LESSON 14

- FUNCTION NAMES AND THEIR ABBREVIATED FORMS

Spatial Arrangements, continued

- SQUARE ROOT DIVISION
- OTHER PRINT LAYOUTS SHOWING DIVISION

Answers to Practice Material

LESSON PREVIEW

Rules regarding function names and their abbreviated forms are presented. Many examples are shown. The study of spatial arrangements continues with a other forms of division problems: square root division, partial quotient layout, synthetic division, and others.

FUNCTION NAMES AND THEIR ABBREVIATED FORMS

[NC Rule 18]

14.1 List of Common Function Names and Their Abbreviated Forms

The most common function names and their abbreviated forms are listed below. Function names that do not appear in this list are subject to the same rules taught in this lesson. Note that abbreviated function names are printed in regular type.

Function Name	Abbreviated Function Name
amplitude	amp
antilogarithm	antilog
arc	arc
argument	arg
cologarithm	colog
cosine	cos
hyperbolic cosine	cosh
cotangent	cot
hyperbolic cotangent	coth
coversine	covers
cosecant	csc
hyperbolic cosecant	csch
cotangent	ctn
hyperbolic cotangent	ctnh
determinant	det
error function	erf
exponential	exp
exsecant	exsec
gradient	grad
haversine	hav
imaginary part	im
infimum	inf
limit	lim
upper limit	$\overline{\text{lim}}$ or $\overline{\text{limit}}$
lower limit	$\underline{\text{lim}}$ or $\underline{\text{limit}}$
natural logarithm	ln
logarithm	log
maximum	max
minimum	min
modulo	mod
real part	re
secant	sec

hyperbolic secant	sech
sine	sin
hyperbolic sine	sinh
supremum	sup
tangent	tan
hyperbolic tangent	tanh
versine	vers

14.2 Code Switching and Punctuation

14.2.1 **Function Names in UEB.** A function name which is not associated with a math term is transcribed in UEB and appropriate contractions and punctuation are used.

Example 14-1

Consider the Law of Sines.

⠠⠒⠎⠊⠙⠻ ⠮ ⠠⠇⠁⠺ ⠷ ⠠⠎⠔⠑⠎⠲

"Sines" is not associated with a mathematical item and so is transcribed in UEB, using appropriate contractions.

Example 14-2

Some trigonometric functions are sine, cosine, and tangent.

⠠⠐⠎ ⠞⠗⠊⠛⠕⠝⠕⠍⠑⠞⠗⠊⠉ ⠋⠥⠝⠉⠰⠝⠎ ⠜ ⠎⠔⠑⠂
⠉⠕⠎⠔⠑⠂ ⠯ ⠞⠁⠝⠛⠢⠞⠲

14.2.2 **Function Names in Nemeth.** A function name which occurs in mathematical context is transcribed in Nemeth, without contractions. A function name used in conjunction with an abbreviated function name is a mathematical term and a switch to Nemeth is required.

⋙ sine α ⠎⠊⠝⠑ ⠨⠁

Because the Greek letter requires Nemeth Code, the associated function name is also transcribed in Nemeth.

⋙ logsine ⠇⠕⠛⠎⠊⠝⠑

"log" is an abbreviated function name, therefore "logsine" requires a switch to Nemeth.

Example 14-3

What is the meaning of logsine?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠍⠂⠝⠬ ⠷ ⠸⠩ ⠇⠕⠛⠎⠊⠝⠑ ⠸⠱⠴

14.2.3 **Abbreviated Function Names.** The abbreviated forms of function names are mathematical items and are transcribed following the rules of the Nemeth Code. An abbreviated function name is punctuated mathematically inside the switches.

Example 14-4

Some trigonometric functions are sin, cos, and tan.

⠠⠐⠎ ⠞⠗⠊⠛⠕⠝⠕⠍⠑⠞⠗⠊⠉ ⠋⠥⠝⠉⠰⠝⠎ ⠜⠑ ⠸⠩ ⠎⠊⠝⠠
⠉⠕⠎⠠ ⠠⠄⠯ ⠞⠁⠝ ⠸⠱⠲

The abbreviated function names are punctuated in mathematical mode.

Example 14-5

The abbreviated form of "logarithm" is "log".

⠠⠮ ⠁⠆⠗⠑⠧⠊⠁⠞⠫ ⠿⠍ ⠷ ⠦⠇⠕⠛⠜⠊⠹⠍⠴ ⠊⠎
⠸⠩ ⠦⠇⠕⠛⠸⠴ ⠸⠱⠲

This function name is in UEB. Its abbreviated form is in Nemeth. A punctuation indicator is required before the closing quotation mark because an abbreviated function name is a mathematical term.

Example 14-6

The inverse sine function is written $\sin^{-1}$.

⠠⠮ ⠔⠧⠻⠎⠑ ⠎⠔⠑ ⠋⠥⠝⠉⠰⠝ ⠊⠎ ⠺⠗⠊⠞⠞⠢
⠸⠩ ⠎⠊⠝⠘⠤⠂ ⠸⠱⠲

14.2.4 **"Arc" in Context.** "Arc" can be a function name, an abbreviated function name, or a word referring to a curve.

Example 14-7

What is the arc sine function?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠜⠉ ⠎⠔⠑ ⠋⠥⠝⠉⠰⠝⠦

The function name "sine" is in UEB, so "arc" is also transcribed in UEB.

Example 14-8

What is the arc sin function?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠸⠩ ⠁⠗⠉ ⠎⠊⠝ ⠸⠱ ⠋⠥⠝⠉⠰⠝⠦

The abbreviated function name "sin" is Nemeth, so "arc" is also transcribed in Nemeth..

Example 14-9

Arc ACB is a major arc in Circle O.

⠀⠀⠠⠜⠉ ⠸⠩ ⠠⠁⠠⠉⠠⠃ ⠸⠱ ⠊⠎ ⠁ ⠍⠁⠚⠕⠗ ⠜⠉ ⠔
⠠⠉⠊⠗⠉⠇⠑ ⠠⠕⠲

"Arc" is a word referring to the curved line ACB.

PRACTICE 14A

1. "sin θ" is pronounced "sine theta".
2. "Arcsin" is the "inverse sine".
3. sin 30° cos 45°
4. The logsine function is related to the logcosine function by $S_n = 2C_n$.

14.3 Spacing of Abbreviated Function Names

Within a mathematical expression, the following spacing rules are observed. These rules apply regardless of the spacing used in the print copy.

a. No space comes <u>before</u> an abbreviated function name unless it follows a sign of comparison or other symbol that requires spacing.

b. A space is required <u>after</u> an abbreviated function name or its inverse (the space follows the superscript). There is one exception – see <u>14.3.2</u>, below.

- ⪢ $\cos 20^\circ$ ⠉⠕⠎ ⠼⠆⠴⠘⠨⠡
- ⪢ $3 \cos 20^\circ$ ⠼⠒⠉⠕⠎ ⠼⠆⠴⠘⠨⠡
- ⪢ $\sin \theta$ ⠎⠊⠝ ⠨⠹
- ⪢ $i \sin \theta$ ⠊⠎⠊⠝ ⠨⠹
- ⪢ $\tan (x)$ ⠞⠁⠝ ⠷⠭⠾
- ⪢ $\tan^{-1}(x)$ ⠞⠁⠝⠘⠤⠂ ⠷⠭⠾
- ⪢ $f(x) = \sin (x)$ ⠋⠷⠭⠾ ⠨⠅ ⠎⠊⠝ ⠷⠭⠾

Example 14-10

For any angle θ, $\sin(\theta + 360^\circ) = \sin \theta$ and $\cos(\theta + 360^\circ) = \cos \theta$.

  ⠠⠿ ⠁⠝⠽ ⠁⠝⠛⠇⠑ ⠸⠩ ⠨⠹⠠
⠎⠊⠝ ⠷⠨⠹⠬⠒⠖⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠎⠊⠝ ⠨⠹ ⠠⠄⠯
⠉⠕⠎ ⠷⠨⠹⠬⠒⠖⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠉⠕⠎ ⠨⠹ ⠸⠱⠲

In print, there is no space before each opening parenthesis. In braille, a space is required following each abbreviated function name.

Example 14-11

$\sin(35^\circ) = \text{Opposite/Hypotenuse}$.

⠸⠩
⠎⠊⠝ ⠷⠒⠢⠘⠨⠡⠐⠾ ⠨⠅ ⠠⠕⠏⠏⠕⠎⠊⠞⠑⠸⠌⠠⠓⠽⠏⠕⠞⠑⠝⠥⠎⠑
⠸⠱⠲

In print, there is no space between sin and (35°). In braille, a space is required following the abbreviated function name.

14.3.1 **Spacing with Operation Symbols.** In braille, an operation symbol is usually unspaced from the symbols which precede and follow it. However, when an abbreviated function name is followed by an operation symbol, a space is required.

> ⫸ $\tan \cdot \sin$
>
> ⠞⠁⠝ ⠡⠎⠊⠝
>
> *"tan" is followed by a space. A space is not required before "sin".*

> ⫸ $1/\cos - \cos = \tan \cdot \sin$
>
> ⠼⠂⠸⠌⠉⠕⠎ ⠤⠉⠕⠎ ⠨⠅ ⠞⠁⠝ ⠡⠎⠊⠝
>
> *Each abbreviated function name is followed by a space.*

14.3.2 **Spacing with Indicators.** A space is not inserted between an abbreviated function name and an indicator which applies to it.

> ⫸ $\frac{1}{\cos}$ ⠹⠂⠌⠉⠕⠎⠼

Example 14-12

Reciprocal Functions: $\frac{1}{\cos} - \cos = \tan \cdot \sin$

⠘⠂⠠⠗⠑⠉⠊⠏⠗⠕⠉⠁⠇ ⠘⠂⠠⠋⠥⠝⠉⠰⠝⠎⠒
⠸⠩ ⠹⠂⠌⠉⠕⠎⠼⠤⠉⠕⠎ ⠨⠅ ⠞⠁⠝ ⠡⠎⠊⠝ ⠸⠱

The abbreviated function name in the denominator is unspaced from the closing fraction indicator. The expression continues, following other spacing rules of the Nemeth Code.

14.3.3 **Examples.** Examine the spacing in the following examples.

Example 14-13

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $\cos\theta = \frac{1}{\sin\theta}$

⠉⠕⠎ ⠨⠹ ⠨⠅ ⠹⠂⠌⠎⠊⠝ ⠨⠹⠼

(2) $\sin(\theta + 90°) = \cos\theta$

⠎⠊⠝ ⠷⠨⠹⠬⠔⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠉⠕⠎ ⠨⠹

(3) $y = 3\tan 2x$

⠽ ⠨⠅ ⠼⠒⠞⠁⠝ ⠼⠆⠭

(4) $a^2 = 2ac\cos\beta + c^2$

(5) $\sin(\alpha + \beta) + \sin(\alpha - \beta)$

(6) $y' = x\cos\varphi - y\sin\varphi$

(7) $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} = C$

(8) $6\sin 2A\cos 4A$

(9) $\cos 203° \csc 203°$

(10) $\dfrac{2\sin\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}}$

(11) $\ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

"ln" is the abbreviated form of "natural logarithm". Refer to the table at the beginning of this lesson.

PRACTICE 14B

(1) $\sin\theta/\cos\theta$

(2) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

(3) $\dfrac{\tan 90°}{\cot 90°}$

(4) $r[3\cos\theta + 4\sin\theta] = 5$

(5) $7(\cos 20° + i\sin 20°)$

(6) $\frac{1}{2}\ln|\sec 2t + \tan 2t| + C$

(7) $a\sin\frac{x}{a}\cdot\frac{1}{a} = \sin\frac{x}{a}$

14.3.4 **Spacing with Consecutive Abbreviated Function Names.** A space is required between two or more consecutive abbreviated function names unless they are clearly unspaced in the print text. When there is doubt concerning the presence of a space between abbreviated function names, a space should be inserted.

⫸ $y = \text{arc sin}\, x$ ⠽ ⠨⠅ ⠁⠗⠉ ⠎⠊⠝ ⠭

Example 14-14

What is cos arctan(−1)?

⠠⠱⠁⠞ ⠊⠎ ⠸⠩ ⠉⠕⠎ ⠁⠗⠉⠞⠁⠝⠷⠤⠂⠾ ⠸⠱⠦

14.3.5 **Examples.** Study the following examples.

Example 14-15

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $n = \log \sin 50°27'$

⠝ ⠨⠅ ⠇⠕⠛ ⠎⠊⠝ ⠼⠢⠴⠘⠨⠡⠐⠆⠶⠄

(2) $\cos\left[2\,\text{Arc csc}\left(-\frac{7}{5}\right)\right]$

⠉⠕⠎ ⠈⠷⠆⠠⠁⠗⠉ ⠉⠎⠉ ⠷⠤⠹⠶⠌⠢⠼⠾⠈⠾

(3) $\cos\left(\text{arc Tan}\, x + \frac{\pi}{3}\right)$

⠉⠕⠎ ⠷⠁⠗⠉ ⠠⠞⠁⠝ ⠭⠬⠹⠨⠏⠌⠒⠼⠾

(4) $\text{Arctan}\, x + \text{Arccot}\, x = \frac{1}{2}\pi$

⠠⠁⠗⠉⠞⠁⠝ ⠭⠬⠠⠁⠗⠉⠉⠕⠞ ⠭ ⠨⠅ ⠹⠂⠌⠆⠼⠨⠏

14.4 Nonuse of the English-letter Indicator

The English-letter indicator is not used with an English letter or a Roman numeral following an abbreviated function name.

⫸ $\sin x$ ⠎⠊⠝ ⠭

⫸ cot I ⠉⠕⠞ ⠠⠊

14.4.1 **Examples.** Examine the English letters and the spacing in the following examples.

Example 14-16

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $\sin x + y$

⠎⠊⠝ ⠭⠬⠽

(2) $\text{ctn} - \text{A} = -\text{ctn A}$

⠉⠞⠝ ⠤⠠⠁ ⠨⠅ ⠤⠉⠞⠝ ⠠⠁

(3) $y = 2\sin x + \sin 2x$

⠽ ⠨⠅ ⠼⠆⠎⠊⠝ ⠭⠬⠎⠊⠝ ⠼⠆⠭

(4) $y = \sqrt{\cot x}$

⠽ ⠨⠅ ⠜⠉⠕⠞ ⠭⠻

(5) $\{\sin x \mid \sin x + 2 \leq +1\}$

⠨⠷⠎⠊⠝ ⠭ ⠳ ⠎⠊⠝ ⠭⠬⠆ ⠐⠅⠱ ⠬⠂⠨⠾

(6) $y = \ln |\tan x|$

⠽ ⠨⠅ ⠇⠝ ⠳⠞⠁⠝ ⠭⠳

14.5 Keep Together

A function name or its abbreviated form and the sign which follows it (known as the "argument") is regarded as a single mathematical item and therefore should not be divided between braille lines. Also, a two-part function name should not be divided between braille lines. These rules also applies in UEB context.

Example 14-17

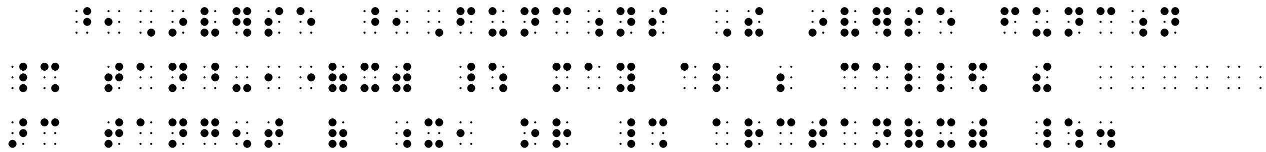
If $\theta = 51°$ is the angle between vectors, determine $\sin\theta$ and $\cos\theta$.

⠠⠊⠋ ⠸⠩ ⠨⠹ ⠨⠅ ⠼⠢⠂⠘⠨⠡ ⠸⠱ ⠊⠎ ⠮ ⠁⠝⠛⠇⠑ ⠆⠞
⠧⠑⠉⠞⠕⠗⠎⠂ ⠙⠑⠞⠻⠍⠔⠑ ⠸⠩ ⠎⠊⠝ ⠨⠹ ⠠⠄⠯
⠉⠕⠎ ⠨⠹ ⠸⠱⠲

"cos θ" is not divided between lines even though "cos" fits on the previous line.

Example 14-18

Inverse Functions The inverse function $\tan^{-1} x$ may also be called the arc tangent of x, or arctan x.



"arc tangent" is not divided between lines even though "arc" fits on the previous line.

14.6 Clarification— Abbreviated Function Names in an Enclosed List

An abbreviated function name and the item which follows it are regarded as a single item. Although the numeric indicator is not used at the beginning of an item in an enclosed list, it must be used before a numeral (or decimal point and a numeral) following an abbreviated function name.

⫸ (2 sin 30°, 3 cos 60°)

PRACTICE 14C

(A) sine x − sine y

(B) $2 \sin x + 3 \cos y$

(C) $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$

(D) The logarithm of sine 18° is written log sin 18°.

(E) $\cos 225° = -\sqrt{\frac{1+\cos 450°}{2}}$

(F) ArcTan[x, y] gives the arc tangent of $\frac{y}{x}$, taking into account in which quadrant the point (x, y) lies.

(G) The arc tangent of the complex number q is written "ArcTan[q]".

(H) Consider the ordered pair (cos .8000, 2 cos .8000).

14.7 Superscripts and Subscripts

When a function name or an abbreviated function name carries a superscript or a subscript, the required space follows the superscript or subscript. A letter, numeral, or other mathematical expression following this space assumes the same level as the function name or abbreviated function name.

In the following three examples, letters "θ" and "x" are printed on the baseline of writing and are unspaced.

- ⫸ $\sin^2\theta$
- ⫸ $\sin^2x$
- ⫸ sine^2x

Example 14-19

The coordinates are the cosine and sine, so we conclude $\sin^2\theta + \cos^2\theta = 1$.

The required space follows the superscript in sin² and cos². There is no space following the plus sign.

Example 14-20

Verify that $1 - \cos\frac{2\pi}{3} = 2\sin^2\frac{4\pi}{3}$.

The required space follows "cos" and "sin²". There is no space following the minus sign.

14.7.1 **Examples.** Examine the spacing in the following examples.

Example 14-21

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $\sin^2 A + \cos^2(B + A)$

(2) $(1 - \sin^2 x)^2 \cos^2 x$

(3) $\sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta} - 1$

(4) $\frac{1 - \frac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$

14.7.2 **Use/Nonuse of the Subscript Indicator.** The subscript indicator is not used when an abbreviated function name carries a numeric subscript on the first level below the baseline of writing. A subscript indicator is required in all other circumstances, including a function name which carries a numeric subscript.

⫸ $\log_3 81 = 4$

The numeral "3" is printed at the subscript level.

⫸ $\text{logarithm}_3\ 81 = 4$

A subscript indicator is required because the subscript applies to a word (a function name is a word).

⫸ $\log_b N \cdot \log_a b$

A subscript indicator is required because each subscript is a letter.

⫸ $\log_{2e} x = -1.4$

A subscript indicator is required because the subscript contains a letter.

14.7.3 **Abbreviated Function Names Within a Superscript or a Subscript.** When an abbreviated function name occurs within a superscript or subscript, the required space following it maintains the level at which the abbreviated function name appears. A restatement of the level indicator is not needed.

⫸ $e^{\sin x}$

"sin x" is in the superscript position.

⫸ $y = e^{\cos^2 x}$

"$\cos^2 x$" is in the superscript position.

14.7.4 **Examples.** Study the following examples.

Example 14-22

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $y = e^{\sin x}$ ⠽ ⠨⠅ ⠑⠘⠎⠊⠝ ⠭

(2) $y = e^{\text{sine}\, x}$ ⠽ ⠨⠅ ⠑⠘⠎⠊⠝⠑ ⠭

(3) $y = (\sin x)^{\tan x}$ ⠽ ⠨⠅ ⠷⠎⠊⠝ ⠭⠾⠘⠞⠁⠝ ⠭

(4) $e^{\ln x - 2\ln y}$ ⠑⠘⠇⠝ ⠭⠤⠆⠇⠝ ⠽

(5) $\mathrm{a}^{\mathrm{g(x)log_a f(x)}}$ ⠁⠘⠛⠷⠭⠾⠇⠕⠛⠘⠰⠁ ⠋⠷⠭⠾

(6) $3^{\log_3 9}$ ⠼⠒⠘⠇⠕⠛⠘⠰⠒ ⠼⠔

Recall from Lesson 6 that a subscript indicator is required in superscript and subscript combinations. The super/sub indicator shows a numeric subscript in the superscript position.

(7) $3^{\log_3 7} + 2^{\log_2 5}$

⠼⠒⠘⠇⠕⠛⠘⠰⠒ ⠼⠶⠐⠬⠆⠘⠇⠕⠛⠘⠰⠆ ⠼⠢

Same note as (6), above.

(8) $a^{\log_a x} = x$

⠁⠘⠇⠕⠛⠘⠰⠁ ⠭ ⠨⠅ ⠭

Recall from Lesson 6 that the space before a comparison sign returns the reader to the baseline.

(9) $e^{\sin x = a} > y$

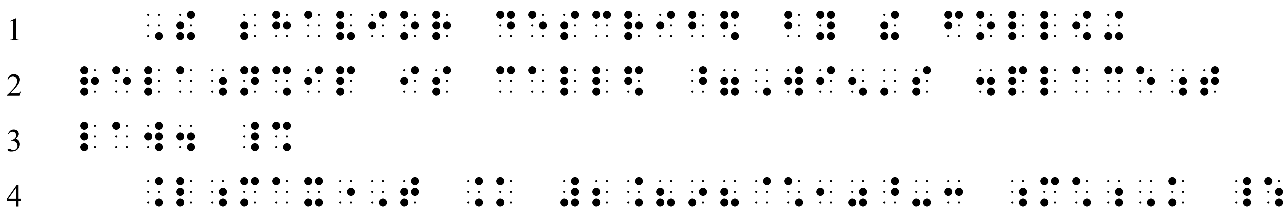
⠑⠘⠎⠊⠝ ⠭ ⠘⠨⠅ ⠁ ⠨⠂ ⠽

Recall from Lesson 6 that when a comparison sign occurs within a superscript, the level is restated before the comparison sign.

Example 14-23

The behavior described by the following relationship is called **Wien's displacement law.**

$$\lambda_{\text{max}}T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$$



Line 3: The switch to Nemeth terminates the UEB bold passage.
Line 4: Since both Nemeth switches do not fit on this line, the opening switch is placed on line 3. The multiplication dot which comes between the two-part abbreviation is unspaced. Review 4.20.1 in Lesson 4.

PRACTICE 14D

1. $\log_n .125 = -.6$
2. $\text{antilog}_{\text{a}}\ \text{x} = \text{N}$
3. $\log_{.0543}\text{x} = -.8$
4. $\cot^{-1} x + \frac{\pi}{2} - \tan^{-1} x$
5. $\sin^2 90° + \cos^2 90° = 1$
6. $e^{x+\ln x}$
7. $e^{\sin x} + e^{\sin y}$
8. $2^{\sec x} = y$

14.8 Modifiers

Modified abbreviated function names are transcribed according to the five-step rule for the transcription of modified expressions introduced in Lesson 12. When an abbreviated function name carries a modifier, the required space after the abbreviated function name follows the termination of the modifier.

➢ $\lim\limits_{x \to a}$ ⠐⠇⠊⠍⠩⠭⠀⠫⠕⠀⠁⠻

➢ $\lim\limits_{x \to a} f(x) = 1$

⠐⠇⠊⠍⠩⠭⠀⠫⠕⠀⠁⠻⠀⠋⠷⠭⠾⠀⠨⠅⠀⠼⠂

14.8.1 **Examples.** Study these additional examples.

Example 14-24

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $\lim\limits_{x \uparrow 4} (x-4)^{-1}$

⠐⠇⠊⠍⠩⠭⠀⠫⠣⠒⠒⠕⠀⠼⠲⠻⠀⠷⠭⠤⠲⠾⠘⠤⠂

(2) $\lim\limits_{\theta \to \theta_0} (\tan \theta)$

⠐⠇⠊⠍⠨⠹⠀⠫⠕⠀⠨⠹⠴⠻⠀⠷⠞⠁⠝⠀⠨⠹⠾

14.8.2 **Special Case—Upper Limit and Lower Limit.** The symbols below denote "upper limit" or "lower limit". The horizontal bar directly over or under "lim" or `limit" is <u>not</u> treated as a modifier.

⠣⠇⠊⠍	`upper limit`	$\overline{\text{lim}}$
⠣⠇⠊⠍⠊⠞	`upper limit`	$\overline{\text{limit}}$

➢ $\overline{\lim\limits_{n \to \infty}} f_n(x)$

⠐⠣⠇⠊⠍⠩⠝⠀⠫⠕⠀⠠⠿⠻⠀⠋⠰⠝⠐⠷⠭⠾

➢ $\overline{\text{limit}}_{n \to \infty} f_n(x)$

⠐⠣⠇⠊⠍⠊⠞⠩⠝⠀⠫⠕⠀⠠⠿⠻⠀⠋⠰⠝⠐⠷⠭⠾

⠩⠇⠊⠍	`lower limit`	$\underline{\text{lim}}$
⠩⠇⠊⠍⠊⠞	`lower limit`	$\underline{\text{limit}}$

⫸ $\underline{\lim}_{n \to \infty} f_n(x)$

⠐⠩⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

⫸ $\underline{\text{limit}}_{n \to \infty} f_n(x)$

⠐⠩⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

PRACTICE 14E

1. Find $\lim_{x \to 0.6} 2^{25x^2 - 10x - 1}$.
2. Formulate a precise definition for $\lim_{x \downarrow -\infty} f(x) = L$.
3. If $\overline{\lim}_{n \to \infty} a_n = A$ and $\overline{\lim}_{n \to \infty} b_n = B$, must it be true that $\overline{\lim}_{n \to \infty} (a_n + b_n) = A + B$?
4. Find $\overline{\text{limit}}_{n \to \infty} a_n$ when $a_n = (-1)_n$.
5. $\lim_{x \to 0} \csc x \ln(1 + x)$

You may wish to revisit the Review of Format for Spatial Arrangements in Lesson 10. NOTE: Code switch indicators are omitted and blank lines are implied in the examples that do not contain narrative.

SQUARE ROOT DIVISION

[NC Rule 25.6]

14.9 Review of Terminology

Radical expressions were presented in Lesson 8. When an answer is shown, a spatial arrangement is required. Here are the names of the parts of a radical expression. The line above the radicand is the vinculum. √ is the radical sign.

$$\begin{array}{rl} 12 & \textit{root} \\ \sqrt{144} & \textit{radicand} \end{array}$$

14.10 Spatial Arrangement for Square Root Problems

In the spatially arranged radical expression, the first cell of the vinculum is placed directly above the radical symbol. The last cell of the vinculum extends one cell beyond the radicand.

⠒⠒⠒ ⠜	Radical Sign (with Vinculum)	$\sqrt{}$

Example 14-25

The square root of 144 is 12, and is written as follows.

$$\begin{array}{r} 12 \\ \sqrt{144} \end{array}$$

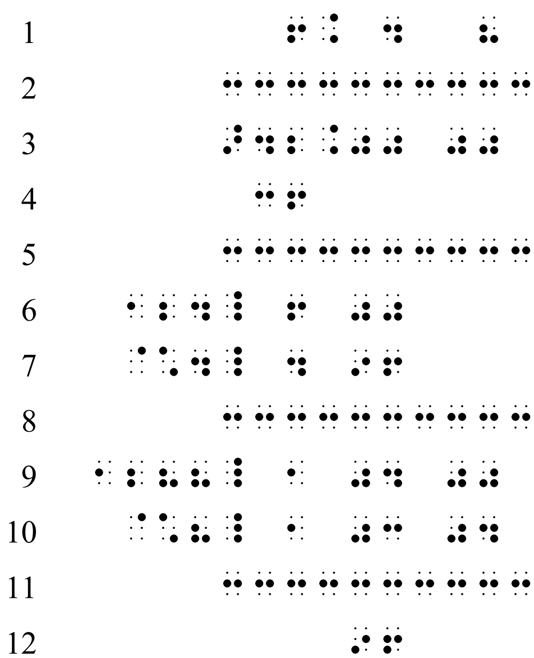
```
⠀⠀⠠⠮⠀⠎⠟⠥⠜⠑⠀⠗⠕⠕⠞⠀⠷⠀⠼⠁⠙⠙⠀⠊⠎⠀⠺⠗⠊⠞⠞⠢⠀⠵
⠋⠕⠇⠇⠪⠎⠲⠀⠸⠩

⠀⠀⠀⠀⠂⠆
⠀⠀⠒⠒⠒⠒⠒
⠀⠀⠜⠂⠲⠲

⠸⠱
```

a. **Solving a Square Root Problem.** The procedures used with long division arrangements are applied to a spatially arranged square root problem. Review alignment and spacing rules for long division in Lesson 10. The vertical line that separates the parts of the problem is represented by dots 456. Spacing between digits replicates spacing in print. Follow print regarding the alignment of the vertical lines.

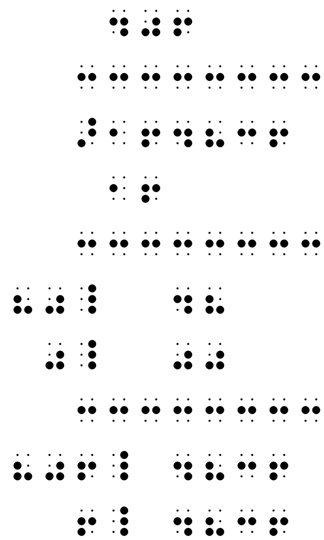
Example 14-26

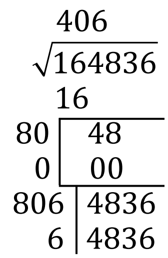


```
          6.  4   8
       √42.00 00
        36
   124 | 6 00
   × 4 | 4 96
  1288 | 1 04 00
   × 8 |  1 03 04
               96
```

All lines: Spacing between digits matches print in order to attain proper vertical alignment.
Line 2: The vinculum begins in the cell above the radical sign and ends one cell beyond the rightmost character in the entire arrangement.
Lines 2, 5, 8, 11: Separation lines are all the same width.
Lines 6, 7, 9, 10: These vertical lines align below the radical sign.
Lines 7, 10: The multiplication cross is unspaced from the multiplier.

Example 14-27



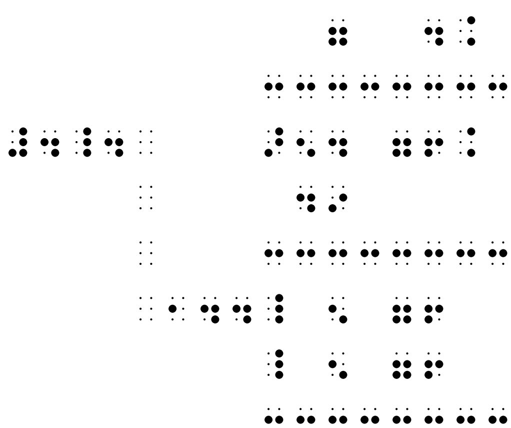


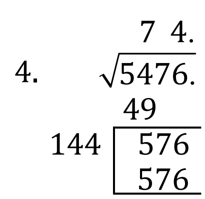
Lines 9-10: These vertical lines are aligned to the right of the vertical lines on lines 6-7, as printed.

14.11 Placement of Identifiers with Spatial Radical Expressions

An identifier, if present, is placed on the line with the radicand. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 14-28





PRACTICE 14F

(A)

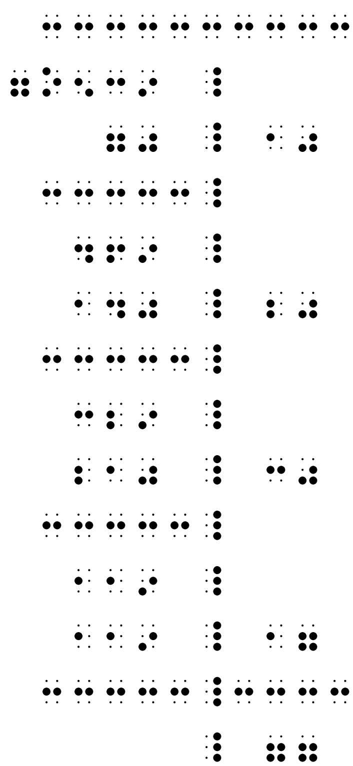
$$
\begin{array}{r|l}
 & \ \ 5.\ \ 7\ \ \ 4 \\
 & \sqrt{33.00\,00} \\
 & 25 \\ \hline
107 & \ \ 800 \\
\times 7 & \ \ 749 \\ \hline
1144 & \ \ 5100 \\
\times 4 & \ \ 4576 \\ \hline
 & \ \ \ \ 524
\end{array}
$$

OTHER PRINT LAYOUTS SHOWING DIVISION

14.12 Partial Quotients [NC Rule 25.5.8]

This layout shows partial quotients printed to the right of the division problem. A vertical line separates the partial quotients from the rest of the problem. The vertical line may be either drawn as a tactile graphic or it may be represented by dots 456. One space (one blank cell) is left between the vertical line and any digit preceding or following it.

Example 14-29



```
   ________
7) 539 |
    70 | 10
   469 |
   140 | 20
   329 |
   210 | 30
   119 |
   119 | 17
       | 77
```

Notice the comparative lengths of the separation lines as well as their vertical alignment.

Instructions: Review Section 10.13.6.d regarding alignment of the minus signs.

PRACTICE 14G

$$
\begin{array}{r|l}
\multicolumn{1}{r}{132} & \\
6\overline{)792} & \\
-600 & 100 \\
\hline
192 & \\
-\ 60 & 10 \\
\hline
132 & \\
-\ 60 & 10 \\
\hline
72 & \\
-\ 60 & 10 \\
\hline
12 & \\
-\ 12 & 2 \\
\hline
0 &
\end{array}
$$

14.13 Synthetic Division [NC Rule 25.7]

Synthetic division is a method of showing division of polynomials. There is not a standard print layout. The transcription replicates the print design, following alignment rules discussed below, and using the standard separation line and vertical line of the Nemeth Code. Here is an example of one possible layout of a synthetic division problem.

$$\begin{array}{r|rrrr} +2 & 1 & -3 & +4 & +5 \\ & & +2 & -2 & +4 \\ \hline & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} \end{array}$$

The parts to this problem are labeled as follows.

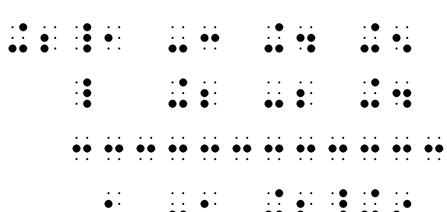
$$\begin{array}{lr|rrrrl} \textit{divisor} & +\,2 & 1 & -3 & +4 & +5 & \textit{dividend} \\ & & & +2 & -2 & +4 & \textit{product} \\ \cline{3-6} \textit{quotient} & & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} & \textit{remainder} \end{array}$$

14.13.1 **Alignment and Spacing.** In the examples which follow, look carefully at the vertical alignment. The numerals in the dividend, product, and quotient are aligned in vertical columns as in the print copy. Signs of operation, if any, are also vertically aligned. At least one blank cell is left between adjacent columns.

14.13.2 **Vertical Line.** Dots 456 represent the vertical line in the print copy. The braille symbol is shown between the divisor and the division arrangement, beginning on the line with the dividend and ending on the line with the product. No space is left between the vertical line and the dividend or divisor. The separation line (dots 25) extends from the vertical line to one cell beyond the entire arrangement. Another unspaced vertical line is transcribed between the quotient and the remainder.

Example 14-30

$$\begin{array}{r|rrrr} +2 & 1 & -3 & +4 & +5 \\ & & +2 & -2 & +4 \\ \hline & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} \end{array}$$



Note the vertical alignment of the numerals and the operation signs.
In this problems, the divisor is +2, the dividend is 1 -3 +4 +5, the product is +2 -2 +4, the quotient is 1 -1 +2, and the remainder is +9.

14.13.3 **Another Print Style—Divisor on the Right.** If the divisor is printed to the right of the overall problem, the same layout is followed in braille. Follow the alignment and spacing rules outlined above, particularly noting that at least one blank cell must be left between adjacent columns. The vertical lines are unspaced from the dividend and the divisor, as well as from the quotient and the remainder.

Example 14-31

```
3   −7    −1    −23 | 3
    +9    +6    +15 |
--------------------
3   +2    +5 |  −8
```

```
⠀⠒⠀⠀⠤⠶⠀⠤⠂⠀⠤⠆⠒⠸⠒
⠀⠀⠀⠬⠔⠀⠬⠖⠀⠬⠂⠢⠸
⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠀⠒⠀⠬⠆⠀⠬⠢⠸⠤⠀⠦
```

14.13.4 **Another Print Style—Boxed Divisor.** If the divisor appears boxed in on two sides, the boxing is omitted. A vertical line between the divisor and the dividend is inserted in order to differentiate the divisor from the rest of the arrangement, even though this vertical line does not appear in print. Follow the same alignment and spacing rules outlined above. The first example shows the divisor at the left; the second shows the divisor at the right.

Example 14-32

```
|−1   1   +2   +2   +4
          −1   −1   −1
      ------------------
      1   +1   +1 | +3
```

```
⠤⠂⠸⠂⠀⠬⠆⠀⠬⠆⠀⠬⠲
⠀⠀⠸⠀⠀⠤⠂⠀⠤⠂⠀⠤⠂
⠀⠀⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠀⠀⠀⠂⠀⠬⠂⠀⠬⠂⠸⠬⠒
```

Example 14-33

```
1    +2    +2    +4   |−2
     −2    +0    −4
_________________________
1    +0    +2    +0
```

```
 ⠂ ⠬⠆ ⠬⠆ ⠬⠲⠸⠤⠆
   ⠤⠆ ⠬⠴ ⠤⠲
⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
 ⠂ ⠬⠴ ⠬⠆ ⠬⠴
```

Note that this example has no remainder.

14.13.5 **Placement of Identifiers with Synthetic Division.** An identifier, if present, is placed on the line with the dividend (the top line of the arrangement, in this case). One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 14-34

```
197.   |+2   1   +6     −1    −30
                 +2    +16    +30
             ____________________
```

```
⠼⠂⠔⠶⠸⠲ ⠬⠆⠸⠂ ⠬⠖ ⠤ ⠂ ⠤⠒⠴
         ⠸  ⠬⠆ ⠬⠂⠖ ⠬⠒⠴
         ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
```

Notice the vertical alignment of the operation signs. The numerals are aligned by place value, with the "1" directly above the "6" of "16".

PRACTICE 14H

Dividing Polynomials: Divide $(3x^4 + 12x^3 - 5x^2 - 18x + 8) \div (x + 4)$

$$\begin{array}{r|rrrr|r} \underline{-4} & 3 & 12 & -5 & -18 & 8 \\ & & -12 & 0 & 20 & -8 \\ \hline & 3 & 0 & -5 & 2 & 0 \end{array}$$

Answer: $3x^2 - 5x - 2$

For further practice, see Appendix A—Reading Practice.

EXERCISE 14

Prepare Exercise 14 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 14A

⠼⠁⠲ ⠸⠩ ⠦⠎⠊⠝ ⠨⠹⠸⠴ ⠸⠱ ⠊⠎ ⠏⠗⠕⠝⠳⠝⠉⠫ ⠦⠎⠔⠑
⠮⠞⠁⠴⠲
⠼⠃⠲ ⠸⠩ ⠦⠠⠁⠗⠉⠎⠊⠝⠸⠴ ⠸⠱ ⠊⠎ ⠮ ⠦⠔⠧⠻⠎⠑ ⠎⠔⠑⠴⠲
⠼⠉⠲ ⠸⠩ ⠎⠊⠝ ⠼⠒⠴⠘⠨⠡ ⠉⠕⠎ ⠼⠲⠢⠘⠨⠡
⠼⠲⠸⠲ ⠠⠄⠠⠮ ⠇⠕⠛⠎⠊⠝⠑ ⠸⠱ ⠋⠥⠝⠉⠰⠝ ⠊⠎ ⠗⠑⠇⠁⠞⠫ ⠞⠕
⠮ ⠸⠩ ⠇⠕⠛⠉⠕⠎⠊⠝⠑ ⠸⠱ ⠋⠥⠝⠉⠰⠝ ⠃⠽
⠸⠩ ⠠⠎⠰⠝ ⠨⠅ ⠼⠆⠠⠉⠔⠝ ⠸⠱⠲

PRACTICE 14B

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⠷⠂⠾ ⠎⠊⠝ ⠼⠒⠴⠘⠨⠡⠐⠉⠕⠎ ⠼⠲⠢⠘⠨⠡
⠷⠆⠾ ⠎⠊⠝ ⠨⠹⠸⠌⠉⠕⠎ ⠨⠹
⠷⠒⠾ ⠎⠊⠝ ⠼⠆⠨⠁ ⠨⠅ ⠼⠆⠎⠊⠝ ⠨⠁⠉⠕⠎ ⠨⠁
⠷⠲⠾ ⠹⠞⠁⠝ ⠼⠔⠴⠘⠨⠡⠐⠌⠉⠕⠞ ⠼⠔⠴⠘⠨⠡⠐⠼
⠷⠢⠾ ⠗⠈⠷⠒⠉⠕⠎ ⠨⠹⠬⠲⠎⠊⠝ ⠨⠹⠈⠾ ⠨⠅ ⠼⠢
⠷⠖⠾ ⠼⠶⠷⠉⠕⠎ ⠼⠆⠴⠘⠨⠡⠐⠬⠊⠎⠊⠝ ⠼⠆⠴⠘⠨⠡⠐⠾
⠷⠶⠾ ⠹⠂⠌⠆⠼⠇⠝ ⠳⠎⠑⠉ ⠼⠆⠞⠬⠞⠁⠝ ⠼⠆⠞⠳⠬⠠⠉
⠷⠦⠾ ⠁⠎⠊⠝ ⠹⠭⠌⠁⠼⠡⠹⠂⠌⠁⠼ ⠨⠅ ⠎⠊⠝ ⠹⠭⠌⠁⠼ ⠸⠱

PRACTICE 14C

⠸⠩

⠷⠠⠁⠾ ⠎⠊⠝⠑ ⠭⠤⠎⠊⠝⠑ ⠽

⠷⠠⠃⠾ ⠼⠇⠎⠊⠝ ⠭⠬⠒⠉⠕⠎ ⠽

⠷⠠⠉⠾ ⠹⠂⠬⠉⠕⠎ ⠭⠌⠎⠊⠝ ⠭⠼⠬⠹⠎⠊⠝ ⠭⠜⠂⠬⠉⠕⠎ ⠭⠼ ⠸⠱

⠐⠣⠰⠠⠙⠐⠜ ⠠⠮ ⠇⠕⠛⠜⠊⠹⠍ ⠷ ⠸⠩ ⠎⠊⠝⠑ ⠼⠂⠦⠘⠨⠡ ⠸⠱

⠊⠎ ⠺⠗⠊⠞⠞⠢ ⠸⠩ ⠇⠕⠛ ⠎⠊⠝ ⠼⠂⠦⠘⠨⠡⠸⠲

⠷⠠⠑⠾

⠉⠕⠎ ⠼⠆⠆⠢⠘⠨⠡ ⠤⠜⠹⠂⠬⠉⠕⠎ ⠼⠲⠢⠴⠘⠨⠡⠐⠌⠆⠼⠻

⠷⠠⠋⠾ ⠠⠁⠗⠉⠠⠞⠁⠝ ⠈⠷⠭⠠ ⠽⠈⠾ ⠸⠱ ⠛⠊⠧⠑⠎ ⠮

⠜⠉ ⠞⠁⠝⠛⠢⠞ ⠷ ⠸⠩ ⠹⠽⠌⠭⠼ ⠸⠱⠂ ⠞⠁⠅⠬ ⠔⠞⠕ ⠁⠒⠨⠞

⠔ ⠱ ⠟⠥⠁⠙⠗⠁⠝⠞ ⠮ ⠏⠕⠔⠞ ⠸⠩ ⠷⠭⠠ ⠽⠾ ⠸⠱ ⠇⠊⠑⠎⠲

⠐⠣⠰⠠⠛⠐⠜ ⠠⠮ ⠜⠉ ⠞⠁⠝⠛⠢⠞ ⠷ ⠮ ⠉⠕⠍⠏⠇⠑⠭ ⠝⠥⠍⠃⠻

⠰⠟ ⠊⠎ ⠺⠗⠊⠞⠞⠢ ⠸⠩ ⠦⠠⠁⠗⠉⠠⠞⠁⠝ ⠈⠷⠟⠈⠾⠸⠴ ⠸⠱⠲

⠐⠣⠰⠠⠓⠐⠜ ⠠⠒⠎⠊⠙⠻ ⠮ ⠕⠗⠙⠻⠫ ⠏⠁⠊⠗

⠸⠩ ⠷⠉⠕⠎ ⠼⠨⠦⠴⠴⠴⠠ ⠆⠉⠕⠎ ⠼⠨⠦⠴⠴⠴⠾ ⠸⠱⠲

PRACTICE 14D

⠸⠩

⠼⠂⠸⠲ [illegible]

⠼⠆⠸⠲ [illegible]

⠼⠒⠸⠲ [illegible]

⠼⠲⠸⠲ [illegible]

⠼⠢⠸⠲ [illegible]

⠼⠖⠸⠲ [illegible]

⠼⠶⠸⠲ [illegible]

⠼⠦⠸⠲ [illegible]

PRACTICE 14E

PRACTICE 14F

PRACTICE 14G

```
⠸⠩
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠀⠀⠂⠒⠆
⠀⠒⠒⠒⠒⠒⠒
⠖⠕⠶⠔⠆⠀⠸
⠀⠤⠖⠴⠴⠀⠸⠀⠂⠴⠴
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠂⠔⠆⠀⠸
⠀⠤⠀⠖⠴⠀⠸⠀⠂⠴
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠂⠒⠆⠀⠸
⠀⠤⠀⠖⠴⠀⠸⠀⠂⠴
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠀⠶⠆⠀⠸
⠀⠀⠤⠖⠴⠀⠸⠀⠂⠴
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠀⠂⠆⠀⠸
⠀⠀⠤⠂⠆⠀⠸⠀⠆
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠀⠀⠴⠀⠸
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠸⠱
```

PRACTICE 14H