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INCORPORATING ALL ERRATA/ADDENDA POSTED SINCE 2/15/2014**

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*—The print examples may be clearer in your original book.*

*—The HOMEWORK exercise is not reproduced here. Use the original  
but note the changes mentioned in this document.*

Print examples that require corrections are highlighted in yellow; the corrections are explained.  
All simbraille examples are included herein; those that have been corrected are identified as such.  
The original page numbers are shown in gray.

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## AN INTRODUCTION TO BRAILLE MATHEMATICS

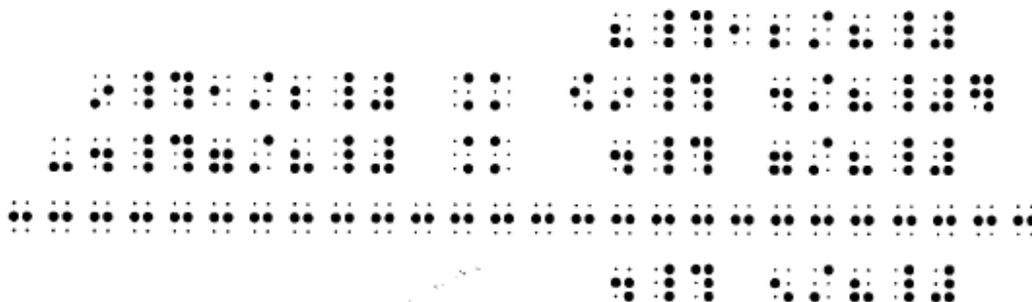
by  
HELEN ROBERTS  
BERNARD M. KREBS  
BARBARA TAFFET

Based on  
the Nemeth Braille Code for Mathematics  
and Science Notation, 1972

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1978



$$(3) \quad \begin{array}{r} 9\frac{1}{2} = 9\frac{4}{8} \\ -4\frac{7}{8} = 4\frac{7}{8} \\ \hline 4\frac{5}{8} \end{array}$$



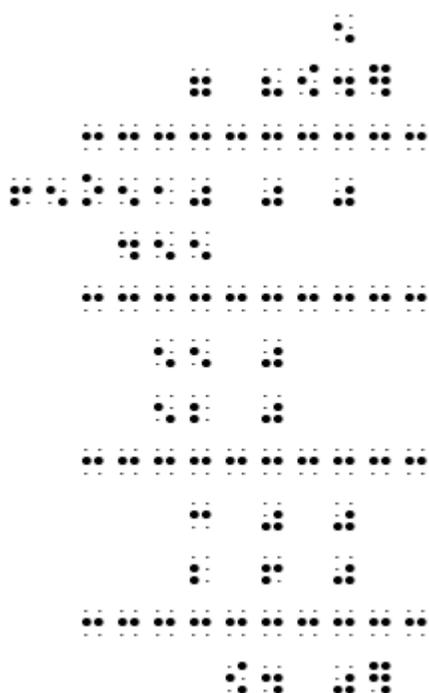
**§175. Cancellation in Long Division:** If cancellation is shown in long division, the canceled material must be enclosed in cancellation indicators, and blank cells must be left, where necessary, for proper alignment.

$$(1) \quad \begin{array}{r} 784 \\ 65 \overline{) 5100} \\ \underline{455} \\ 550 \\ \underline{520} \\ 300 \\ \underline{260} \\ 40 \end{array}$$

Example (1): Correct the following in the print: The dividend should have an additional zero 51000 and the quotient should be arranged so the numerals 7 8 and 4 are directly above the three zeros in the dividend.

$$\begin{array}{r} 784 \\ \hline 51000 \end{array}$$

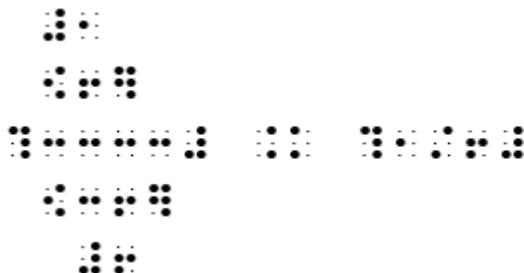
Corrected simbraille is shown below:



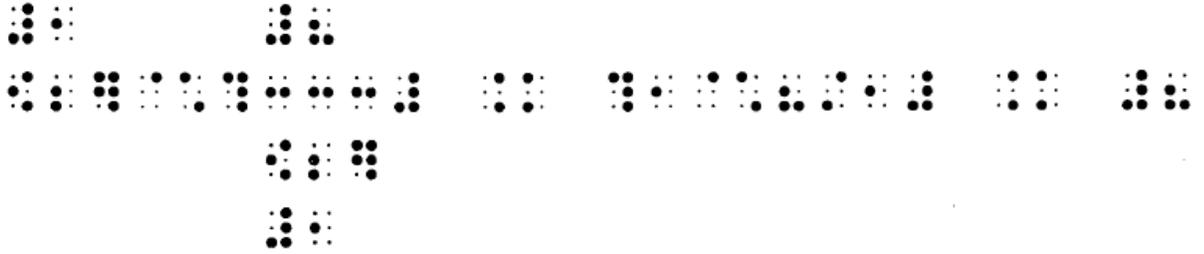
**§176. Cancellation with Fractions:** Where cancellation is shown *within* fractions, a spatial arrangement must be used. In a spatial fraction arrangement, the fraction line (dots 2-5) should be as long as the longest line of braille above or below it. The opening and closing fraction indicators should be placed at the ends of the fraction line. Items canceled individually in print should be enclosed in separate pairs of cancellation indicators. Since fractions do not contain material aligned for computation, the numeric indicator and English letter indicator must be used, where necessary. Fractions without canceled items should be transcribed linearly. Terms of the fraction must be centered above or below their fraction lines. The result of cancellation must be centered with respect to the canceled term. An identifying numeral or letter, if present, is placed on the line with the fraction line.

$$(1) \frac{1}{\cancel{36}} = \frac{1}{6}$$

Corrected simbraille is shown below:

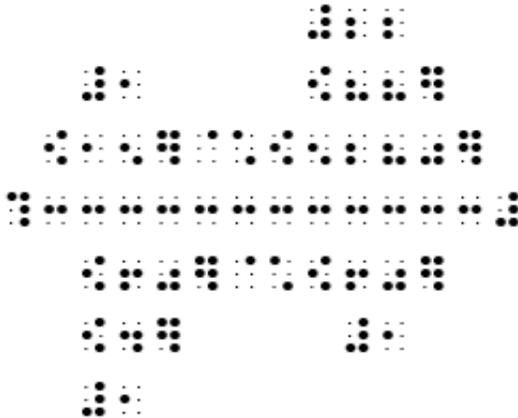


$$(2) \frac{1}{2} \times \frac{8}{2} = \frac{1 \times 8}{1} = 8$$

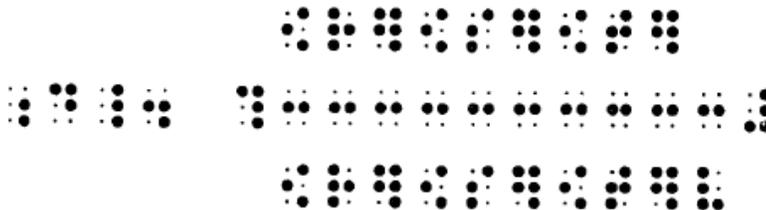


$$(3) \frac{15 \times 5280}{60 \times 60} = 1$$

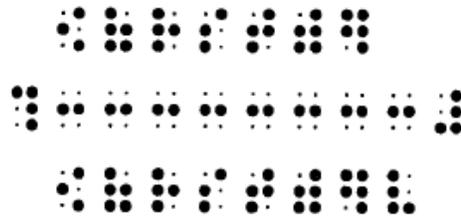
Corrected simbraille is shown below:



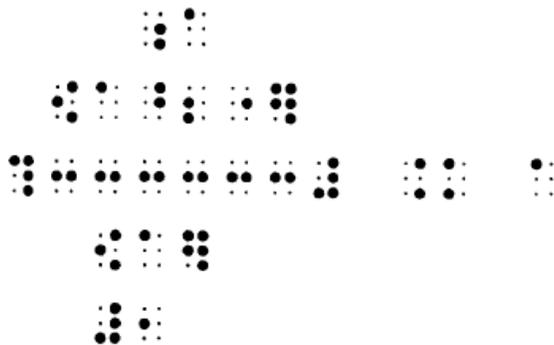
$$(4) d. \frac{xy}{yz}$$



$$(5) \frac{(pst)}{(pst)v}$$

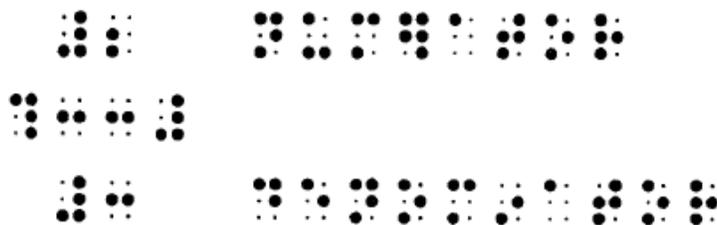


$$(6) \frac{a}{1} = a$$

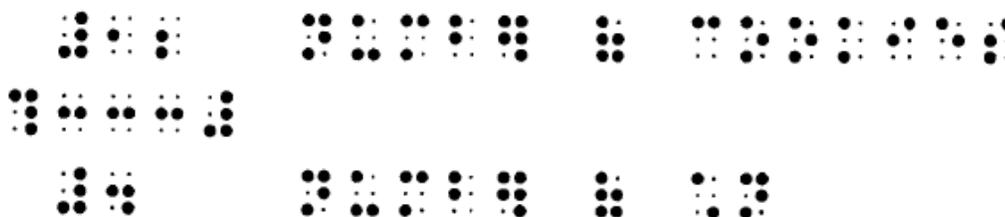


**§177. Simple Fractions Arranged Spatially for Illustration:** It is often helpful to use a spatial arrangement if the parts of a simple fraction are explained or identified. The numeric and English letter indicator must be used, where necessary, within the arrangement.

$$(1) \frac{2 \text{ numerator}}{3 \text{ denominator}}$$



(2)  $\frac{12}{4}$  number of cookies  
number of children



§178. Hypercomplex Fractions:

Hypercomplex Fraction Indicators

Opening      ⠠⠠⠠

Closing      ⠡⠡⠡

Horizontal Hypercomplex Fraction Line      ⠠⠠⠠⠡

a. A hypercomplex fraction is one whose numerator or denominator, or both, contain at least one complex fraction. A fraction is not a hypercomplex fraction if the only complex fractions it contains are at the superscript or subscript level.

(1)  $\frac{1}{\frac{\frac{a}{b}}{2^d}}$

(this is not a hypercomplex fraction)

(2)  $\frac{\frac{1}{2}}{\frac{\frac{3}{4}}{5}}$

(this is a hypercomplex fraction)

b. Hypercomplex fraction indicators must be used with hypercomplex fractions. Although it is permissible to use an entirely spatial arrangement or an entirely linear arrangement, it is preferable to use a linear arrangement within a spatial arrangement.

$$(1) \frac{\frac{1}{\frac{mn}{m+n} (\bar{x}_1 - \bar{x}_2)^2}}{\sum (x_{1i} - \bar{x}_1)^2 + \sum (x_{2j} - \bar{x}_2)^2}}$$

Corrected simbraille is shown below:

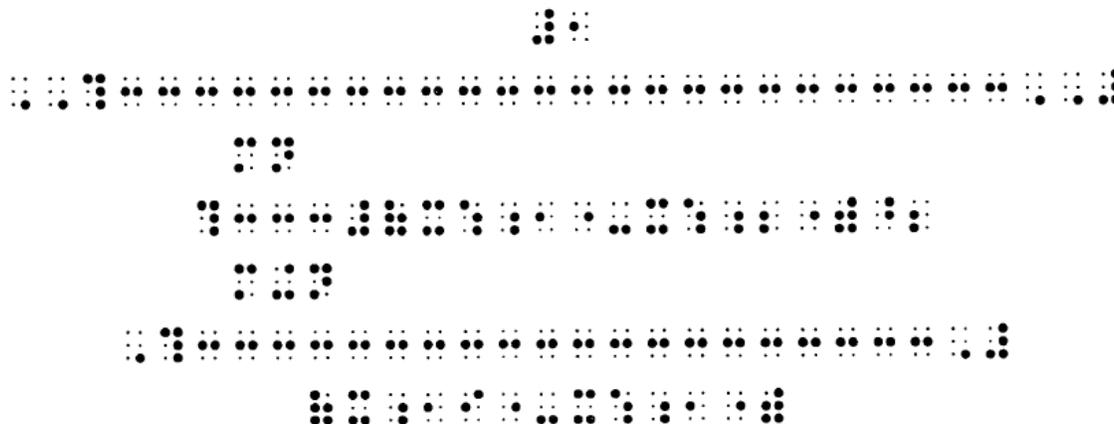
The Braille representation of equation (1) consists of four lines of Braille cells. The first line contains the number 1. The second line contains the fraction  $\frac{mn}{m+n} (\bar{x}_1 - \bar{x}_2)^2$ . The third line contains the denominator  $\sum (x_{1i} - \bar{x}_1)^2 + \sum (x_{2j} - \bar{x}_2)^2$ . The fourth line contains the division bar.

$$(2) \frac{\frac{1}{\frac{mn}{m+n} (\bar{x}_1 - \bar{x}_2)^2}}{\sum (x_{1i} - \bar{x}_1)^2 + \sum (x_{2j} - \bar{x}_2)^2}}$$

Corrected simbraille is shown below:

The Braille representation of equation (2) consists of four lines of Braille cells. The first line contains the number 1. The second line contains the fraction  $\frac{mn}{m+n} (\bar{x}_1 - \bar{x}_2)^2$ . The third line contains the denominator  $\sum (x_{1i} - \bar{x}_1)^2 + \sum (x_{2j} - \bar{x}_2)^2$ . The fourth line contains the division bar.

$$(3) \frac{1}{\frac{\frac{mn}{m+n} (\bar{x}_1 - \bar{x}_2)^2}{(x_1 - \bar{x}_1)}}$$



(complete spatial arrangement)

Note: Hypercomplex fractions of higher order may be transcribed in the manner described above. Dot 6 should be added the appropriate number of times before the fraction indicators and the matching fraction lines.

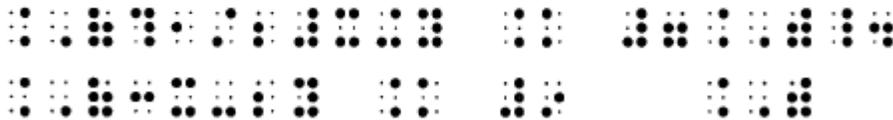
**§179. Continued Fractions:** A continued fraction is one in which each denominator is the sum of a whole number and a fraction. Such a fraction must be transcribed entirely in a spatial arrangement. Each fraction line should be proportionately the same length shown in print. No fraction indicators may be used within a continued fraction. Each numerator must be centered with respect to the fraction line below it.

$$(1) \quad n = 1 - \frac{7}{3 + \frac{3}{2 + \frac{2}{2 - \frac{2}{3}}}}$$

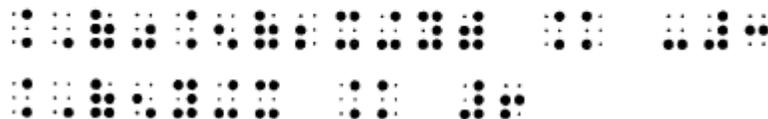




$$(2) \begin{cases} \frac{1}{2}x + y = 7 \\ 3x - 2y = 9 \end{cases}$$



$$(3) \begin{cases} 0.5(2x + y) = -3 \\ 5y + x = 6 \end{cases}$$

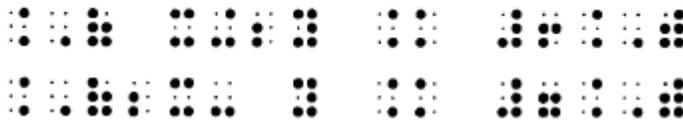


$$(4) \begin{cases} \frac{x}{3} - \frac{y}{2} = 2 \\ 5x + 3y = 51 \end{cases} = (?, ?)$$



b. In a unified system of equations, the numeric indicator must be used or must not be used according to the rules of the code.

$$(1) \begin{cases} x + 2y = 6 \\ 2x - y = 7 \end{cases}$$



$$(2) \begin{cases} x + 0y = 4 \\ 0x + y = 1 \end{cases}$$

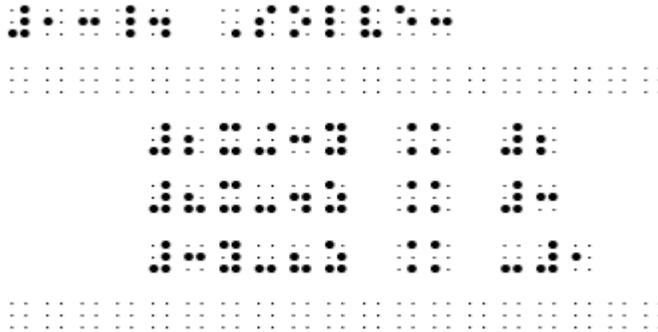


c. Ununified Systems of Equations: When a system of equations is arranged on two or more lines and is not joined by any sign of grouping, the arrangement is considered to be spatial and blank lines must be left above and below it.

(1) 13. Solve:

$$\begin{aligned}2x + 3y &= 2 \\ 8x - 4z &= 3 \\ 3y - 8z &= -1\end{aligned}$$

Corrected simbraille is shown below:

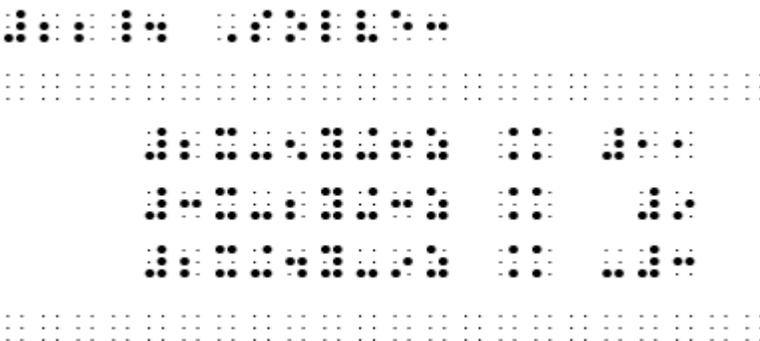


The image shows the corrected simbraille for the system of equations in problem 13. It consists of three lines of equations, each with a coefficient, a variable, an operator, and a constant. The equations are: 2x + 3y = 2, 8x - 4z = 3, and 3y - 8z = -1. The simbraille uses standard Braille characters for numbers, letters, and operators, with appropriate spacing and line structure.

(2) 22. Solve:

$$\begin{aligned}2x - 5y + 6z &= 11 \\ 3x - 2y + 3z &= 9 \\ 2x + 4y - 9z &= -3\end{aligned}$$

Corrected simbraille is shown below:



The image shows the corrected simbraille for the system of equations in problem 22. It consists of three lines of equations, each with a coefficient, a variable, an operator, and a constant. The equations are: 2x - 5y + 6z = 11, 3x - 2y + 3z = 9, and 2x + 4y - 9z = -3. The simbraille uses standard Braille characters for numbers, letters, and operators, with appropriate spacing and line structure.



**§183. Determinants and Matrices:**

**a.** Determinants and matrices, including those with only one row, are spatial arrangements. Thus, a line must be left above and below each determinant or matrix. In a determinant or matrix that appears on more than one line of braille, at least one enlarged grouping symbol must appear on each braille line. However, when a determinant or matrix appears on only one line in braille, a regular, not enlarged grouping symbol is used.

**b.** Each entry must be moved as far up as possible in the row to which it applies. Each entry must be moved as far left as possible in its column, and one column of blank cells must be left between the columns of the arrangement. Centering and other forms of alignment are not permitted.

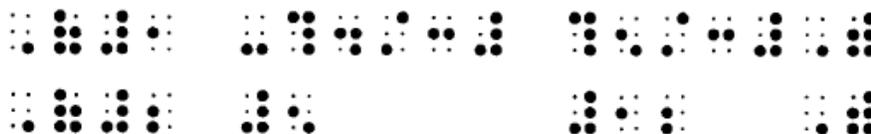
**c.** Each opening sign of grouping must be in direct contact with an entry in the determinant or matrix. In the same way, at least one closing sign of grouping must appear in direct contact with an entry in the determinant or matrix.

**d.** The numeric indicator must be used with numeric entries in a determinant or matrix, even when such entries are in direct contact with an opening grouping sign.

(1)  $\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

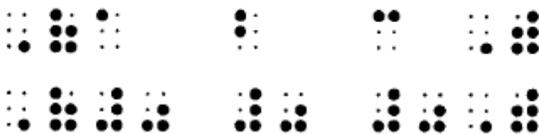


(2)  $\begin{pmatrix} 1 & -\frac{4}{3} & \frac{5}{3} \\ 2 & 5 & 12 \end{pmatrix}$

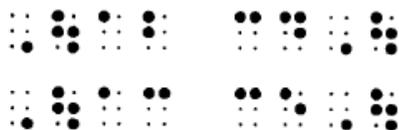


**e.** The English letter indicator must not be used with any letter or combination of letters in a determinant or matrix.

(1)  $\begin{pmatrix} a & b & c \\ 0 & 0 & 0 \end{pmatrix}$

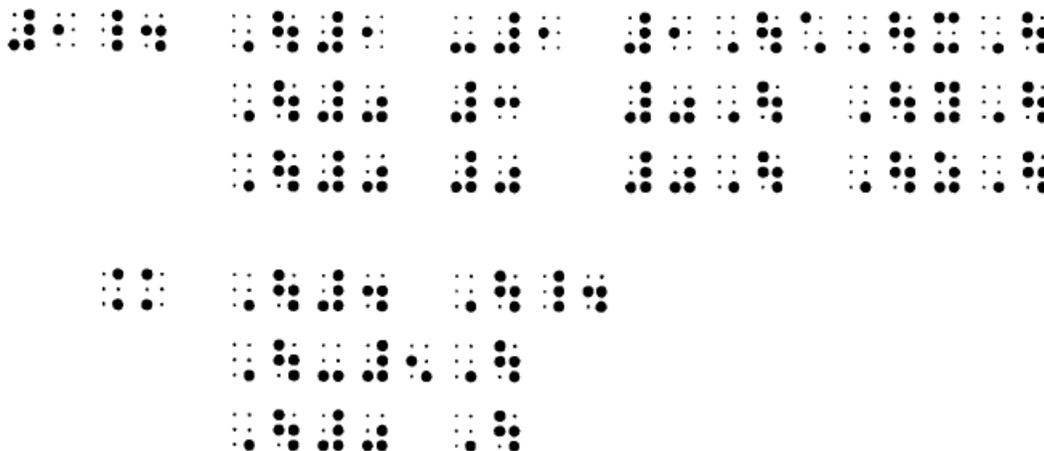


$$(2) \begin{vmatrix} ab & cd \\ ac & ce \end{vmatrix}$$



f. Material outside the determinant or matrix, such as identifiers, punctuation, signs of operation, or signs of comparison, should be placed on the top line of the arrangement, even though it is centered in print.

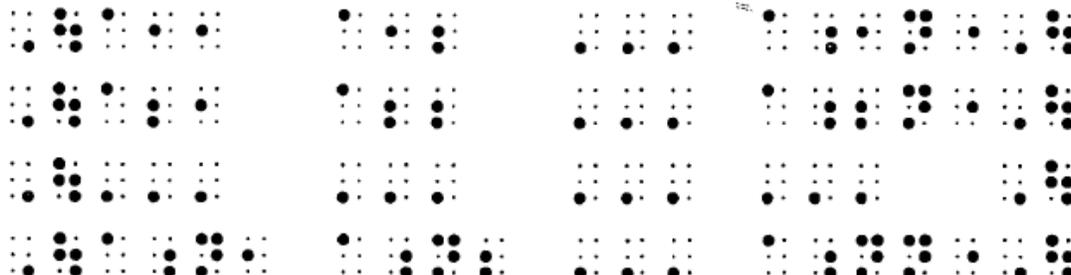
$$(1) \quad 1. \begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 4 \\ -5 \\ 0 \end{vmatrix} .$$



g. When dots are shown in a determinant or matrix to indicate omission of one or more rows, a series of dots 3 is used in braille.

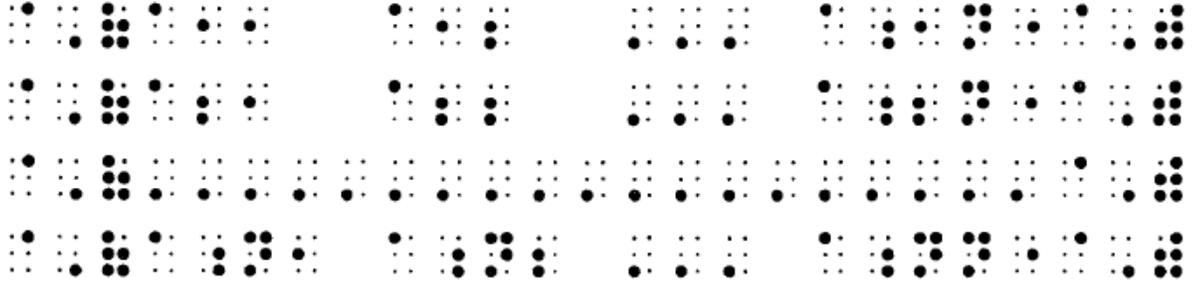
i. If at least one dot appears in each column and no dots appear between columns, an ellipsis (...) is placed as far left as possible in each column.

$$(1) \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \dots & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$



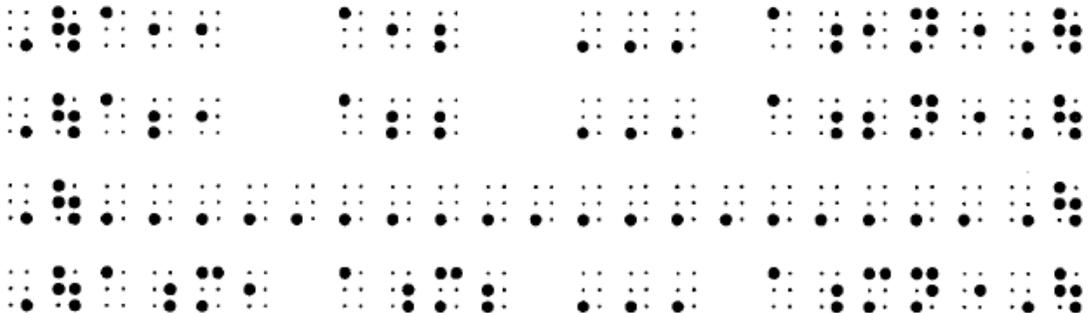
ii. When dots appear completely across a row and occupy space between the columns as well as in the columns, a sequence of dots 3 must be used, beginning in the first cell of the first column and extending to the end of the longest entry in the last column.

$$(1) \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$



iii. If some of the columns contain no dots, a line of dots 3 must be used as in ii above.

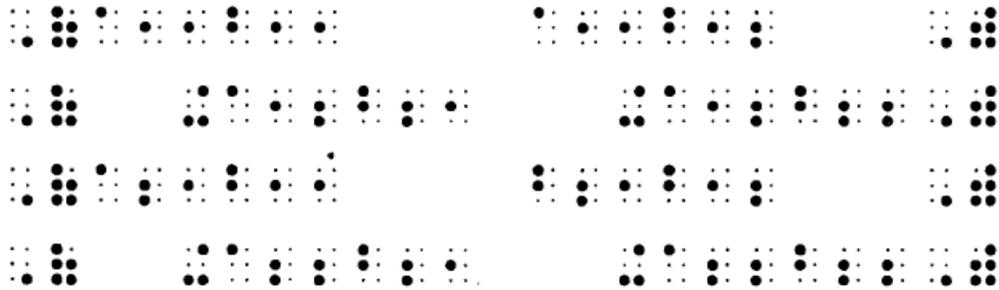
$$(1) \begin{array}{cccc} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{array}$$



**h.** Sometimes space-saving techniques must be adopted to confine the arrangement to one braille page.

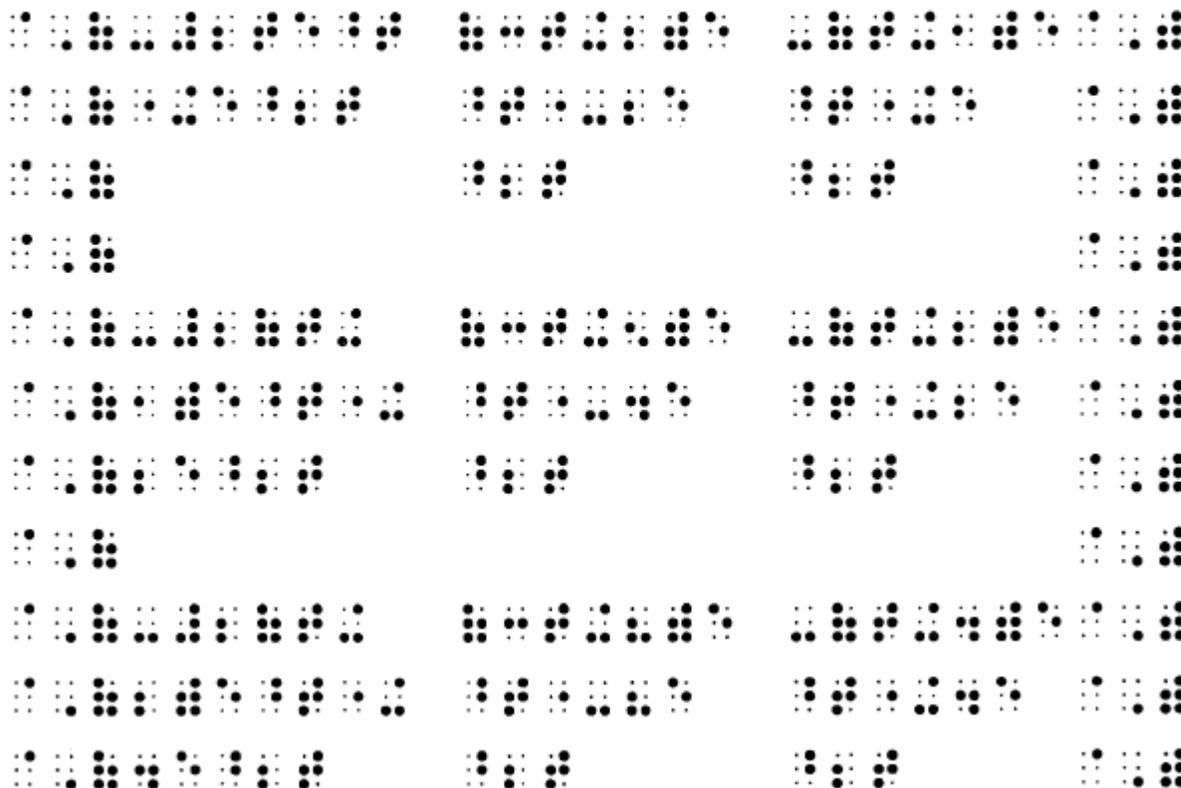
**i.** Entries may be run over to new lines. Each runover should be indented two cells from the first cell of the first line of the entry. No lines should be skipped between entries. If necessary, rules for preference in runovers can be ignored. No baseline indicator should be used before a closing grouping symbol unless it is the end of the item and touches the grouping symbol. This is the preferred space-saving technique.

$$(1) \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & b_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$



ii. Entries may be run over to new lines with no indentations. In this case, a line should be skipped between each row in the arrangement. Preference rules for runovers need not be observed if space would be saved.

$$(1) \begin{bmatrix} -2te^t + e^{2t} & (3t+2)e^t - 2e^{2t} & -(t+1)e^t + e^{2t} \\ -2(t+1)e^t + 2e^{2t} & (3t+5)e^t - 4e^{2t} & -(t+2)e^t + 2e^{2t} \\ -2(t+2)e^t + 4e^{2t} & (3t+8)e^t - 8e^{2t} & -(t+4)e^t + 4e^{2t} \end{bmatrix}$$



iii. Grouping symbols may be drawn in place of the braille equivalents.

iv. Fractions may be shown spatially. In this case, lines should be skipped above and below the rows containing the fraction.

Example 1: In the print example, the first entry in the last row, there should be no space between the "3" and the "e."

$$3e^{2x}$$

$$(1) \left| \begin{array}{cccc} \frac{3}{8}e^{2x} & - \frac{3}{8} & + \frac{1}{4} & - \frac{3}{4}x^2 \\ - \frac{3}{8}e^{2x} & + \frac{3}{8} & + \frac{3}{4} & + \frac{3}{4}x^2 \\ \mathbf{3e^{2x}} & - \frac{3}{8} & - \frac{3}{4} & + \frac{3}{4}x^2 \end{array} \right|$$

In the braille example, the first entry in the last row  $\dots$  should be placed on the first braille line of its row, as shown below:

v. When no other method saves the required space, the technique of keying, discussed in §186, should be used.

## TABLES

§184. **Tables:** In transcribing tables, the rules provided in the most recent revision of *Braille Formats: Principles of Print to Braille Transcription* should be followed.

**a. Letters in Tables:** When letters appear in tables, whether as entries or headings, the English letter indicator must be used or must not be used as though the letters were not part of the table.

**b. Numbers in Table Headings:** When numerals appear in table headings, the rules for the use and nonuse of the numeric indicator must be followed.

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**c. Numbers in Table Entries:** The numeric indicator must not be used when the entries in a table consist entirely of numerals, including interior commas and decimal points. If the entries in a table contain words, letters, signs of operation, signs of comparison, or any other mathematical signs, the numeric indicator must be used throughout the table. If guide dots are needed within any column, the numeric indicator is required for all numeric entries.

(1)

e	e <sup>2</sup>	S
1	1	6
2	4	24
3	9	54
4	16	96

Corrected simbraille is shown below:

The Braille representation shows the table with the following structure:

- Row 1: English letter indicator followed by 'e', English letter indicator followed by 'e<sup>2</sup>', and English letter indicator followed by 'S'.
- Row 2: Numeric indicator followed by '1', numeric indicator followed by '1', and numeric indicator followed by '6'.
- Row 3: Numeric indicator followed by '2', numeric indicator followed by '4', and numeric indicator followed by '24'.
- Row 4: Numeric indicator followed by '3', numeric indicator followed by '9', and numeric indicator followed by '54'.
- Row 5: Numeric indicator followed by '4', numeric indicator followed by '16', and numeric indicator followed by '96'.

(2)	R	T	D
	30	t+2	30(t+2)
	45	t	45t

Corrected simbraille is shown below:

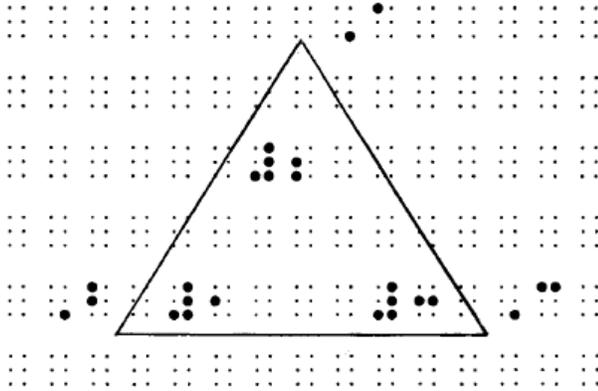
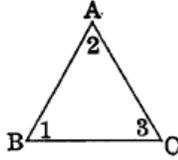
⠠⠠⠠	⠠⠠⠠	⠠⠠⠠
⠠⠠⠠	⠠⠠⠠	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
⠠⠠⠠	⠠⠠⠠	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
⠠⠠⠠	⠠⠠	⠠⠠⠠⠠⠠

## LABELS FOR FIGURES AND DIAGRAMS

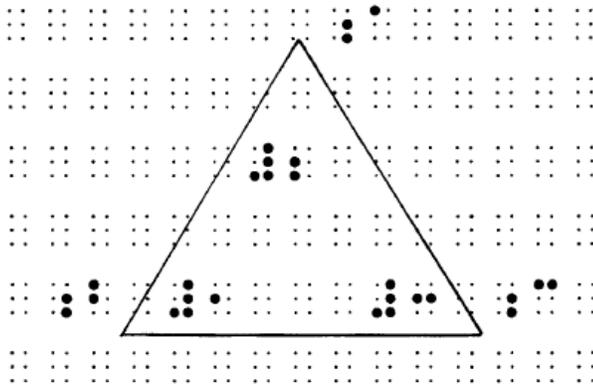
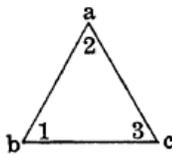
**§185. Labels for Figures and Diagrams:** In labeling diagrams, the numeric indicator must be used with numerals except for the coordinate labels in number lines. When a single English letter in regular type is used as a label in a diagram, the English letter indicator is required if the letter is in lower case, but omitted if the letter is capitalized.

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(1)



(2)



## KEYING

Follow the most recent revision of *Braille Formats: Principles of Print to Braille Transcription*, and the BANA 2011 Nemeth Code Updates (Replacement Section 187 of the Code) for keying rules, the text of which is reproduced below as §186 a.-f. of this lesson manual.

**§186. Keying:** A numeric or alphabetic key may be substituted where there is not enough space for determinants, matrices, column headings, table entries, figure labels, etc.

**a.** When space does not permit the inclusion of labels, headings, entries, etc., in a figure, determinant, matrix, or table as shown in ink print, one or more of the labels, headings, entries, etc., may be replaced by a numeric or alphabetic key. Two items which are identical should have the same key assigned to them.

**b.** A numeric key should consist of one or more numerals brailled in the upper part of the braille cell. This number must be preceded by the numeric indicator and must not be punctuated. Numbered keys should be listed in numeric order. These numbers should be placed in the figure, determinant, matrix, or table in the same position as the material which they replace.

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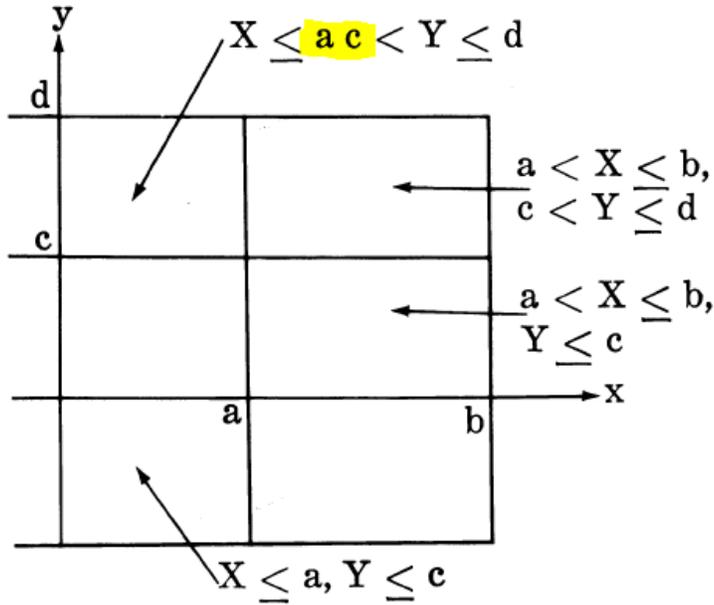
**c.** An alphabetic key must consist of two lower-case English letters, one of which contains a dot 3 or dot 6, and, if possible, the combination should be suggestive of the item it represents. Contractions and letter combinations that correspond to short form words may not be used. An alphabetic key may not be used if any of the print entries in the table are made up of two lower case letters. In that case, a numeric key must be used.



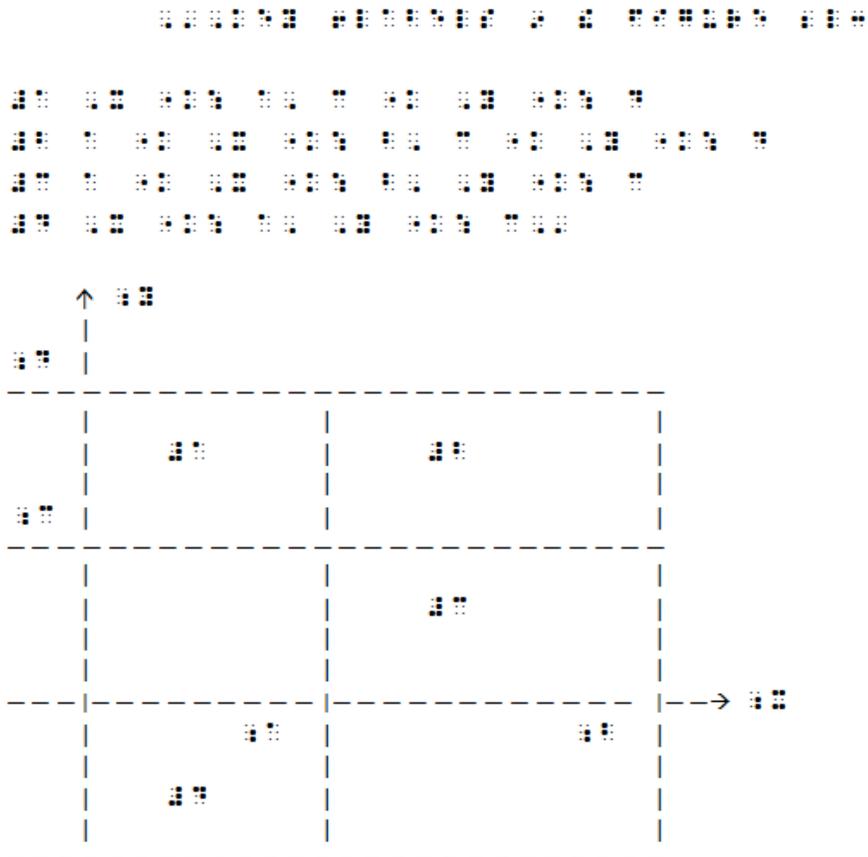


Example (3): A comma and a space have been omitted. Print should read as follows:

$$X \leq a, c < Y \leq d$$



Corrected simbraille is shown below:



d. A list of numeric and/or alphabetic keys and their meanings must be included in a transcriber's note as described in the most recent revision of *Braille Formats: Principles of Print to Braille Transcription*.

e. **Tables, Lists, Columned Material:** Subject to the provisions of §17, §30, §188 [of the Nemeth Code], and the above, tables, lists and columned material must be transcribed in accordance with the provisions of the most recent revision of *Braille Formats: Principles of Print to Braille Transcription*.

f. **Figures:** Subject to the provisions of §17, §30 [of the Nemeth Code], and the above, figures must be produced in accordance with the provisions of the BANA *Guidelines and Standards for Tactile Graphics, 2010*.

**FORMAT (CONTINUED)**

**§187. Spatial Arrangements with Main Divisions and Subdivisions:** When spatial arrangements contain both main divisions and subdivisions, the first main division should begin at the margin, and the first subdivision should follow on the same braille line if there no material between the main division number and the first subdivision number. As many subdivisions as possible should be placed on line. If additional subdivisions are left, they should be transcribed beginning in cell 3, after leaving a blank line below the longest arrangement above.

Example (1): Add a period following the print word "Solve". Like this: **Solve.**

(1)

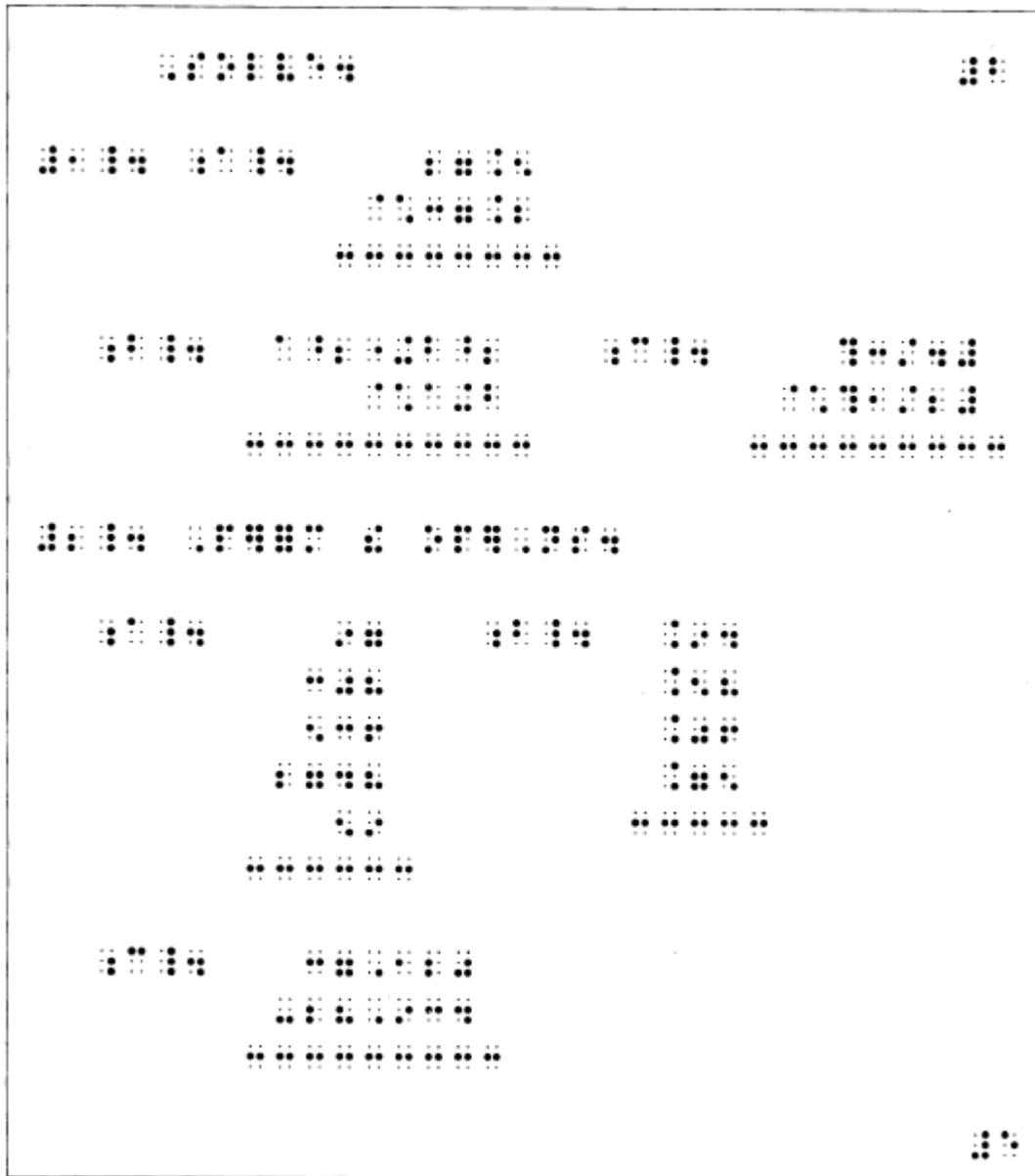
**Solve**

2

<p>1. a. <math display="block">\begin{array}{r} 27.5 \\ \times 37.2 \\ \hline \end{array}</math></p>	<p>b. <math display="block">\begin{array}{r} a^2 + b^2 \\ \times a + b \\ \hline \end{array}</math></p>	<p>c. <math display="block">\begin{array}{r} \frac{3}{4} \\ \times \frac{1}{2} \\ \hline \end{array}</math></p>
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2. Perform the operations.

<p>a. <math display="block">\begin{array}{r} 97 \\ 308 \\ 536 \\ 2748 \\ \hline 59 \end{array}</math></p>	<p>b. <math display="block">\begin{array}{r} .94 \\ .58 \\ .06 \\ \hline .75 \end{array}</math></p>	<p>c. <math display="block">\begin{array}{r} 37,120 \\ -28,934 \\ \hline \end{array}</math></p>
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**HOMEWORK**

With the exception of the four items listed below, the HOMEWORK printed on pages 332-334 are printed correctly. Use the original book to prepare **EXERCISE 16**.

#7, #8, #9: These mathematical expressions should be indented more to show they are displayed expressions.

7. Check for correctness.

$$\sqrt{3} = 1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}$$

8. Solve and check

$$\begin{cases} 3c = 4d + 17 \\ 2c + 3d = 0 \end{cases}$$

9. The system

$$\begin{cases} x + 2y = 8 \\ 2x - 3y = 2 \end{cases}$$

has the solution set  $\{(4, 2)\}$ .

#13: The second line of the displayed expression should show a comma, not a plus sign, in the expression, like this:

$$(0, 0) + (v_1, v_2) = (0 + v_1, 0 + v_2) = (v_1, v_2)$$

13. That  $(0, 0)$  is the identity element for vector addition is shown below.

$$\begin{aligned} (v_1, v_2) + (0, 0) &= (v_1 + 0, v_2 + 0) = (v_1, v_2) \\ (0, 0) + (v_1, v_2) &= (0 + v_1, 0 + v_2) = (v_1, v_2) \end{aligned} \quad \text{For all real numbers } v_1 \text{ and } v_2.$$

After page 334, insert the following:

## ***FINAL WORD***

This completes the coursework. *The Nemeth Braille Code For Mathematics and Science Notation, 1972* Revision, should now become your primary source reference for technical materials. Subject matter from several lessons may be grouped into one section in the codebook, giving new perspective and understanding of the rules. Appendix B offers a useful index of braille symbols. The Index will help you learn the vocabulary used in this work.

Updates to our source books, including *English Braille American Edition* and *Braille Formats*, have applicability to your Nemeth certification exam and any work you will be doing after certification. Once approved, errata and addenda to the codebooks are posted on the BANA website.

### How to download the BANA Updates and additional materials

- Go to [www.brailleauthority.org](http://www.brailleauthority.org)
- Click on "Codebooks and Guidelines" and then on "Mathematics and Science Braille"
- Scroll down to the heading **Updates**
- Each document is listed by the year which it was posted. Click on the format you prefer (pdf, html, or brf) to download each document. You can save the document on your computer and/or print a copy.
- Also find the document titled "*Application of the Formats Guidelines to Nemeth Transcriptions* " on the Braille Formats page by returning to the "Codebooks and Guidelines" screen and clicking on "Braille Formats"

*Keep all of your resources up-to-date by visiting the other links to Literary Braille, Braille Formats, Computer Braille, and Tactile Graphics. All codes have applicability in our work.*

Periodically, new updates are added to the BANA site. It is your responsibility to check back occasionally for further updates. You can receive e-mail notifications of new postings by joining the BANA contact list. Click on "Contact Us" and then follow the instructions under the heading **Sign up for BANA-announce Listserv.**

### How to obtain a print copy

If you prefer, a print copy of the Updates can be obtained free of charge from Judith Dixon at the Braille Authority of North America, 1805 North Oakland Street, Arlington, VA 22207; phone: 202-707-0722.