Question: Is the subscript y^x or $_y x$?

This transcription shows left subscript "y" with a sub-superscript "x".

This transcription shows left subscript "x" with a left sub-subscript "y".

$\gg x_y n$ n is on the baseline of writing and has a left subscript.

Question: Is the subscript x_y or xy ?

This transcription shows left subscript "x" with a sub-subscript "y".

A 3x7 grid of dots. The first three columns contain only small dots, while the last four columns contain a mix of small and large dots.

This transcription shows left subscript "y" with a left sub-superscript "x".

8.15 Consecutive Superscripts and Consecutive Subscripts: The appropriate indicator is repeated before a left superscript or a left subscript when two are consecutive—each superscript or subscript has its own indicator. One indicator applies to the expression preceding it and the other to the expression following it.

The space that appears in print helps visually differentiate the two factors, but no space comes between factors in braille unless other spacing rules take precedence.

$$\begin{aligned} \gg x_1^2 & \quad \text{Braille: } \mathbf{x}_1^2 \\ \gg a^k_m & \quad \text{Braille: } \mathbf{a}^k_m \\ \gg y^n_x & \quad \text{Braille: } \mathbf{y}^n_x \\ \gg {}^b_a n & \quad \text{Braille: } \mathbf{{}^b_a n} \end{aligned}$$

Note: A magnifier and a straightedge can help determine whether superscripts or subscripts are simultaneous or nonsimultaneous. If in doubt whether the expression shows nonsimultaneous super/subscripts or if, instead, the super/subscripts have super/subscripts of their own, compare to the surrounding text for clues.

8.18 Detached Superscripts and Subscripts: When an entire superscript or subscript stands alone, it is brailled without a level indicator. A transcriber's note explains its print position.

Example 8.18-1 In x^2 , the 2 is the exponent.

$$\text{Braille representation of } x^2 \text{ showing the exponent } 2 \text{ as a detached superscript.}$$

8.19 Literary Symbols and Level Indicators: If a literary symbol is used mathematically and appears in a superscript or a subscript, its level must be indicated.

Example 8.19-1 A dagger may be used as a superscript in quantum mechanics, for example, A^\dagger .

$$\text{Braille representation of } A^\dagger \text{ showing the dagger symbol as a superscript with a level indicator.}$$

Summary

Here is a summary of the difference in print among four types of superscripts and subscripts as defined in the Nemeth Code.

- **Consecutive:** A right super/subscript belongs to the preceding character and a left super/subscript belongs to the next character.
- **Simultaneous:** A character has both a superscript and a subscript. The superscript is printed directly above the subscript.
- **Nonsimultaneous:** A character has both a superscript and a subscript. The two scripts are not printed directly above and below each other.
- **Detached:** A super/subscript stands alone without being associated with a letter or number. It is printed slightly above or below the baseline and is smaller than the rest of the text.

PRACTICE 8J

1. Here are some expressions with left superscripts: 3x , nx , ${}^{-2} + {}^{-4} = {}^{-6}$, $(-3)^{-2+{}^{+2}}$.
 2. ${}^{12}_6\text{C}$ and ${}^{12}\text{C}$ represent the same carbon isotope.
 3. $\text{D}_2{}^{18}\text{O}$ is the doubly labeled water isotopologue!
 4. In CO_2 , the subscript $_2$ means "two oxygen atoms".
 5. ${}_nP_r = K({}_{n-1}P_{r-1})$
 6. $a_1^2 + b_1^2 + c_1^2$
 7. $[t]_0^4$
 8. $2 \times 10_6^2 + 3 \times 10_6^1 + 2$
 9. $P_{xy}Q$
 10. $\text{NH}_4^+ + \text{Cl}^- + \text{H}_2\text{O}$
 11. *Temperature Conversion:* What is 0 K in °C? in °F? (Answers: -273.15° C; -459.67° F.)
-

GROUPING SIGNS, *cont.*

8.20 Review of Rules: The following rules have been discussed in earlier lessons. A review of the cited sections is recommended.

- In mathematical context, enclosure symbols are not considered to be punctuation; they are signs of grouping. Grouping symbols that are part of a mathematical expression must be brailled as Nemeth braille symbols. [3.9]
- The numeric indicator is not used before a numeral which immediately follows an opening sign of grouping unless such numeral is in non-regular type. [3.10, 7.7]
- The English letter indicator is not used when a single English letter or a Roman numeral is entirely enclosed within signs of grouping unless such a letter, or Roman numeral is in non-regular type. This rule does not apply to abbreviations. [4.12.2, 5.3.5]
- Single-letter abbreviations without a related period, or abbreviations whose letters correspond to a shortform without a related period, require an English letter indicator even when enclosed between signs of grouping. [5.19, 5.20]
- When a single English letter, or a Roman numeral is in direct contact with only its opening or only its closing sign of grouping, the English letter indicator is or is not used as though the grouping signs were absent. If the grouping sign carries a prime or other modifying symbol, the English letter indicator is not used. [5.16]

- Regarding the main topic in this lesson—superscripts and subscripts—another point can be added.

- $$\gg (R_H T_H) + (R_S T_S)$$

$$\gg (a_1, a_2, a_3)$$

Example 8.21-1 Solve: $(x^2 + y^2) - (x^2 + y^2)$

Example 8.21-2 In this case, $x_{(a,b)} + y_a$.

The figure consists of 10 sub-diagrams, each showing a 5x5 grid of dots. Black dots represent the 'on' state of a cell, while white dots represent the 'off' state. The sequence shows a pattern that grows from a small cluster in the top-left corner towards the bottom-right corner, exhibiting a fractal-like growth pattern.

- Diagram 1: A small cluster of 4 black dots in the top-left corner.
- Diagram 2: The cluster grows to 6 dots.
- Diagram 3: The cluster grows to 8 dots.
- Diagram 4: The cluster grows to 10 dots.
- Diagram 5: The cluster grows to 12 dots.
- Diagram 6: The cluster grows to 14 dots.
- Diagram 7: The cluster grows to 16 dots.
- Diagram 8: The cluster grows to 18 dots.
- Diagram 9: The cluster grows to 20 dots.
- Diagram 10: The cluster grows to 22 dots.

Example 8.21-3 Solve for a and b : $(a^2)^8(+2b)^3$

Review the examples in 8.6.1 with this in mind.

8.22 Grouping Symbols with Super/Subscripts:

$$\gg S_a^b \quad \begin{array}{ccccccc} \bullet & & \bullet & & \bullet & & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} \quad \text{Think: } S_a^b \quad \begin{array}{ccccccc} \bullet & & \bullet & & \bullet & & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

8.23 More Signs of Grouping: Grouping signs w

Barred Brackets

I



1



Barred Braces

 $\{$ 

I



Half Brackets

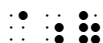
┐ or ┐



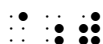
or





_____ or _____



└ or ─


$$\gg A_{n \sqcup i} \quad \begin{array}{ccccccc} & \bullet & & \bullet & \bullet & \bullet & \\ : & : & : & : & : & : & \\ \bullet & : & \bullet & \bullet & : & \bullet & \end{array}$$
$$\gg \lfloor x \rfloor \quad \begin{array}{cccccc} \bullet & & \bullet & \bullet & & \bullet \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \bullet & \bullet & \cdot & \bullet \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

Example 8.23-1 Integer division can be defined as $a \setminus b \equiv \lfloor a/b \rfloor$, where $/$ denotes normal division and $\lfloor x \rfloor$ is the floor function.

Example 8.24-1 To illustrate order of operations with $18 + 64 \times 5 - 6 \div 2$, Angie drew starbursts around each grouping: $18 + \textcircled{64 \times 5} - \textcircled{6 \div 2}$

PRACTICE 8K

- (1) $\{f_n\}$
- (2) $|a_m - a_n|$
- (3) $(x_1y_1 + x_2y_2)$
- (4) $([\text{CH}_3]_2\text{CH})$
- (5) $\text{I}_{\text{ue}}^{2''} = (\text{H}'_{44}\text{x}'_{\text{ve}})^{+'}$
- (6) $[x] = m$ if and only if $m \leq x < m + 1$; $[x] = n$ if and only if $n - 1 < x \leq n$.
- (7) The dagger and the asterisk are used as superscripts in quantum mechanics: A^\dagger , $(x^\dagger)^\dagger = x$, $\langle \phi | \psi \rangle^* = \langle \psi | \phi \rangle$.

INTRODUCTION TO DISPLAYED MATHEMATICAL EXPRESSIONS

FORMAT

8.25 Displayed Mathematical Expressions: Up to this point in the lesson material, mathematical expressions in the examples have appeared in line with the narrative. These are referred to as *embedded expressions*. When mathematical material is set apart from the body of the text in the print copy, it is referred to as a *displayed expression*. Various layouts in the print copy are used to set the material apart, for example, skipped lines, centering or other indentation, or off to the side. In braille, margins for displayed mathematical material depend upon the margins of the surrounding text and are transcribed in one of the following formats.

- *In unitemized explanatory portions of the text*, displayed mathematical material begins in cell 3. Runovers begin in cell 5. In other words: text (3-1); displayed material (**3-5**).
- *In itemized text without subdivisions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. In other words: text (1-3); displayed material (**5-7**).
- *In itemized text with subdivisions*, displayed mathematical material begins in cell 7. Runovers begin in cell 9. These margins apply to both items and subdivisions, to whatever depth. In other words: main division text (1-5); displayed material (**7-9**). Subdivision text (3-5); displayed material (**7-9**).
- *Within or following instructions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. In other words: instructions (5-3); displayed material (**5-7**).

Notice that in all four layouts, the first cell of the displayed material is indented two cells to the right of the runover cell of the preceding material. These margins apply regardless of the presence of runover material in the preceding paragraph. A line is not skipped above or below displayed mathematical material unless the preceding or following material requires a blank line. [Refer to **8.26** when the context is not mathematical.]





















8.25.1 Placement of Code Switch Indicators: When displayed mathematical material is both preceded and followed by UEB text, the expression and the two switch indicators may be placed all together on one line if they will fit within current margins. If more than one line is required for the expression, the opening Nemeth Code indicator is placed at the end of the text line preceding the displayed material. The Nemeth Code terminator is placed at the completion of the displayed expression. In either case, if the indicator will not fit on the current line, it is placed on the following line in the runover position.

Math displayed to unitemized text: start in cell 3, runover to cell 5.

Example 8.25-1 (Math is displayed to unitemized text.)

The expression $a(b + c) - d(b + c)$ has the form $ax - dx$ where $x = b + c$. Thus $ax - dx = x(a - d)$ and therefore

$$a(b + c) - d(b + c) = (b + c)(a - d).$$

1                    


















Lines 1-3: Narrative paragraph (3-1) with embedded math.


















***Line 4: Because the entire displayed expression will fit on one line with its opening switch indicator and terminator, the open indicator is the first character in the display cell.*


















Example 8.25-2 (Math is displayed to unitemized text.)


















The behavior described by the following relationship is called **Wien's displacement law**:

$$\lambda_{\max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

1                 

2                 

3                 

4                 

Lines 1-3: Narrative paragraph (3-1).

Line 3: The opening Nemeth Code indicator is placed at the end of the line of text preceding the displayed expression. (Reminder: The switch to Nemeth Code terminates the UEB bold passage.)

***Line 4: Displayed to unitemized text is placed in cell 3. The Nemeth Code terminator follows the end of the math expression.*

Example 8.25-3 (Math is displayed to unitemized text.)

A sequence $a_1, a_2, a_3, \dots, a_n$ is said to *converge* if, for each $h > 0$, there exists

a positive number M such that $|a_n - A| < h$, for all $n > M$.

A sequence that does not converge is said to *diverge*.

1
2
3
4
5
6
7
8

Lines 1-3: Narrative paragraph (3-1) with embedded math.

****Lines 4-6:** The displayed math expression begins in cell 3, with runovers in cell 5.

Lines 7-8: Continuation of narrative paragraph.

Example 8.25-4 (Math is displayed to unitemized text.)

Now we will find the root of the following polynomial equation:

$$x^5 + x^4 - 18x^3 + 4x^2 + 88x - 96 = 0$$

How many roots to you predict we will find?

1
2
3
4
5

Lines 1-2, 5: Narrative paragraph begins (3-1) and continues on line 5 in runover cell (1).

****Lines 3-4:** Displayed math begins in cell 3. Since both switch indicators do not fit on this line, the opening Nemeth Code indicator is on line 2 (in the narrative paragraph), and the Nemeth Code terminator falls in the runover position of the current text – cell 5.

Line 5: Continuation of 3-1 paragraph (cell 1).

Example 8.25-5 (*Math is displayed to itemized text.*)

5. Solve for x if $y = 9$.

6. Explain why the answer to #5 is the same if $y = -9$.

Lines 1-2: Itemized material with no subdivisions (1-3).

***Line 4: The displayed item begins in cell 5 even though the text to which it applies does not have a runover.*

Lines 5-6: Itemized material continues (1-3).

1. Find the area of the triangle whose vertices are located at the coordinates given below.

2. What is the 3 x 3 identity matrix?























Lines 1-2 and 4-5: Itemized material with no subdivisions (1-3).























***Line 3: Cell 5 is the starting cell for math displayed to itemized text.*























Math displayed to subitems: start in cell 7, runover to cell 9.























Example 8.25-7 (Math is displayed to subitems.)























5. Give two examples illustrating
- (a) the associative law for addition
$$(a + b) + c = a + (b + c).$$
 - (b) the associative law for multiplication
$$(a \times b) \times c = a \times (b \times c).$$





















1                      

2                      

3                      

4                      

5                      

6                    











Line 1: Main item begins in cell 1.











Lines 2 and 4-5: Each subitem begins in cell 3. Line 5 shows a runovers in cell 5.











***Lines 3 and 6: Displayed math is in cell 7, which is two cells in from the runover of subitems, whether or not runovers occur.*











Example 8.25-8 (Math is displayed to subitems.)











2. Now solve each of the following equations.
- a. $3(x + 5) = 6x + 6$ b. $x^2 - 25 = 0$
- $x = \underline{\hspace{2cm}}$ $x = \underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$











1          

2          

3          

4          

5          

6          

Lines 1-2: Main item begins in cell 1. The opening Nemeth Code indicator is placed at the end of the line of text preceding the math items. Because there is no room for the switch indicator on line 1, it falls in the runover position of the current text (cell 5).

Lines 3 and 5: Each subitem begins in cell 3, not side-by-side as printed.

***Lines 4 and 6: Displayed math begins in cell 7, which is two cells in from the runover of subitems, whether or not runovers occur. (Subitems are 3-5.)*

Line 6: Nemeth Code is terminated at the end of the displayed material.

Example 8.25-9 (Math is displayed to unitemized text.)

$$27 = 9(4 - 1) \quad 1 + 1 = 3 \quad x + 7 = 50$$
[illegible]

Lines 6-7: Continuation of narrative paragraph, in the runover cell (cell 1).































When punctuation is used to separate displayed items, the items may continue on the same line, running over as necessary in the appropriate runover cell.

Example 8.25-10 (Math is displayed to itemized text.)

21. Use a number line to show the following inequalities:

$$-6 < -5, \quad 0 < +6, \quad -8 < +2, \quad -1 > -5$$

22. ...

1                              

Lines 1-2 and 5: Itemized material with no subdivisions (1-3).

***Lines 3-4: The displayed math begins in cell 5 and continues in the runover cell (cell 7).
(In print, the inequalities are separated by commas and are all printed on one line.)*

When sentence-structure punctuation is included in the displayed math, each expression may begin in the primary display cell if doing so more closely matches the print layout.

Example 8.25-11 (Two math items are displayed to unitemized text.)

The result of nested floor or ceiling functions is the innermost function:

$$\begin{aligned} \left\lceil \left\lfloor x \right\rfloor \right\rceil &= \lceil x \rceil, \\ \left\lfloor \left\lceil x \right\rceil \right\rfloor &= \lfloor x \rfloor. \end{aligned}$$

Lines 1-2: Narrative paragraph (3-1).

***Line 3: Math displayed to unitemized text begins in cell 3.*

****Line 4: New displayed math item also begins in cell 3.**

Example 8.26-1

You can guess what your friends are thinking by learning to "operate" your way into their minds! For example, try this math magic trick.

Think of a number. Multiply the number by 8, divide by 2, add 5, and then subtract 4 times the original number.

No matter what number you choose, the answer will always be 5. Try another number and see. You can use what you know about variables to prove it.

[illegible]

Lines 1, 5, and 9: According to Nemeth Code rules, the first line of each paragraph is indented two cells from the paragraph's left margin.

Lines 4 and 8: According to Braille Formats guidelines, displayed literary material is preceded and followed by a blank line.

Lines 5-7: According to Braille Formats guidelines, cell 3 is the adjusted left margin in narrative.

PRACTICE 8L

Solve this polynomial using basic algebra. *Hint:* First factor out "x" to make it a quadratic equation.

$$x^3 + 2x^2 - x = x(x^2 + 2x - 1)$$

Do you notice a familiar pattern?

● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●

The figure shows a sequence of 10 diagrams, each representing a 5x5 grid with black dots. The dots represent the state of a system at discrete time steps. The pattern starts as a small cluster of dots and grows into a larger, more complex shape over 10 steps.

The figure consists of 10 small diagrams arranged horizontally, each showing a pattern of black dots on a grid. The patterns evolve from left to right, starting with a small cluster of dots and growing into a larger, more complex shape.

The figure shows a sequence of 10 diagrams, each representing a 5x5 grid of dots. The dots are black, and the background is white. The sequence shows a pattern of dots growing from left to right. In each diagram, the dots form a shape that is roughly rectangular, with the width increasing by one unit in each step. The height of the shape is constant at 5 units. The sequence starts with a single dot at (1,1) and ends with a full 5x5 grid of dots.

The figure displays a sequence of 12 diagrams, each showing a pattern of black dots on a grid. The diagrams are arranged in two rows of six. The top row shows the initial pattern and its first five iterations. The bottom row shows the next five iterations. The pattern consists of a central cluster of dots that grows and changes shape over time, with some dots appearing to move or change state.

The figure consists of 10 sub-diagrams, each showing a 10x10 grid of dots. Black dots represent the 'on' state of a binary system. The sequence shows a pattern that starts small and grows in a complex, non-linear fashion, characteristic of a fractal or chaotic system. The growth is not uniform, with some areas expanding more rapidly than others.

PRACTICE 8H

1. $x_{1,2} \neq x^{i,j}$
2. $P_{q_{r,s}}$
3. $x_{n-1,n-1}, x_{n-1,n}, x_{n,n-1}$
4. A^{n+n+n} all n 's are equal.
5. $s]_{t=a}$
6. $e^{1,000}$
7. $a^{m+k} \div a^m = a^k$
8. $P_{s_1 \dots s_2}$
9. $10_{\underline{\hspace{0.5cm}}} = 6_8$
10. a'_1, a'_2, \dots, a'_n are the inverses.

• • • • •

The figure consists of 12 small diagrams arranged horizontally. Each diagram shows a set of black dots on a white background, representing a pattern. The patterns are as follows:

- Diagram 1: A single dot.
- Diagram 2: Two dots arranged vertically.
- Diagram 3: Three dots arranged in a horizontal line.
- Diagram 4: Four dots arranged in a 2x2 square.
- Diagram 5: Five dots arranged in a cross shape.
- Diagram 6: Six dots arranged in a hexagonal pattern.
- Diagram 7: Seven dots arranged in a more complex, irregular shape.
- Diagram 8: Eight dots arranged in a larger, more complex shape.
- Diagram 9: Nine dots arranged in a complex shape.
- Diagram 10: Ten dots arranged in a complex shape.
- Diagram 11: Eleven dots arranged in a complex shape.
- Diagram 12: Twelve dots arranged in a complex shape.

