

➤ $144 \text{ ft} + 144 \text{ ft}$



A space precedes and follows the abbreviation "ft". The operation sign is unspaced from the number which follows.

➤ $144 \text{ ft}^2 + 144 \text{ ft}^2$



A space precedes the abbreviation "ft". The superscript is unspaced from the abbreviation. There is no space before the baseline indicator.

PRACTICE 8C

- (1) $4(x - y)^3 - 2(x - y)^3$
- (2) $3a^3b + 6a^6b^2 + 9a^9b^3$
- (3) 98.6° F
- (4) $V = 2\pi^2 Rr^2$
- (5) x^n -dimensional system
- (6) $6^2 \times 6^3 = 6^{2+3} = 6^5$
- (7) $144 \text{ ft}^2 + 144 \text{ ft}^2 = \dots$
- (8) Given that $(x + 2)$ and $(x - 3)$ are factors of $f(x)$, show that $a = 3$ and $b = -10$.

8.6 Higher Levels of Writing: Superscripts may carry superscripts of their own. In such cases, the superscript indicator is doubled, tripled, etc. to indicate superscripts on the second, third, or higher levels of writing.

Superscript with Superscript		<i>(two levels above the baseline)</i>
Superscript with Superscript with Superscript		<i>(three levels above the baseline)</i>

➤ n^{a^b}

➤ $n^{a+1^{b+1}^{c+1}}$

PRACTICE 8G

- 1) These expressions need subscript indicators in braille: y_{-2} , x_{2+k} , a_{m1} , x_{3n} , x_{y_2} .
 - 2) These expressions do not use a subscript indicator in braille: x_1 , ax_2 , CO_2 , $z_{.7}$, β_2 .
 - 3) Decide whether these expressions require a subscript indicator and braille them correctly: shape_4 , Q'_2 , $C_6Fe_2O_{12}$, n_k , x_{2k} , $P_{r_{st}}$, D_{56} , $G_{9,999}$, $2E6B_{16}$.
 - 4) Use $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ to denote the direction vectors v_1 and v_2 .
 - 5) *Chemistry*: While Na_2ZnCl_4 could be cooled in the normal way, $Na_2[CoCl_4]$ had to be quenched in the liquid N_2 .
 - 6) $f_1(x) = g(x) \cdot q_2(x) + f_2(x)$
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8.11 Spaces Within Superscripts and Subscripts: A space usually returns the reader to the baseline. Various strategies are used to retain the level in effect when a space occurs within a superscript or a subscript.

8.11.1 Commas: As noted above, the effect of a level indicator is terminated by a comma unless the comma occurs in a long numeral (a "segmenting" comma). *Recall:* A segmenting comma maintains the level in effect,

$\gg x_{1,000}$


The numeral 1,000 is a right subscript to the letter x.

but a comma followed by a space re-establishes the baseline. The return to the baseline starts at the comma. No baseline indicator is needed.

$\gg x^2, y^2, z^2$


$\gg (a_1, a_{1i}, a_{2i})$


Example 8.11-1 Add: 2_{five} , 3_{five} , and 4_{five} .



The level extends through the required spaces before and after an ellipsis or a long dash.

$$\gg x^{1+3+5+\dots+(2n-1)}$$


The superscript is 1 + 3 + 5 + ... + (2n - 1). The superscript level extends through the spaces before and after the ellipsis.

Exception: If an ellipsis or long dash in a superscript or subscript is followed by a sign of comparison or by literary text, reading returns to the baseline level without the need for a baseline indicator.

$$\gg 10^{7+—} = 10^{21}$$


The first superscript is 7+—. The superscript level extends through the required space before the long dash. The sign of comparison after the dash is assumed to be at the baseline level.

$$\gg 10^{7+—} \text{ equals } 10^{21}$$


The first superscript is 7+—. The superscript level extends through the required space before the long dash. The word after the dash is assumed to be at the baseline level.

8.11.5 Ellipsis or Long Dash on the Baseline of Writing: Because the space before an ellipsis or long dash maintains the level in effect, an indicator is required to return to the baseline when the symbol is printed there. The baseline indicator takes the place of the required space.

$$\gg a^1 b^2 c^3 d^4 \dots z^n$$


There is no need to indicate a return to the baseline after a numeric subscript that does not require a subscript indicator.

$$\gg r_1 \dots r_n$$


The ellipsis is printed on the baseline. The subscripts are 1 and n.

8.11.6 Segmented Numbers: The effect of a level indicator extends through the space inserted in a numeral for the purpose of dividing it into short regular groups of digits.

$$\gg e^{3.14159 26535}$$


The superscript is 3.14159 26535

Instructions: Translate the simulated braille by writing the mathematical characters in the blank space below each line. Show spaces and levels clearly. Compare your translation to the print copy in the answer section.

PRACTICE 8H

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PRACTICE 8J

1. Here are some expressions with left superscripts: 3x , nx , ${}^{-2} + {}^{-4} = {}^{-6}$, $(-3)^{-2+{}^{+2}}$.
 2. ${}^{12}_6\text{C}$ and ${}^{12}\text{C}$ represent the same carbon isotope.
 3. $\text{D}_2{}^{18}\text{O}$ is the doubly labeled water isotopologue!
 4. In CO_2 , the subscript ${}_2$ means "two oxygen atoms".
 5. ${}_nP_r = K({}_{n-1}P_{r-1})$
 6. $a_1^2 + b_1^2 + c_1^2$
 7. $[t]_0^4$
 8. $2 \times 10_6^2 + 3 \times 10_6^1 + 2$
 9. $P_{xy}Q$
 10. $\text{NH}_4^+ + \text{Cl}^- + \text{H}_2\text{O}$
 11. *Temperature Conversion:* What is 0 K in °C? in °F? (Answers: -273.15°C ; -459.67°F .)
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GROUPING SIGNS, cont.

8.20 Review of Rules: The following rules have been discussed in earlier lessons. A review of the cited sections is recommended.

- In mathematical context, enclosure symbols are not considered to be punctuation; they are signs of grouping. Grouping symbols that are part of a mathematical expression must be brailled as Nemeth braille symbols. [3.9]
- The numeric indicator is not used before a numeral which immediately follows an opening sign of grouping unless such numeral is in non-regular type. [3.10, 7.7]
- The English letter indicator is not used when a single English letter or a Roman numeral is entirely enclosed within signs of grouping unless such a letter, or Roman numeral is in non-regular type. This rule does not apply to abbreviations. [4.12.2, 5.3.5]
- Single-letter abbreviations without a related period, or abbreviations whose letters correspond to a shortform without a related period, require an English letter indicator even when enclosed between signs of grouping. [5.19, 5.20]
- When a single English letter, or a Roman numeral is in direct contact with only its opening or only its closing sign of grouping, the English letter indicator is or is not used as though the grouping signs were absent. If the grouping sign carries a prime or other modifying symbol, the English letter indicator is not used. [5.16]

- The appropriate alphabetic indicator must be used with any letter from the German, Greek, Hebrew, or Russian alphabets even when enclosed within, or in contact with, signs of grouping. [5.8.1]
- In mathematical context, non-mathematical symbols of enclosure can be transcribed as Nemeth Code signs of grouping if switching out of Nemeth Code would be cumbersome. [3.13]
- A sequence of mathematical items enclosed in signs of grouping (an "enclosed list") must use the Nemeth braille enclosure symbols. [5.15]
- When square brackets or vertical bars are printed in mathematically-significant boldface, dots 456 are used before the grouping symbol. [7.13.3]

Regarding the main topic in this lesson—superscripts and subscripts—another point can be added.

- When a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed before the sign of grouping.

$$\gg (R_H T_H) + (R_S T_S)$$



If no subscript indicator is used, a return to the baseline is implied.

$$\gg (a_1, a_2, a_3)$$



8.21 Grouping Symbols and Level Indicators: Be watchful when determining the level of a grouping sign. Because the bottom margin of grouping signs may extend below the level in effect, it is better to compare the center of the symbol to the surrounding text if its printed level is in question.

Example 8.21-1 Solve: $(x^2 + y^2) - (x^2 + y^2)$



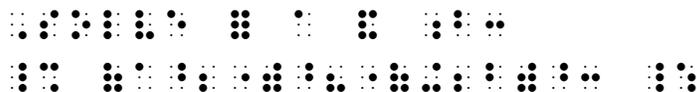
If the grouping symbol starts a different level, the appropriate level indicator is placed before the sign of grouping to properly identify its location.

Example 8.21-2 In this case, $x_{(a,b)} + y_a$.



The opening grouping symbol begins the subscript level.

Example 8.21-3 Solve for a and b : $(a^2)^8 (+2b)^3$

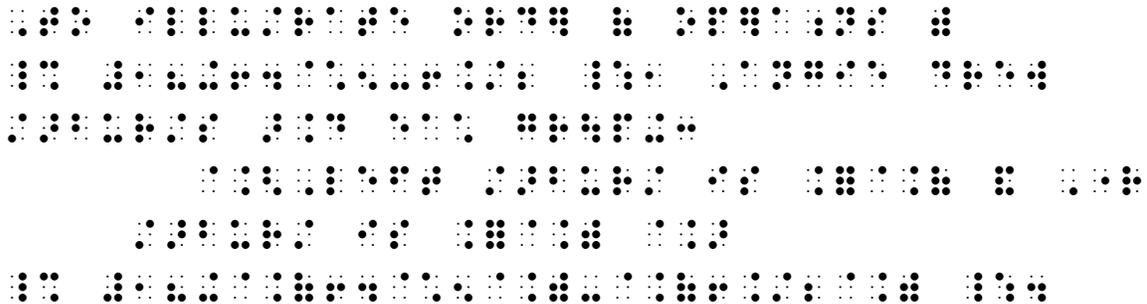


All of these grouping symbols are on the baseline of writing.

Review the examples in **8.6.1** with this in mind.

8.24 Transcriber-Devised Grouping Symbols: If a mathematical sign of grouping is not represented in the Nemeth Code, the transcriber is to devise one. Use two or more braille symbols whose last cell is ⠠ for the opening sign and whose last cell is ⠡ for the closing sign. The transcriber-devised symbol should be identified in a transcriber's note or listed on the Special Symbols page at the beginning of the volume according to the guidelines given in *Braille Formats*.

Example 8.24-1 To illustrate order of operations with $18 + 64 \times 5 - 6 \div 2$, Angie drew starbursts around each grouping: $18 + \textcircled{64 \times 5} - \textcircled{6 \div 2}$



Instructions: Treat the vertical bar in sentence (7) as an operation sign.

PRACTICE 8K

- (1) $\{f_n\}$
- (2) $|a_m - a_n|$
- (3) $(x_1y_1 + x_2y_2)$
- (4) $([\text{CH}_3]_2\text{CH})$
- (5) $I_{ue}^{2''} = (H'_{44}x'_{ve})^{+'}$
- (6) $[x] = m$ if and only if $m \leq x < m + 1$; $[x] = n$ if and only if $n - 1 < x \leq n$.
- (7) The dagger and the asterisk are used as superscripts in quantum mechanics: A^\dagger , $(x^\dagger)^\dagger = x$, $\langle \phi | \psi \rangle^* = \langle \psi | \phi \rangle$.

INTRODUCTION TO DISPLAYED MATHEMATICAL EXPRESSIONS

FORMAT

8.25 Displayed Mathematical Expressions: Up to this point in the lesson material, mathematical expressions in the examples have appeared in line with the narrative. These are referred to as *embedded expressions*. When mathematical material is set apart from the body of the text in the print copy, it is referred to as a *displayed expression*. Various layouts in the print copy are used to set the material apart, for example, skipped lines, centering or other indentation, or off to the side. In braille, margins for displayed mathematical material depend upon the margins of the surrounding text and are transcribed in one of the following formats.

- *In unitemized explanatory portions of the text*, displayed mathematical material begins in cell 3. Runovers begin in cell 5. In other words: text (3-1); displayed material (**3-5**).
- *In itemized text without subdivisions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. In other words: text (1-3); displayed material (**5-7**).
- *In itemized text with subdivisions*, displayed mathematical material begins in cell 7. Runovers begin in cell 9. These margins apply to both items and subdivisions, to whatever depth. In other words: main division text (1-5); displayed material (**7-9**). Subdivision text (3-5); displayed material (**7-9**).
- *Within or following instructions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. In other words: instructions (5-3); displayed material (**5-7**).

Notice that in all four layouts, the first cell of the displayed material is indented two cells to the right of the runover cell of the preceding material. These margins apply regardless of the presence of runover material in the preceding paragraph. A line is not skipped above or below displayed mathematical material unless the preceding or following material requires a blank line. [Refer to **8.26** when the context is not mathematical.]

8.25.1 Placement of Code Switch Indicators: When displayed mathematical material is both preceded and followed by UEB text, the expression and the two switch indicators may be placed all together on one line if they will fit within current margins. If more than one line is required for the expression, the opening Nemeth Code indicator is placed at the end of the text line preceding the displayed material. The Nemeth Code terminator is placed at the completion of the displayed expression. In either case, if the indicator will not fit on the current line, it is placed on the following line in the runover position.

8.25.2 Look For Context Clues: Each separate expression should start on a new line in the main display cell, even if printed widely-spaced on one line. Notice paragraph patterns in the print copy to determine whether the narrative following displayed material is a continuation of the preceding text or if it is the start of a new paragraph.

Example 8.25-9 (Math is displayed to unitemized text.)

A sentence with an equal sign is called an **equation**. Here are three examples.

$$27 = 9(4 - 1) \quad 1 + 1 = 3 \quad x + 7 = 50$$

Which equation is true? Which equation is false? Which equation may be either true or false?

1
2
3
4
5
6
7

Lines 1-2: Narrative (3-1).

***Lines 3-5: Three separate displayed expressions, each in cell 3. (In print, the three equations are on one line with extra spacing between them.)*

Lines 6-7: Continuation of narrative paragraph, in the runover cell (cell 1).

