LESSON 8

- INTRODUCTION TO FRACTIONS
    - Simple Fractions
    - Mixed Numbers
    - Complex Fractions
    - More Fraction Rules
- RADICAL EXPRESSIONS

Format

- LINKED EXPRESSIONS
    - Division of Linked Expressions
    - Nested Linked Expression

Answers to Practice Material

LESSON PREVIEW

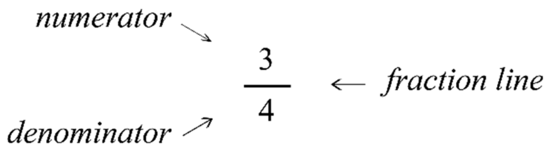
Fractions, mixed numbers, and radical expressions are studied. The "linked expression" is defined and its format is discussed in detail, including the special case of the nested linked expression.

Note: The font size in some of the examples is larger than standard, for clarity. Some of the larger symbols may appear to be bold. Do not consider the size difference or the darker image to be a variant typeform.

INTRODUCTION TO FRACTIONS

Two types of fractions are presented in this lesson: simple fractions (including mixed numbers) and complex fractions.

A fraction is composed of three parts: a numerator, a denominator, and a fraction line.



Fractions are printed in a variety of ways. The numerator may be printed above the denominator or they may be printed on the same level. The fraction line may be horizontal or diagonal. Here are three examples of the fraction "three fourths" printed in different styles.

$$\frac{3}{4} \qquad {}^{3}/_{4} \qquad 3/4$$

The numerator and/or denominator may also consist of or contain words or abbreviations. Here are four examples.

$\mathrm{m/s}$	(*Spoken:* meters per second)
${}^{\text{ft.}}/_{\text{sec.}}$	(*Spoken:* feet per second)
$\frac{\text{rise}}{\text{run}}$	(*Spoken:* rise over run)
$3.5\%/\text{year}$	(*Spoken:* 3.5 percent per year)

In a technical transcription, fractions are transcribed in Nemeth.

Simple Fractions

[NC 13.1]

8.1 Definition

For the purposes of the Nemeth Code, a simple fraction is one whose numerator and denominator contain no fractions, except possibly at the superscript or subscript level. These are simple fractions:

$$\frac{1}{2} \qquad \frac{a^2}{b^2} \qquad \frac{\text{72 mi.}}{\text{4 hr.}} \qquad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}} \qquad {}^{5}/_{12}$$

This is not a simple fraction because the numerator contains a fraction: $\frac{1/3}{5}$

8.2 Simple Fraction Indicators

Regardless of print layout, unless otherwise stated, the fraction is transcribed linearly so that the numerator, the fraction line, and the denominator are written horizontally across one braille line. When the numerator is printed at a higher level than the denominator, braille fraction indicators are used.

⠹	Opening Simple Fraction Indicator
⠼	Closing Simple Fraction Indicator

These fractions will use fraction indicators. $\frac{3}{4}$ $^{3}/_{4}$

This fraction will not use fraction indicators. 3/4 (See 8.4.2, below.)

8.3 The Horizontal Simple Fraction Line

The type of fraction line used in the print copy (horizontal or diagonal) is replicated in the braille transcription. In a simple fraction, the horizontal fraction line is transcribed as the symbol shown below. Note that the horizontal fraction line symbol consists of one braille cell.

⠌	Horizontal Simple Fraction Line	—

⋙ $\frac{3}{4}$ ⠹⠒⠌⠲⠼

⋙ $\frac{d}{t}$ ⠹⠙⠌⠞⠼

a. **Spacing.** The numerator and denominator are unspaced from the fraction indicators and from the fraction line. Spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction.

b. **Keep Together.** A fraction must not be divided between braille lines. Fractions which do not fit on the line will be discussed in Lesson 15.

Example 8-1

Terry has 32 candy bars. She shares $\frac{3}{4}$ of them with her class.

⠀⠀⠠⠞⠻⠗⠽ ⠓⠁⠎ ⠼⠉⠃ ⠉⠯⠽ ⠃⠜⠎⠲ ⠠⠩⠑ ⠩⠜⠑⠎
⠸⠩ ⠹⠒⠌⠲⠼ ⠸⠱ ⠷ ⠮⠍ ⠾ ⠓⠻ ⠉⠇⠁⠎⠎⠲

Example 8-2

Rate formula: rate $= \frac{\text{distance}}{\text{time}}$ or $r = \frac{d}{t}$.

⠀⠀⠠⠗⠁⠞⠑ ⠿⠍⠥⠇⠁⠒
⠸⠩ ⠗⠁⠞⠑ ⠨⠅ ⠹⠙⠊⠎⠞⠁⠝⠉⠑⠌⠞⠊⠍⠑⠼ ⠠⠄⠕⠗
⠗ ⠨⠅ ⠹⠙⠌⠞⠼ ⠸⠱⠲

Reminder: Words in Nemeth are transcribed without contractions.

Example 8-3

Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{\Delta y}{\Delta x}$.

⠀⠀⠠⠎⠇⠕⠏⠑ ⠿⠍⠥⠇⠁⠒ ⠸⠩ ⠍ ⠨⠅ ⠹⠽⠆⠤⠽⠂⠌⠭⠆⠤⠭⠂⠼
⠠⠄⠕⠗ ⠍ ⠨⠅ ⠹⠨⠠⠙⠽⠌⠨⠠⠙⠭⠼ ⠸⠱⠲

Reminder: A numeric subscript does not need a subscript indicator when it is a right subscript to a letter.

Example 8-4

Multiplying fractions is easy! $\frac{3}{4} \cdot \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$

⠀⠀⠠⠍⠥⠇⠞⠊⠏⠇⠽⠬ ⠋⠗⠁⠉⠰⠝⠎ ⠊⠎ ⠑⠁⠎⠽⠖
⠸⠩ ⠹⠒⠌⠲⠼⠡⠹⠂⠌⠆⠼ ⠨⠅ ⠹⠒⠡⠂⠌⠲⠡⠆⠼ ⠨⠅ ⠹⠒⠌⠦⠼ ⠸⠱

Spacing: There is no space before or after the operation signs (multiplication dots); there is a space before and after the comparison signs (equals signs).

Example 8-5

Use the reciprocal of` the coefficient to solve for x in $\frac{3}{8}x = 72$.

⠀⠀⠠⠥⠎⠑ ⠮ ⠗⠑⠉⠊⠏⠗⠕⠉⠁⠇ ⠷ ⠮ ⠉⠕⠑⠖⠊⠉⠊⠢⠞ ⠞⠕
⠎⠕⠇⠧⠑ ⠿ ⠰⠭ ⠔ ⠸⠩ ⠹⠒⠌⠦⠼⠭ ⠨⠅ ⠼⠶⠆ ⠸⠱⠲

Spacing: The coefficient (fraction) is unspaced from the variable (x).

Example 8-6

Anderson sprinted $\frac{2}{3}$ of the $\frac{1}{4}$–mile track.

⠠⠯⠻⠎⠕⠝ ⠎⠏⠗⠔⠞⠫ ⠸⠩ ⠹⠆⠌⠒⠼ ⠸⠱ ⠷ ⠮
⠸⠩ ⠹⠂⠌⠲⠼⠤⠍⠊⠇⠑ ⠸⠱ ⠞⠗⠁⠉⠅⠲

Spacing: The fraction in this hyphenated expression is unspaced from the hyphen.

Instructions: Transcribe the first two lines as a paragraph, with one space between expressions. A blank line must precede the itemized portion. When you proofread, check that you closed each fraction, that you returned to the baseline after each superscript, that displayed expressions are placed in the proper cell, and that you terminated Nemeth Code where appropriate.

PRACTICE 8A

Horizontal Simple Fraction Line

Here are some examples of simple fractions. $\frac{1}{2}$ $\frac{15}{16}$ $\frac{x}{y}$ $\frac{a+b}{c+d}$ $\frac{\Delta y}{\Delta x}$ $\frac{(x+y)}{(x-y)}$ $\frac{9}{12}$ $\left(\frac{3}{2}a+\frac{1}{2}b\right)$ $\frac{3x}{17y}$ $x-\frac{1}{4}(x-2x)$

1. $V=\frac{1}{3}\pi r^2 h$
2. $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$
3. $\left|\frac{a}{b}\right|=\frac{|a|}{|b|}$
4. Write an equation to show that $\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$.
5. $x^2\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4x^2-x-2}{(x+1)(y+1)}$
6. Solve this differential equation:

   $$x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=e^{x^2}$$

7. The number π is the ratio of the circumference of a circle to its diameter. That is,

   $$\pi=\frac{\text{circumference}}{\text{diameter}}.$$

8. $\frac{12}{33}=\frac{m}{11}$
9. $\frac{4}{32}=\frac{10.5}{x}$
10. $\frac{1}{4}+\frac{3}{4}-\frac{1}{2}=\frac{1}{2}$

8.4 The Diagonal Simple Fraction Line

The type of fraction line used in the print copy (horizontal or diagonal) is replicated in the braille transcription. In a simple fraction, the diagonal fraction line is transcribed as the symbol shown below. Note that the diagonal simple fraction line consists of two braille cells.

⠸⠌	Diagonal Simple Fraction Line	/

When a diagonal fraction line is printed, it may not be clear where the fraction begins and where it ends. The transcriber must not attempt to analyze the math. Instead, application of the following rules will prevent misinterpretation of the expression.

8.4.1 **Use of Simple Fraction Indicators with the Diagonal Simple Fraction Line.** When the numerator and denominator are printed at different levels of writing on either side of the diagonal line, the construction is a fraction and so simple fraction indicators are used. Do not confuse this type style with superscripts and subscripts. In this example, the numeral 3 is the numerator and the numeral 4 is the denominator.

⫸ ${}^{3}/_{4}$ ⠹⠒⠸⠌⠲⠼

Example 8-7

Terry has 32 bags of M&Ms. She shares ${}^{3}/_{4}$ of the bags with her class.

⠠⠞⠻⠗⠽ ⠓⠁⠎ ⠼⠉⠃ ⠃⠁⠛⠎ ⠷ ⠠⠍⠈⠯⠠⠍⠎⠲ ⠠⠩⠑ ⠩⠜⠑⠎
⠸⠩ ⠹⠒⠸⠌⠲⠼ ⠸⠱ ⠷ ⠮ ⠃⠁⠛⠎ ⠾ ⠓⠻ ⠉⠇⠁⠎⠎⠲

8.4.2 **Type Size on the Baseline of Writing.** If the numerator and denominator are printed at the same level of writing on either side of the diagonal line, the transcriber must notice the type size.

a. **Fraction in Smaller Type.** If the type is in a different size from that normally used for similar expressions throughout the text, identify this as a fraction by using simple fraction indicators. In the example below, note that the fraction is printed on the baseline of writing—it is not a subscript.

⫸ $2\ 1/y$ ⠼⠆⠹⠂⠸⠌⠽⠼

The numeral 1 is smaller than the numeral 2. The space between the coefficient and the fraction is not shown in braille.

Example 8-8

In the expression $2\ 1/y$, 2 is the coefficient of the fraction.

⠠⠔ ⠮ ⠑⠭⠏⠗⠑⠎⠨⠝ ⠸⠩ ⠼⠆⠹⠂⠸⠌⠽⠼ ⠸⠱⠂ ⠼⠃ ⠊⠎ ⠮
⠉⠕⠑⠖⠊⠉⠊⠢⠞ ⠷ ⠮ ⠋⠗⠁⠉⠰⠝⠲

b. **Fraction in Normal Sized Type.** When the numerator and denominator are printed at the same level of writing on either side of the diagonal line and the type size is normal when compared to similar expressions, fraction indicators are not used. This is a straightforward transcription of number-slash-number. A switch to Nemeth Code is required. (Review 5.4.8 "Slash".)

⋙ $3/4$ ⠼⠒⠸⠌⠲

⋙ $1/y$ ⠼⠂⠸⠌⠽

Example 8-9

Jean has 16 bags of Skittles. She shares 3/4 of the bags with her class.

⠠⠚⠂⠝ ⠓⠁⠎ ⠼⠁⠋ ⠃⠁⠛⠎ ⠷ ⠠⠎⠅⠊⠞⠞⠇⠑⠎⠲ ⠠⠩⠑
⠩⠜⠑⠎ ⠸⠩ ⠼⠒⠸⠌⠲ ⠸⠱ ⠷ ⠮ ⠃⠁⠛⠎ ⠾ ⠓⠻ ⠉⠇⠁⠎⠎⠲

Compare the next example to Example 8-8. The first fraction is printed on the baseline but is in smaller type. Fraction indicators are used. The second fraction, however, is printed in normal type size and the numerator and denominator are at the same level of writing on either side of the diagonal line. This meets the requirements for nonuse of simple fraction indicators, even though the text refers to it as a "fraction."

Example 8-10

In the expression 2 $1/y$, 2 is the coefficient of the fraction $1/y$.

⠠⠔ ⠮ ⠑⠭⠏⠗⠑⠎⠨⠝ ⠸⠩ ⠼⠆⠹⠂⠸⠌⠽⠼ ⠸⠱⠂ ⠼⠃ ⠊⠎ ⠮
⠉⠕⠑⠖⠊⠉⠊⠢⠞ ⠷ ⠮ ⠋⠗⠁⠉⠰⠝ ⠸⠩ ⠼⠂⠸⠌⠽ ⠸⠱⠲

c. **Fraction in a Subscript.** In the next example, the fraction is a subscript. A subscript is printed in a smaller type. This fraction meets the requirements for nonuse of simple fraction indicators because (a) the numerator and denominator are printed at the same level of writing on either side of the diagonal line; and (b) the type size is normal for a subscript. Simple fraction indicators are not used.

⋙ $3_{x/y}$ ⠼⠒⠰⠭⠸⠌⠽

Example 8-11

Notice the fraction in the subscript. $3_{x/y}$

⠠⠝⠕⠞⠊⠉⠑ ⠮ ⠋⠗⠁⠉⠰⠝ ⠔ ⠮ ⠎⠥⠃⠎⠉⠗⠊⠏⠞⠲
⠸⠩ ⠼⠒⠰⠭⠸⠌⠽ ⠸⠱

According to Nemeth rules, fraction indicators are not used here, even though the text refers to the subscript as a "fraction".

8.4.3 **A Reminder About the UEB Slash.** When the slash is serving as a separator between words or numbers in literary context, it is not mathematical.

and/or, c/o	⠯⠸⠌⠕⠗⠂ ⠉⠸⠌⠕	(UEB)
true/false test	⠞⠗⠥⠑⠸⠌⠋⠁⠇⠎⠑ ⠞⠑⠌	(UEB)
6/6/44 (date)	⠼⠋⠸⠌⠼⠋⠸⠌⠼⠙⠙	(UEB)

Example 8-12

On 11/15/59 U.S. Patent #14–5/2806ds-XRX was renamed *xerography* and Haloid/Xerox launched the world of photocopying into a new era with the Xerox 914/01.

⠠⠕⠝ ⠼⠁⠁⠸⠌⠼⠁⠑⠸⠌⠼⠑⠊ ⠠⠥⠲⠠⠎⠲ ⠠⠏⠁⠞⠢⠞
⠸⠹⠼⠁⠙⠠⠤⠼⠑⠸⠌⠼⠃⠓⠚⠋⠰⠙⠎⠤⠠⠠⠭⠗⠭ ⠴ ⠗⠑⠐⠝⠙
⠨⠂⠭⠻⠕⠛⠗⠁⠏⠓⠽ ⠯ ⠠⠓⠁⠇⠕⠊⠙⠸⠌⠠⠭⠻⠕⠭ ⠇⠁⠥⠝⠡⠫ ⠮ ⠸⠺
⠷ ⠏⠓⠕⠞⠕⠉⠕⠏⠽⠬ ⠔⠞⠕ ⠁ ⠝⠑⠺ ⠻⠁ ⠾ ⠮ ⠠⠭⠻⠕⠭
⠼⠊⠁⠙⠸⠌⠼⠚⠁⠲

Numerals printed with slashes such as dates, model numbers, etc. are not mathematical and therefore the rules of UEB are followed.

a. **Which Code?** When a slash represents a mathematical operation meaning *per*, *over*, or *divided by*, Nemeth fraction rules are followed. When a slash does not mean *per*, *over*, or *divided by*, the construction is transcribed in UEB unless it is part of a larger mathematical expression.

Example 8-13

The input/output ratio is equal.

⠠⠮ ⠔⠏⠥⠞⠸⠌⠳⠞⠏⠥⠞ ⠗⠁⠞⠊⠕ ⠊⠎ ⠑⠟⠥⠁⠇⠲

The slash means "to" and so is transcribed in UEB.

Example 8-14

The red/white probabilities are the same.

⠠⠮ ⠗⠫⠸⠌⠱⠊⠞⠑ ⠏⠗⠕⠃⠁⠃⠊⠇⠊⠞⠊⠑⠎ ⠜⠑ ⠮ ⠎⠁⠍⠑⠲

The slash means "or" and so is transcribed in UEB.

8.4.4 **Spacing.** Words or abbreviations in the numerator or denominator are unspaced from the fraction indicators and are also unspaced from the fraction line.

Example 8-15

Express m/s in mph.

⠠⠑⠭⠏⠗⠑⠎⠎ ⠸⠩ ⠰⠍⠸⠌⠰⠎ ⠸⠱ ⠔ ⠍⠏⠓⠲

Because the slash means "per" ("meters per second"), Nemeth is required.

Example 8-16

Express $^{\text{ft.}}/_{\text{sec.}}$ in mph.

⠠⠑⠭⠏⠗⠑⠎⠎ ⠸⠩ ⠹⠋⠞⠲⠸⠌⠎⠑⠉⠲⠼ ⠸⠱ ⠔ ⠍⠏⠓⠲

Because the numerator is printed higher than the denominator, fraction indicators are used.

Example 8-17

Express the rate in miles/hour.

⠠⠑⠭⠏⠗⠑⠎⠎ ⠮ ⠗⠁⠞⠑ ⠔ ⠸⠩ ⠍⠊⠇⠑⠎⠸⠌⠓⠕⠥⠗ ⠸⠱⠲

Because the slash means "per" ("miles per hour"), Nemeth is required. No contractions are used.

8.4.5 **Code Switch Reminders.** Review these points from Lesson 3.

a. **Words.** As part of a math problem expressed in symbols and words, the words are included in the switch. Compare the next two examples.

Example 8-18

$^1/_4$ of 24 is 6

⠸⠩ ⠹⠂⠸⠌⠲⠼ ⠸⠱ ⠷ ⠼⠃⠙ ⠊⠎ ⠼⠋

Only the fraction requires a switch to Nemeth Code because the math statement is expressed in words.

Example 8-19

$^1/_4$ of $24 = 6$

⠸⠩ ⠹⠂⠸⠌⠲⠼ ⠕⠋ ⠼⠆⠲ ⠨⠅ ⠼⠖ ⠸⠱

The presence of the equals sign turns the entire expression into a Nemeth construction. The word "of" is part of the equation and is uncontracted in Nemeth.

b. **Abbreviations.** An abbreviation associated with a Nemeth item is included inside the switches.

Example 8-20

Now add 2/3 c. cornstarch—Oobleck!

⠠⠝⠪ ⠁⠙⠙ ⠸⠩ ⠼⠆⠸⠌⠒ ⠰⠉⠲ ⠸⠱ ⠉⠕⠗⠝⠌⠜⠡⠠⠤
⠠⠕⠕⠃⠇⠑⠉⠅⠖

The abbreviation is included inside the switch, along with its associated fraction. Fraction indicators are not used because the numerator and denominator are printed on the baseline of writing.

Example 8-21

To convert feet to inches, change $^{1}/_{2}$ ft. to 6 in. Now you can add 6 in. to 3 in. to get 9 in.

⠠⠞⠕ ⠒⠧⠻⠞ ⠋⠑⠑⠞ ⠞⠕ ⠔⠡⠑⠎⠂ ⠡⠁⠝⠛⠑ ⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠸⠩ ⠹⠂⠸⠌⠆⠼ ⠋⠞⠲ ⠸⠱ ⠞⠕ ⠼⠋ ⠊⠝⠲ ⠠⠝⠪ ⠽ ⠉ ⠁⠙⠙
⠼⠋ ⠊⠝⠲ ⠞⠕ ⠼⠉ ⠊⠝⠲ ⠞⠕ ⠛⠑⠞ ⠼⠊ ⠊⠝⠲

Recall that, in braille, the abbreviation must fall on the same line as its associated value.

Example 8-22

Equivalent ratios of $^{y}/_{x}$ (or $^{x}/_{y}$) can be seen in Table 4.1.

⠠⠑⠟⠥⠊⠧⠁⠇⠢⠞ ⠗⠁⠞⠊⠕⠎ ⠷ ⠸⠩ ⠹⠽⠸⠌⠭⠼ ⠷⠠⠄⠕⠗
⠹⠭⠸⠌⠽⠼⠾ ⠸⠱ ⠉ ⠆ ⠎⠑⠢ ⠔ ⠠⠞⠁⠃⠇⠑ ⠼⠙⠲⠁⠲

A single-word switch indicator is used for the word "or". Nemeth grouping symbols are used for the parentheses here because we have not switched out of Nemeth Code.

Instructions: Determine whether the slash is used mathematically, that is, does it require a switch to Nemeth? If it does, then determine if fraction indicators are required.

PRACTICE 8B

Diagonal Simple Fraction Line

A) How many ${}^{2}/_{3}$'s are there in ${}^{5}/_{6}$?

B) Energy is absorbed at the rate of 880 J/s for each square meter of the surface.

C) $y(0) = \pi/4$

D) $1\ {}^{\text{ft}}/_{\text{sec}} \approx 0.6818$ mph

E) In y 1/5, y is the coefficient of the fraction 1/5.

F) True/False: The rise/run ratio is 5 in graph A.

G) ${}^{a}/_{b} \cdot {}^{c}/_{d} = {}^{ac}/_{bd}$

H) A 5-year CD went from earning interest at the rate of 12.06%/year in 1984 to earning less than 0.87%/year in 2015.

Mixed Numbers

[NC 13.4]

8.5 Definition of Mixed Number

A mixed number is an expression composed of a whole number followed by a simple fraction whose numerator and denominator are both numerals. Numerals in a mixed number may be represented by omission signs. An expression is not a mixed number if it contains any letters, even though the expression appears to be in the form of a mixed number. Here are some examples.

$1\frac{2}{3}$ (*Spoken:* one and two-thirds)

$99\ {}^{15}/_{16}$ (*Spoken:* ninety-nine and fifteen-sixteenths)

8.5.1 **Use of Mixed Number Fraction Indicators.** The opening and closing mixed number fraction indicators encase the fractional part of a mixed number.

Mixed Number Fraction Indicators

⠸⠹	Opening
⠸⠼	Closing

The fractional part of the mixed number uses simple fraction lines, either horizontal or diagonal, according to the fraction line style used in print.

⠌	Horizontal Simple Fraction Line	——
⠸⠌	Diagonal Simple Fraction Line	/

The examples shown above are transcribed as follows.

⫸ $1\frac{2}{3}$ ⠼⠂⠸⠹⠆⠌⠒⠸⠼

⫸ $99\ {}^{15}/_{16}$ ⠼⠔⠔⠸⠹⠂⠢⠸⠌⠂⠖⠸⠼

Example 8-23

Russ is making a small flowerbed that is $3\frac{1}{2}$ feet by $1\frac{1}{2}$ feet.

  ⠠⠗⠥⠎⠎ ⠊⠎ ⠍⠁⠅⠬ ⠁ ⠎⠍⠁⠇⠇ ⠋⠇⠪⠻⠃⠫ ⠞ ⠊⠎
⠸⠩ ⠼⠒⠸⠹⠂⠌⠆⠸⠼ ⠸⠱ ⠋⠑⠑⠞ ⠃⠽ ⠸⠩ ⠼⠂⠸⠹⠂⠌⠆⠸⠼ ⠸⠱
⠋⠑⠑⠞⠲

a. **Linear Print Representation of a Mixed Number.** When the fractional part of a mixed number is printed on the baseline, the mixed number fraction indicators are used. Read the surrounding text to be sure this is a mixed number.

⫸ 1 1/2 ⠼⠂⠸⠹⠂⠌⠆⠸⠼

b. **Keep Together.** A whole number must not be divided from its fractional part.

8.5.2 **Mixed Numbers and Omissions.** If any part of a mixed number contains a sign of omission, the mixed number fraction indicators are used.

⫸ $\frac{7}{4} = 1\frac{?}{4}$ ⠹⠶⠌⠲⠼ ⠨⠅ ⠼⠂⠸⠹⠿⠌⠲⠸⠼

⫸ $1{}^{15}/_{25} = ?\,{}^{3}/_{5}$ ⠼⠂⠸⠹⠂⠢⠸⠌⠆⠢⠸⠼ ⠨⠅ ⠿⠸⠹⠒⠸⠌⠢⠸⠼

8.5.3 **Nonuse of Mixed Number Fraction Indicators.** If the fractional part of the expression contains a letter, it no longer suits the definition of "mixed number." Appropriate fraction indicators are used (or are not used) according to the rules.

⫸ $\frac{7}{4} = 1\frac{x}{4}$ ⠹⠶⠌⠲⠼ ⠨⠅ ⠼⠂⠹⠭⠌⠲⠼

⫸ $3\,x/y$ ⠼⠒⠹⠭⠸⠌⠽⠼

PRACTICE 8C

Mixed Numbers

1. Find the premium for a 1½–yr. policy at the yearly rate of 24¢ per \$100.

2. $2\frac{1}{2}$ ft + 8 in = ? inches

3. $\left(\frac{1}{2} \times 3\frac{1}{2}\right) + \left(3\frac{1}{2} \times 2\right)$

4. $13\,{}^{1}/_{2} + 2\,{}^{2}/_{3} = 16\,{}^{1}/_{6}$

5. $7/4 = 1\,{}^{?}/_{4}$

6. $\frac{9}{4} = 2\frac{x}{4}$

Fraction Review

Compute each **unit rate** (price/pound).

a. \$1.50 for 2/3 pound of potatoes

b. \$4.20 for ${}^{1}/_{2}$ pound of Edam cheese

c. \$6.00 for $\frac{3}{4}$ pound of deli smoked turkey

d. \$12.50 for $1\frac{1}{2}$ pounds of sliced ham

Complex Fractions

[NC 13.5]

8.6 Definition of Complex Fraction

A complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers.

$$\begin{array}{ll} \textit{numerator} & \dfrac{4\frac{3}{4}}{5} \longleftarrow \textit{complex fraction line} \\ \textit{denominator} & \end{array}$$

Here are more examples of complex fractions.

$$\frac{\frac{4}{3}}{12} \qquad \frac{\frac{a}{b}-\frac{c}{d}}{\frac{a}{b}+\frac{c}{d}} \qquad \frac{1}{3/8} \qquad \frac{1}{2}\Big/\frac{3}{4}$$

Reminder: A fraction is not a complex fraction if the only fractions it contains are at the superscript or subscript level. Such a fraction is a simple fraction.

This is a simple fraction, not a complex fraction: $\dfrac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

8.6.1 **Use of Complex Fraction Indicators and Complex Fraction Lines.** The opening and closing complex fraction indicators are used to enclose a complex fraction.

```
Complex Fraction Indicators
  ⠠⠹    Opening
  ⠠⠼    Closing
```

The main complex fraction line is represented by its appropriate braille symbol—either horizontal or diagonal.

⠠⠌	Horizontal Complex Fraction Line	—
⠠⠸⠌	Diagonal Complex Fraction Line	/

The examples shown above are transcribed as follows. Simple fraction indicators enclose each simple fraction when required; complex fraction indicators enclose the entire complex fraction. We suggest that you underline the complex fraction indicators and the complex fraction line in each example in order to analyze each transcription.

⫸ $\frac{4\frac{3}{4}}{5}$ ⠠⠹⠲⠸⠹⠒⠌⠲⠸⠼⠠⠌⠢⠠⠼

⫸ $\frac{\frac{4}{3}}{12}$ ⠠⠹⠹⠲⠌⠒⠼⠠⠌⠂⠆⠠⠼

⫸ $\frac{1}{2}\Big/\frac{3}{4}$ ⠠⠹⠹⠂⠌⠆⠼⠠⠸⠌⠹⠒⠌⠲⠼⠠⠼

⫸ $\frac{\frac{a}{b}-\frac{c}{d}}{\frac{a}{b}+\frac{c}{d}}$ ⠠⠹⠹⠁⠌⠃⠼⠤⠹⠉⠌⠙⠼⠠⠌⠹⠁⠌⠃⠼⠬⠹⠉⠌⠙⠼⠠⠼

Although the denominator in the next example does not require simple fraction indicators (see 8.4.2.b), it is still a fraction and so the overall construction is a complex fraction.

⫸ $\frac{1}{3/8}$ ⠠⠹⠂⠠⠌⠒⠸⠌⠦⠠⠼

Instructions: Begin each complex fraction on a new braille line, not side by side as printed. Read left to right.

PRACTICE 8D

Complex Fractions

$$\frac{\frac{1}{8}+\frac{3}{4}}{7} \qquad \frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{7}{9}} \qquad \frac{1/3+1/4}{4/5-1/2} \qquad \frac{\frac{\pi}{8}}{2\pi}$$

$$\frac{a}{\frac{b}{c}} \qquad \frac{33\frac{1}{3}}{100} \qquad \frac{3/5}{6} \qquad \frac{3}{5}\Big/\frac{7}{9}$$

More Fraction Rules

8.7 Fractions and the Baseline Indicator

When a fraction is on the baseline level, assure that the components of the fraction (the fraction indicators and the fraction line) are notated on the baseline of writing. When there is a level in effect at the end of a numerator or a denominator, correct placement of the baseline indicator assures accurate reading.

⫸ $\frac{a^2}{b}$ ⠹⠁⠘⠆⠐⠌⠃⠼

The baseline indicator precedes the fraction line following the superscript in the numerator.

⫸ $\frac{a}{b^2}$ ⠹⠁⠌⠃⠘⠆⠐⠼

The baseline indicator precedes the closing fraction indicator following the superscript in the denominator.

⫸ $$\frac{\frac{x}{x-1}-1^2}{\frac{x}{x+1}+1^2}$$

⠠⠹⠹⠭⠌⠭⠤⠂⠼⠤⠂⠘⠆⠐⠠⠌⠹⠭⠌⠭⠬⠂⠼⠬⠂⠘⠆⠐⠠⠼

These fractions are all on the baseline.

⫸ $x^{\frac{1}{2}} \times y$ ⠭⠘⠹⠂⠌⠆⠼⠐⠈⠡⠽

This fraction is in the superscript position.

⫸ $\frac{y_2 - y_1}{x_2 - x_1}$ ⠹⠽⠆⠤⠽⠂⠌⠭⠆⠤⠭⠂⠼

Recall that a return to the baseline after each numeric subscript is assumed when a baseline indicator is not used.

8.8 Further Observations Regarding Spacing

Spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction. No space is left between a fraction and a letter, a numeral, a sign of grouping, a braille indicator, or another fraction when these items are part of the same expression.

Example 8-24

Explain why $\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$.

⠠⠑⠭⠏⠇⠁⠔ ⠱⠽
⠸⠩ ⠹⠂⠌⠆⠼⠬⠹⠒⠌⠲⠼ ⠨⠅ ⠼⠂⠸⠹⠂⠌⠲⠸⠼ ⠸⠱⠲

The plus sign is unspaced from the fractions before and after it; there is a space before and after the equals sign. There is no space between the components of a mixed number.

Example 8-25

Multiply the fractions. $\left(\frac{5}{12}\right)\left(\frac{4}{12}\right) = \_\_\_$

⠠⠍⠥⠇⠞⠊⠏⠇⠽ ⠮ ⠋⠗⠁⠉⠰⠝⠎⠲
⠸⠩ ⠷⠹⠢⠌⠂⠆⠼⠾⠷⠹⠲⠌⠂⠆⠼⠾ ⠨⠅ ⠤⠤⠤⠤ ⠸⠱

No space is left between factors even though one may appear in print.

Example 8-26

Differentiating the first two terms, $^1\!/_2\, x^{1/2} + {}^5\!/_6\, x^{-3/2}$.

⠠⠙⠊⠖⠻⠢⠞⠊⠁⠞⠬ ⠮ ⠋⠌ ⠞⠺⠕ ⠞⠻⠍⠎⠂
⠸⠩ ⠹⠂⠸⠌⠆⠼⠭⠘⠂⠸⠌⠆⠐⠬⠹⠢⠸⠌⠖⠼⠭⠘⠤⠒⠸⠌⠆ ⠸⠱⠲

In the braille transcription, no spaces occur in this long math expression.

8.9 Fractions and the Ellipsis and Long Dash

a. **Spacing Next to a Fraction Indicator.** No space is left between an opening or closing fraction indicator and an ellipsis or long dash in the numerator or denominator of a fraction.

⫸ $\frac{\ldots\times 5}{2\times 10} = \frac{15}{20}$

⠹⠄⠄⠄ ⠈⠡⠢⠌⠆⠈⠡⠂⠴⠼ ⠨⠅ ⠹⠂⠢⠌⠆⠴⠼

The space following this ellipsis is required. Review 1.12 in Lesson 1.

⫸ $$\frac{3\times 5}{2\times \_\_} = \frac{15}{20}$$

⠹⠒⠈⠡⠢⠌⠆⠈⠡ ⠤⠤⠤⠤⠼ ⠨⠅ ⠹⠂⠢⠌⠆⠴⠼

The space preceding this long dash is required. Review 1.12 in Lesson 1.

b. **Spacing Next To a Fraction Line.** A space is left between a fraction line and an ellipsis or long dash.

⫸ $$\frac{2+4+6+\ldots}{1+3+5+\ldots}$$

⠹⠆⠬⠲⠬⠖⠬ ⠄⠄⠄ ⠌⠂⠬⠒⠬⠢⠬ ⠄⠄⠄⠼

c. **Spacing Between Fractions.** A space is left between a fraction and an ellipsis or long dash preceding or following the fraction.

⫸ $\frac{1}{10} \cdots \frac{10}{10}$ ⠹⠂⠌⠂⠴⠼ ⠄⠄⠄ ⠹⠂⠴⠌⠂⠴⠼

Example 8-27

Fill in the blanks with the missing fractions. $\frac{1}{10}$ —— —— $\frac{4}{10}$

⠠⠋⠊⠇⠇ ⠔ ⠮ ⠃⠇⠁⠝⠅⠎ ⠾ ⠮ ⠍⠊⠎⠎⠬ ⠋⠗⠁⠉⠰⠝⠎⠲ ⠸⠩
⠹⠂⠌⠂⠴⠼ ⠤⠤⠤⠤ ⠤⠤⠤⠤ ⠹⠲⠌⠂⠴⠼ ⠸⠱

8.10 Fractions in an Enclosed List

Fractions and mixed numbers may be part of an enclosed list. (Review the rules for "enclosed list" in Lesson 4.)

⫸ $$\left\{0,\ \frac{1}{2},\ 1,\ 1\frac{1}{2}\right\}$$

⠨⠷⠴⠠ ⠹⠂⠌⠆⠼⠠ ⠂⠠ ⠂⠸⠹⠂⠌⠆⠸⠼⠨⠾

Instructions: Determine the formatting before beginning your transcription. Where does each paragraph begin? Which expressions are embedded and which are displayed? What is the proper cell placement for the displayed expressions?

PRACTICE 8E

These are simple fractions:

$$\frac{1}{2} \qquad \frac{a^2}{b^2} \qquad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$$

This is not a simple fraction: $\frac{1/3}{2/3}$

Review the rules in 6.12.5 regarding an ellipsis on the baseline of writing when it follows a superscript.

$$x^{\frac{1}{2}} \ldots\ x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \ldots \frac{x^{\frac{1}{2}}+1}{y^{\frac{1}{2}}-1}$$

Plot the points $\left(-\frac{1}{2},\ 4\right)$, $\left(3,\ 4\frac{1}{4}\right)$, and $\left(-9,\ \frac{3}{4}\right)$. Then express $\frac{a^{3/4}}{b^{5/4}}$ in radical form.

RADICAL EXPRESSIONS

[NC Rule 16]

8.11 Terminology

Here are the parts of a radical expression.

$\sqrt{144}$	√ is the *radical sign*.
	144 is the *radicand*.
	The horizontal bar above the radicand is the *vinculum*. The vinculum shows the extent to which the radical sign applies.

There may be a figure placed to the left and slightly above the radical sign, called the *index* of the radical. For example, this radical sign has an index of three. $\sqrt[3]{}$ When there is no index, the radical sign may be referred to as the "square root" sign.

8.12 The Termination Indicator

When a radical expression has a vinculum, the radical sign is placed before the radicand and the termination indicator is placed after the radicand.

⠜	`Radical Sign`	√
⠻	`Termination Indicator`	

- ⫸ $\sqrt{x}$ ⠜⠭⠻
- ⫸ $\sqrt{64}$ ⠜⠖⠲⠻

Reminders: An English-letter indicator is not needed for an English letter (in regular type) which occurs in an unspaced sequence of mathematical symbols. A numeric indicator is not used when a numeral is not preceded by a space.

a. **No Vinculum.** When a vinculum is not shown in print, or when the radical sign occurs without a radicand, a termination indicator is not used.

- ⫸ $\sqrt{}(x-1)$ ⠜⠷⠭⠤⠂⠾
- ⫸ √ ⠜

Example 8-28

The √ is called a "radical sign."

⠠⠮ ⠸⠩ ⠜ ⠸⠱ ⠊⠎ ⠉⠁⠇⠇⠫ ⠁ ⠦⠗⠁⠙⠊⠉⠁⠇ ⠎⠊⠛⠝⠲⠴

8.13 Spacing

The spacing before and after a radical expression is subject to the spacing rules for the signs preceding or following the radical expression.

⫸ $\sqrt{9}-\sqrt{4}=1$ ⠜⠔⠻⠤⠜⠲⠻ ⠨⠅ ⠼⠂

No space is left between a radical expression and a letter, a numeral, a fraction, a sign of grouping, a braille indicator, or another radical expression when these items are unspaced in print and belong to the same expression.

⫸ $\sqrt{5}y$ ⠜⠢⠻⠽

⫸ $\sqrt{x}^{2}$ ⠜⠭⠻⠘⠆

⫸ $2a\sqrt{4ab}$ ⠼⠆⠁⠜⠲⠁⠃⠻

⫸ $\sqrt{4}\sqrt{87}=2\sqrt{87}$ ⠜⠲⠻⠜⠦⠶⠻ ⠨⠅ ⠼⠆⠜⠦⠶⠻

⫸ $\sqrt{y}\,\mathrm{d}x+(1+x)\,\mathrm{d}y=0,\ y(0)=1$

⠜⠽⠻⠙⠭⠬⠷⠂⠬⠭⠾⠙⠽ ⠨⠅ ⠼⠴⠠ ⠽⠷⠴⠾ ⠨⠅ ⠼⠂

Reminder: In print, derivative notation dx, dy, etc. is often preceded and followed by a space within an expression, for clarity. In braille, the terms are not spaced unless a space is required with the item preceding or following them. (Review 4.15.1 in Lesson 4.)

Example 8-29

Simplify. $\dfrac{2-\sqrt{\frac{1}{4}}}{3-\sqrt{\frac{1}{2}}}$

⠠⠎⠊⠍⠏⠇⠊⠋⠽⠲

⠸⠩ ⠠⠹⠆⠤⠜⠹⠂⠌⠲⠼⠻⠠⠌⠒⠤⠜⠹⠂⠌⠆⠼⠻⠠⠼ ⠸⠱

8.14 Index of Radical

A small number or letter that may appear next to the radical sign is the *index* of the radical. This print example shows an index "3".

$$\sqrt[3]{27}$$

In braille, the index-of-radical indicator and the index precede the radical sign.

⠣	Index-of-Radical Indicator

⋙ $\sqrt[3]{27} = 9$ ⠣⠒⠜⠆⠶⠻ ⠨⠅ ⠼⠔

⋙ $\sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$

⠣⠍⠜⠹⠁⠌⠃⠼⠻ ⠨⠅ ⠹⠣⠍⠜⠁⠻⠌⠣⠍⠜⠃⠻⠼

Example 8-30

Simplify as follows. $\frac{\sqrt{9}}{\sqrt[3]{27}} = \frac{3}{3} = 1$

⠠⠎⠊⠍⠏⠇⠊⠋⠽ ⠵ ⠋⠕⠇⠇⠪⠎⠲
⠸⠩ ⠹⠜⠔⠻⠌⠣⠒⠜⠆⠶⠻⠼ ⠨⠅ ⠹⠒⠌⠒⠼ ⠨⠅ ⠼⠂ ⠸⠱

PRACTICE 8F

Radical Expressions

1. $\sqrt{\frac{b}{a} + \frac{a}{b}}$
2. $\sqrt{c/d}$
3. $\frac{1}{4}\sqrt{\frac{1}{2}}$
4. $\sqrt{\frac{10}{16}} = \sqrt{10}/4$
5. $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$
6. $2\sqrt{2} + 7\sqrt{2} = (2 + 7)\sqrt{2} = 9\sqrt{2}$
7. $\frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{15}}{2}$
8. $\frac{\sqrt{2} - \sqrt{\frac{1}{3}}}{\sqrt{3} - \sqrt{\frac{1}{2}}}$
9. $\sqrt{48x^3y}$
10. $\surd(y - 1) + \surd(2y) = 1$
11. $\sqrt[n]{d}$
12. $\sqrt[a+b]{z - y}$
13. $\sqrt[4]{729} + \sqrt[6]{27} = \sqrt[?]{?}$
14. $7\sqrt[3]{125} \cdot 7\sqrt[5]{2}$
15. $\sqrt[5]{m}\,\sqrt[5]{n} = \sqrt[5]{mn}$

8.15 Nested Radical Expressions [NC 16.3]

When radical expressions are nested one within the other, the appropriate number of order-of-radical indicators shows the depth of each inner radical expression.

Order-of-Radical Indicators

⠨	First Inner Radical
⠨⠨	Second Inner Radical
⠨⠨⠨	Third Inner Radical
⠻	Termination Indicator

The appropriate order-of-radical indicator is placed before its radical sign and before its termination indicator.

≫ $\sqrt{x+\sqrt{x+y}+z}$ ⠜⠭⠬⠨⠜⠭⠬⠽⠨⠻⠬⠵⠻

When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

≫ $\sqrt{x+\sqrt{y+\sqrt{z}}}$ ⠜⠭⠬⠨⠜⠽⠬⠨⠨⠜⠵⠨⠨⠻⠨⠻⠻

8.15.1 **Nested Radical Expression with an Index.** If an inner radical expression has an index, the appropriate order-of-radical indicator is placed before the index-of-radical indicator as well as before its termination indicator. When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

≫ $\sqrt{\sqrt[3]{16}}=\sqrt[3]{\sqrt{16}}$

⠜⠨⠣⠒⠜⠂⠖⠨⠻⠻ ⠨⠅ ⠣⠒⠜⠨⠜⠂⠖⠨⠻⠻

≫ $\sqrt{\sqrt[3]{\sqrt[4]{10}}}$

⠜⠨⠣⠒⠜⠨⠨⠣⠲⠜⠂⠴⠨⠨⠻⠨⠻⠻

8.16 Radical Expressions and the Baseline Indicator

When a radical expression is on the baseline level, assure that the components of the construction (the radical symbol or the indicators) are notated on the baseline of writing. Place the baseline indicator before the component when it follows a superscript or a subscript.

≫ $\left(r^2\sqrt{r}\right)^2$ ⠷⠗⠘⠆⠐⠜⠗⠻⠾⠘⠆

≫ $\sqrt{x^2+y^2}$ ⠜⠭⠘⠆⠐⠬⠽⠘⠆⠐⠻

The baseline indicator is not used after a numeric subscript that does not require a subscript indicator.

⋙ $\sqrt{x_1+y_2}$ ⠜⠭⠂⠬⠽⠆⠻

8.17 Radical Expressions and the Ellipsis and Long Dash

When an ellipsis or a long dash occurs within a radical, no space should be left between the ellipsis or long dash and the termination indicator. However, a space must be left between the radical sign and an ellipsis or long dash.

⋙ $\sqrt{a+b+c+\cdots}$

⠜⠁⠬⠃⠬⠉⠬ ⠄⠄⠄⠻

⋙ $\sqrt{x+\sqrt{x+\sqrt{\ldots}}}$

⠜⠭⠬⠨⠜⠭⠬⠨⠨⠜ ⠄⠄⠄⠨⠨⠻⠨⠻⠻

A space is required between a radical expression and an ellipsis or long dash preceding or following the expression.

⋙ $\sqrt{4}$ ... $\sqrt{64}$ ⠜⠲⠻ ⠄⠄⠄ ⠜⠖⠲⠻

8.18 Radical Expressions and Abbreviations

When an abbreviation occurs within a radical, no space is left between the abbreviation and the termination or order-of-radical indicator following it.

⋙ $\sqrt{9\text{ ft}}$ ⠜⠔ ⠋⠞⠻

However, a space is required between a radical sign and an abbreviation.

⋙ $\sqrt{\text{ft.}}$ ⠜ ⠋⠞⠲⠻

A space is also required between a radical expression and an abbreviation preceding or following the expression.

⋙ $2\sqrt{12}$ sq. in. ⠼⠆⠜⠂⠆⠻ ⠎⠟⠲ ⠊⠝⠲

8.19 Enclosed Lists with Radical Expressions

Radical expressions may be part of an enclosed list.

⋙ $(\sqrt{9},\ 3,\ \sqrt{4},\ 2\sqrt{6})$

⠷⠜⠔⠻⠠ ⠒⠠ ⠜⠲⠻⠠ ⠆⠜⠖⠻⠾

PRACTICE 8G

Nested Radical Expressions

(1) $\sqrt{-\frac{1}{2}-i\frac{\sqrt{3}}{2}}$

(2) $\sqrt{\sqrt{13}+\sqrt{15}+\sqrt{117}}$

(3) $\sqrt{1-\sqrt{a-b}}\times\sqrt{1+\sqrt{a-b}}$

(4) $\sqrt{b\sqrt[3]{b\sqrt{b}}}$

(5) $\sqrt[a]{\sqrt[b]{\sqrt[c]{abc}}}$

(6) $\sqrt{a^2}\sqrt{b^4}\sqrt{c} = ab^2\sqrt{c}$

(7) $\left(s^2\sqrt[3]{s^4}\right)^2$

(8) $\sqrt[3]{x^2\sqrt{64x^6}}$

(9) $\sqrt{\sqrt[3]{\sqrt[4]{\sqrt[5]{b^{48}}}}}$

(10) $\sqrt{x_1+\sqrt{x_2}}$

(11) $q^{\sqrt{r}}+s$

(12) $\sqrt{c+d+e+\cdots}$

LINKED EXPRESSIONS

[NC 26.5]

8.20 Definition of Linked Expression

A linked expression contains at least one sign of comparison. The part preceding the first sign of comparison is called the *anchor*. Each remaining part, beginning with a sign of comparison and ending before the next sign of comparison, is called a *link*. In its simplest form, $x = y$ is a linked expression where x is the anchor and $= y$ is the link.

$12.5\% > \frac{1}{10}$

The anchor is 12.5% and the link is $> \frac{1}{10}$

$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$

The anchor is 6×245, *followed by three links each beginning with an equals sign.*

8.21 Division of Linked Expressions

Recall that a mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. A linked expression is considered to be one expression. If the anchor and its link will fit on one line, do not divide it. When there is more than one link, if the anchor and all of its links will fit on one line, do so. However, it is often the case that an anchor and more than one link will not fit on one line. There are rules to follow, which result in smoother reading when a mathematical expression must be divided.

(1) If the entire embedded linked expression will fit on one braille line, do not divide it. See Example 8-31.

(2) If an anchor and its link not fit on one braille line, begin the runover line with the link. See Example 8-32.

(3) If the expression contains more than one link, use as much of the line as possible before dividing. It is not necessary to divide at every link. See Example 8-33.

(4) If the anchor or a link will not fit on one braille line, further rules apply. Those rules will be covered in Lesson 15.

(5) If the linked expression is printed in the nested layout discussed in the next section, other rules apply. (See 8.22.)

Example 8-31

Follow the steps. $12.5\% = .125 = \frac{125}{1000} = \frac{1}{8}$.

  ⠠⠋⠕⠇⠇⠪ ⠮ ⠌⠑⠏⠎⠲ ⠸⠩
⠼⠂⠆⠨⠢⠈⠴ ⠨⠅ ⠼⠨⠂⠆⠢ ⠨⠅ ⠹⠂⠆⠢⠌⠂⠴⠴⠴⠼ ⠨⠅ ⠹⠂⠌⠦⠼
⠸⠱⠲

Although the anchor and the first link will fit on line 1, since the entire linked expression fits on one line, the anchor begins on line 2.

Example 8-32

How many hours? 3/8 of a day + 1/2 of a day = ___ hours. (*Hint*: A day is 24 hours.)

```
  ⠠⠓⠪ ⠸⠍ ⠓⠳⠗⠎⠦
⠸⠩ ⠼⠒⠸⠌⠦ ⠕⠋ ⠁ ⠙⠁⠽⠬⠂⠸⠌⠆ ⠕⠋ ⠁ ⠙⠁⠽
⠨⠅ ⠤⠤⠤⠤ ⠓⠕⠥⠗⠎ ⠸⠱ ⠐⠣⠨⠂⠠⠓⠔⠞⠒ ⠠⠁ ⠐⠙ ⠊⠎ ⠼⠃⠙
⠓⠳⠗⠎⠲⠐⠜
```

This is a narrative paragraph (3-1) with an embedded equation. The opening switch indicator will fit on the line with the anchor, in the runover position, cell 1. The link begins on the next line with its comparison sign.

Example 8-33

Break the problem down into parts. Can you use mental math?

$$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$$

```
⠼⠁⠲ ⠠⠃⠗⠂⠅ ⠮ ⠏⠗⠕⠃⠇⠑⠍ ⠙⠪⠝ ⠔⠞⠕ ⠐⠏⠎⠲ ⠠⠉ ⠽
  ⠥⠎⠑ ⠍⠢⠞⠁⠇ ⠍⠁⠹⠦ ⠸⠩
    ⠼⠖⠈⠡⠆⠲⠢ ⠨⠅ ⠷⠖⠈⠡⠆⠴⠴⠾⠬⠷⠖⠈⠡⠲⠴⠾⠬⠷⠖⠈⠡⠢⠾
      ⠨⠅ ⠼⠂⠆⠴⠴⠬⠆⠲⠴⠬⠒⠴ ⠨⠅ ⠼⠂⠲⠶⠴ ⠸⠱
```

The anchor starts in cell 5, displayed to itemized text. The first link fits on the same line. The second link starts on a new line in cell 7, the runover cell in this displayed format. The third link fits on this line as well.

8.21.1 **Restrictions.** In order to divide a long expression before a comparison sign, the comparison sign must be on the baseline of writing. Furthermore, do not divide before a comparison sign that is part of an item enclosed in grouping symbols, between fraction indicators, or within radical signs. This topic will be explored in Lesson 15.

8.21.2 **Other Considerations.** A transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines.

➢ $\left(\frac{a^2}{b}\right)^2 \div \left(\frac{b^2}{a}\right)^3 = \frac{a^7}{b^8}$

```
⠷⠹⠁⠘⠆⠐⠌⠃⠼⠾⠘⠆⠐⠨⠌⠷⠹⠃⠘⠆⠐⠌⠁⠼⠾⠘⠒
  ⠨⠅ ⠹⠁⠘⠶⠐⠌⠃⠘⠦⠐⠼
```

A return to the baseline after the superscript "3" is triggered by the presence of the following comparison sign even though it is on the next line.

In order to save a line, if the opening Nemeth Code indicator will not fit on the same line as the preceding text it may be placed in front of the anchor in a displayed linked expression even if the entire linked expression will not fit on one line. Compare two transcriptions of the next example. Either transcription is acceptable.

Example 8-34

1. This simple linear equation has 3 unknowns.

$$\frac{x}{2}+\frac{1}{3}\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{6}\right)=\frac{1}{2}(x+y+z)$$

```
⠼⠁⠲ ⠠⠹ ⠎⠊⠍⠏⠇⠑ ⠇⠔⠑⠜ ⠑⠟⠥⠁⠰⠝ ⠓⠁⠎ ⠼⠉ ⠥⠝⠐⠅⠝⠎⠲
  ⠸⠩
    ⠹⠭⠌⠆⠼⠬⠹⠂⠌⠒⠼⠷⠹⠭⠌⠆⠼⠬⠹⠽⠌⠒⠼⠬⠹⠵⠌⠖⠼⠾
      ⠨⠅ ⠹⠂⠌⠆⠼⠷⠭⠬⠽⠬⠵⠾ ⠸⠱
```

The opening switch does not fit at the end of the line of text. It is placed in the runover position for itemized material, cell 3. The displayed expression begins on the next line in cell 5. The link begins in the runover position on the following line (cell 7).

Alternate Transcription

```
⠼⠁⠲ ⠠⠹ ⠎⠊⠍⠏⠇⠑ ⠇⠔⠑⠜ ⠑⠟⠥⠁⠰⠝ ⠓⠁⠎ ⠼⠉ ⠥⠝⠐⠅⠝⠎⠲
    ⠸⠩ ⠹⠭⠌⠆⠼⠬⠹⠂⠌⠒⠼⠷⠹⠭⠌⠆⠼⠬⠹⠽⠌⠒⠼⠬⠹⠵⠌⠖⠼⠾
      ⠨⠅ ⠹⠂⠌⠆⠼⠷⠭⠬⠽⠬⠵⠾ ⠸⠱
```

This transcription begins the displayed material with the opening switch, even though the expression requires a runover.

PRACTICE 8H

Linked Expressions

1. Is the following inequality true? $\frac{3}{5}\left(\frac{2}{3}x-\frac{1}{2}\right)>\frac{2}{5}\left(\frac{1}{4}x+\frac{1}{3}\right)$
2. $33\frac{1}{3}\%+40\%+61\frac{2}{3}\%=134\frac{3}{3}\%=135\%$
3. In multiplying $5\frac{3}{4}\times 46$, recall that $5\frac{3}{4}=5+\frac{3}{4}$. Therefore, $46\times\left(5+\frac{3}{4}\right)=$ $(46\times 5)+\left(46\times\frac{3}{4}\right)=264\frac{1}{2}$.

8.22 Special Case—Nested Linked Expressions [NC 26.5.3.b]

An expression with two or more links may be subject to special Nemeth format rules if it appears in print in a particular arrangement as described in this section.

Print Layout

- The expression is displayed.
  - There is an exception regarding itemized problems – see b, below.
- The first line contains only the anchor or only the anchor and the first link.
- Each following link begins on a new line, and the comparison signs beginning each link are vertically aligned.
  - An exception applies to the last line – see a, below.

The following linked expression meets the three requirements.

> To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.
>
> $$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c). \end{aligned}$$

a. It is common for the last line of the expression to contain more than one link. As long as the other conditions are met, this layout meets the requirements for this nested format.

> We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:
>
> $$\begin{aligned} 12\tfrac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \frac{125}{1000} = \frac{1}{8} \end{aligned}$$

b. The next example shows an itemized problem with no narrative. Although the expression is not displayed, this arrangement follows all of the other print layout specifications: the comparison signs are vertically aligned, and – other than on the first and last line – no sign of comparison is preceded by any expression on its left. Rules regarding this layout are outlined in Section 8.22.4.

1. $$\begin{aligned} 12\tfrac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \tfrac{125}{1000} = \tfrac{1}{8} \end{aligned}$$

When the print layout meets the definition of a nested linked expression, one of the following Nemeth formats is applied.

8.22.1 **Margin Requirements for a Nested Linked Expression.** The margins which are applied to a nested linked expression follow a reliable pattern, which can be generalized as follows.

- The anchor begins two cells to the right of the runover margin of the material to which it is displayed.
- Each link that starts on a new line begins two cells to the right of the anchor cell.
- Runovers to anchor or links begin four cells to the right of the anchor cell.

Note: Rules regarding how to divide a link that will not fit on the line will be discussed in Lesson 15. In this lesson, in order to illustrate runovers within a nested linked expression, a runover line will begin with a sign of operation.

8.22.2 **Nested Linked Expression Displayed to Narrative.** When a nested linked expression occurs in unitemized explanatory portions of the text (3-1), the anchor begins in cell 3 and each link begins in cell 5. In braille, each link begins on a new line, even when the print copy shows more than one link on the last line.

Reminder: A line is not skipped above or below displayed mathematical material unless a blank line is required under other rules or guidelines.

Example 8-35

To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c)\,. \end{aligned}$$

```
  ⠠⠞⠕ ⠋⠁⠉⠞⠕⠗ ⠸⠩ ⠁⠃⠬⠉⠘⠆⠐⠬⠁⠉⠬⠃⠉ ⠸⠱⠂ ⠮ ⠞⠻⠍⠎
⠉ ⠆ ⠛⠗⠳⠏⠫ ⠔ ⠏⠁⠊⠗⠎ ⠾ ⠁ ⠉⠕⠍⠍⠕⠝ ⠋⠁⠉⠞⠕⠗⠲ ⠸⠩
  ⠁⠃⠬⠉⠘⠆⠐⠬⠁⠉⠬⠃⠉
    ⠨⠅ ⠷⠁⠃⠬⠁⠉⠾⠬⠷⠃⠉⠬⠉⠘⠆⠐⠾
    ⠨⠅ ⠁⠷⠃⠬⠉⠾⠬⠉⠷⠃⠬⠉⠾
    ⠨⠅ ⠷⠁⠬⠉⠾⠷⠃⠬⠉⠾ ⠸⠱⠲
```

Lines 1-2: Narrative paragraph (3-1). The opening switch indicator is placed at the end of the narrative.
Lines 3-6: Nested linked expression.
Line 3: The *anchor is in cell 3 (two cells to the right of the runover cell of the preceding material).*
Lines 4-6: Each link begins in cell 5 (two cells to the right of the anchor), regardless of the amount of available space on the preceding line.

Example 8-36

We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

$$\begin{aligned} 12\tfrac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \tfrac{125}{1000} = \tfrac{5}{40} = \tfrac{1}{8} \end{aligned}$$

```
  ⠠⠺⠑ ⠉ ⠗⠫⠥⠉⠑ ⠸⠩ ⠼⠂⠆⠸⠹⠂⠌⠆⠸⠼⠈⠴ ⠸⠱ ⠞⠕ ⠇⠪⠑⠌
⠞⠻⠍⠎ ⠔ ⠮ ⠋⠕⠇⠇⠪⠬ ⠺⠁⠽⠒ ⠸⠩
  ⠼⠂⠆⠸⠹⠂⠌⠆⠸⠼⠈⠴
    ⠨⠅ ⠼⠂⠆⠨⠢⠈⠴
    ⠨⠅ ⠼⠨⠂⠆⠢
    ⠨⠅ ⠹⠂⠆⠢⠌⠂⠴⠴⠴⠼
    ⠨⠅ ⠹⠢⠌⠲⠴⠼ ⠸⠱
    ⠨⠅ ⠹⠂⠌⠦⠼ ⠸⠱
```

Lines 1-2: Narrative paragraph (3-1). The opening switch indicator is placed at the end of the narrative.
Lines 3-7: Nested linked expression.
Line 3: T*he anchor is in cell 3 (two cells to the right of the runover cell of the preceding material).*
Lines 4-8: Each link begins in cell 5 (two cells to the right of the anchor), regardless of the amount of available space on the preceding line and despite the fact that the last line in print shows three links.

a. **Runovers Within a Nested Linked Expression.** If the anchor or a link will not fit on the current line, the examples in this lesson divide the expression before a sign of operation. The runover line is indented four cells from the anchor, or two cells from the link. In this case, that is cell 7. Anchor: 3-7; link: 5-7.

Example 8-37

We can find the product of 6 and 44,444 by applying the distributive principle:

$$\begin{aligned} 6 \times 44{,}444 &= 6 \times (40{,}000 + 4{,}000 + 400 + 40 + 4) \\ &= (6 \times 40{,}000) + (6 \times 4{,}000) + (6 \times 400) + (6 \times 40) + (6 \times 4) \\ &= 240{,}000 + 24{,}000 + 2{,}400 + 240 + 24 \\ &= 266{,}664 \end{aligned}$$

```
1    ⠠⠺⠑ ⠉ ⠋⠔⠙ ⠮ ⠏⠗⠕⠙⠥⠉⠞ ⠷ ⠼⠋ ⠯ ⠼⠙⠂⠙⠙⠙ ⠃⠽
2  ⠁⠏⠏⠇⠽⠬ ⠮ ⠲⠞⠗⠊⠃⠥⠞⠊⠧⠑ ⠏⠗⠔⠉⠊⠏⠇⠑⠒ ⠸⠩
3    ⠼⠖⠈⠡⠲⠲⠠⠲⠲⠲
4        ⠨⠅ ⠼⠖⠈⠡⠷⠲⠴⠠⠴⠴⠴⠬⠲⠠⠴⠴⠴⠬⠲⠴⠴⠬⠲⠴⠬⠲⠾
5        ⠨⠅ ⠷⠖⠈⠡⠲⠴⠠⠴⠴⠴⠾⠬⠷⠖⠈⠡⠲⠠⠴⠴⠴⠾⠬⠷⠖⠈⠡⠲⠴⠴⠾
6          ⠬⠷⠖⠈⠡⠲⠴⠾⠬⠷⠖⠈⠡⠲⠾
7        ⠨⠅ ⠼⠆⠲⠴⠠⠴⠴⠴⠬⠆⠲⠠⠴⠴⠴⠬⠆⠠⠲⠴⠴⠬⠆⠲⠴⠬⠆⠲
8        ⠨⠅ ⠼⠆⠖⠖⠠⠖⠖⠲ ⠸⠱
```

Lines 1-2: Narrative paragraph (3-1).
Lines 3-8: Nested linked expression.
Line 3: The anchor is in cell 3.
Lines 4, 5, 7, 8: Each link begins in cell 5.
Line 6: The second link requires a runover. The runover begins in cell 7. The runover begins with a plus sign outside of the grouped expresssions.

PRACTICE 8I

Recognizing a Nested Linked Expression

To test the equation $R_t = \frac{R}{n}$, use four 1000-Ω resistors wired in series to predict a total resistance of 250 Ω .

$$R_t = \frac{R}{n} = \frac{1000\ \Omega}{4}$$

$$\frac{1000\ \Omega}{4} = 250\ \Omega$$

Then, by using the resistance theory equation

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n},$$

plug the 150-Ω and 1000-Ω resistors into the equation as R_1 and R_2.

$$\begin{aligned} \frac{1}{R_t} &= \frac{1}{150\ \Omega} + \frac{1}{1000\ \Omega} \\ &= 0.007 + 0.001 \\ &= 0.008 \end{aligned}$$

$$R_t = \frac{1}{0.008} = 125\ \Omega$$

8.22.3 **Nested Linked Expression Displayed to Itemized Material.** Apply the general pattern when a nested linked expression is displayed to itemized material: begin the anchor two cells to the right of the current runover margin; begin each two cells to the right of the anchor cell; begin runovers four cells to the right of the anchor cell.

a. **Itemized Text with No Subdivisions.** When a nested linked expression occurs in itemized text containing no subdivisions (1-3), the anchor begins in cell 5 and each link begins in cell 7. Runovers begin in cell 9.

b. **Itemized Text with Subdivisions.** When a nested linked expression occurs in itemized text containing subdivisions (1-5; 3-5), the anchor begins in cell 7 and each link begins in cell 9. Runovers begin in cell 11.

Example 8-38

1. The example below shows a way of finding 6 × 245 .

$$\begin{aligned} 6 \times 245 &= (6 \times 200) + (6 \times 40) + (6 \times 5) \\ &= 1200 + 240 + 30 \\ &= 1470 \end{aligned}$$

Is there another way to find 6 × 245 ?

```
1  ⠼⠁⠲ ⠠⠮ ⠑⠭⠁⠍⠏⠇⠑ ⠆⠇ ⠩⠪⠎ ⠁ ⠺⠁⠽ ⠷ ⠋⠔⠙⠬
2    ⠸⠩ ⠼⠖⠈⠡⠆⠲⠢⠸⠲
3      ⠼⠖⠈⠡⠆⠲⠢
4        ⠨⠅ ⠷⠖⠈⠡⠆⠴⠴⠾⠬⠷⠖⠈⠡⠲⠴⠾⠬⠷⠖⠈⠡⠢⠾
5        ⠨⠅ ⠼⠂⠆⠴⠴⠬⠆⠲⠴⠬⠒⠴
6        ⠨⠅ ⠼⠂⠲⠶⠴ ⠸⠱
7    ⠠⠊⠎ ⠐⠮ ⠁⠝⠕⠮⠗ ⠺⠁⠽ ⠞⠕ ⠋⠔⠙ ⠸⠩ ⠼⠖⠈⠡⠆⠲⠢ ⠸⠱⠦
```

Observation: The nested linked expression is displayed to 1-3 itemized text.
Lines 1-2: Itemized text with no subdivisions. (1-3)
Lines 3-6: Nested linked expression.
Line 3: The anchor is in cell 5.
Lines 4-6: Each link begins in cell 7, regardless of the amount of available space on the preceding line.
Line 7: Text continues in the runover cell to itemized text, cell 3.

Example 8-39

2. What is the function of the parentheses in the following problem?

$$\begin{aligned} &(4 \times 10{,}000) + (4 \times 1{,}000) + (4 \times 100) + (4 \times 10) + (4 \times 1) \\ &\qquad = 40{,}000 + 4{,}000 + 400 + 40 + 4 \\ &\qquad = 44{,}444 \end{aligned}$$

```
1  ⠼⠃⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠋⠥⠝⠉⠰⠝ ⠷ ⠮ ⠏⠜⠢⠮⠎⠑⠎ ⠔ ⠮
2    ⠋⠕⠇⠇⠪⠬ ⠏⠗⠕⠃⠇⠑⠍⠦ ⠸⠩
3      ⠷⠲⠈⠡⠂⠴⠠⠴⠴⠴⠾⠬⠷⠲⠈⠡⠂⠠⠴⠴⠴⠾⠬⠷⠲⠈⠡⠂⠴⠴⠾
4          ⠬⠷⠲⠈⠡⠂⠴⠾⠬⠷⠲⠈⠡⠂⠾
5        ⠨⠅ ⠼⠲⠴⠠⠴⠴⠴⠬⠲⠠⠴⠴⠴⠬⠲⠴⠴⠬⠲⠴⠬⠲
6        ⠨⠅ ⠼⠲⠲⠠⠲⠲⠲ ⠸⠱
```

Observation: The anchor has a runover.
Lines 1-2: Itemized text with no subdivisions. (1-3)
Line 2: The opening switch indicator is placed at the end of the narrative.

Lines 3-6: Nested linked expression.
Line 3: The anchor is in cell 5.
Line 4: The anchor requires a runover. The runover begins in cell 9. The runover begins with a plus sign outside of the grouped expressions.
Lines 5 and 6: Each link begins in cell 7.

Example 8-40

2. Factor $2a(b-c)-3x(c-b)$.

   (a) Factors $b-c$ and $c-b$ are divisible by $(b-c)$ since

   $$c-b=(-1)(b-c).$$

   (b) Thus

   $$\begin{aligned} 2a(b-c)-3x(c-b) &= 2a(b-c)+3x(b-c) \\ &= (2a+3x)(b-c). \end{aligned}$$

```
1  ⠼⠃⠲ ⠠⠋⠁⠉⠞⠕⠗ ⠸⠩ ⠼⠆⠁⠷⠃⠤⠉⠾⠤⠒⠭⠷⠉⠤⠃⠾⠸⠲
2    ⠷⠁⠾ ⠠⠄⠠⠋⠁⠉⠞⠕⠗⠎ ⠃⠤⠉ ⠠⠄⠯ ⠉⠤⠃ ⠸⠱ ⠜⠑
3      ⠙⠊⠧⠊⠎⠊⠃⠇⠑ ⠃⠽ ⠸⠩ ⠷⠃⠤⠉⠾ ⠠⠄⠎⠔⠉⠑
4        ⠉⠤⠃ ⠨⠅ ⠷⠤⠂⠾⠷⠃⠤⠉⠾⠸⠲
5    ⠷⠃⠾ ⠠⠄⠠⠹⠥⠎
6        ⠼⠆⠁⠷⠃⠤⠉⠾⠤⠒⠭⠷⠉⠤⠃⠾
7          ⠨⠅ ⠼⠆⠁⠷⠃⠤⠉⠾⠬⠒⠭⠷⠃⠤⠉⠾
8          ⠨⠅ ⠷⠆⠁⠬⠒⠭⠾⠷⠃⠤⠉⠾ ⠸⠱⠲
```

Observation: The nested linked expression is displayed to a 3-5 subdivision.
Line 1: Itemized text begins in cell 1.
Lines 2-3: Subdivision (3-5).
Line 4: Displayed math begins in cell 7. This is not a nested linked expression. The link continues on the same line as the anchor.
Line 5: The next subdivision begins in cell 3.
Lines 6-8: Nested linked expression.
Line 6: The anchor is in cell 7 (two cells to the right of the runover cell of the preceding material).
Lines 7-8: Each link begins in cell 9 (two cells to the right of the anchor), regardless of the amount of available space on the preceding line.

Example 8-41

7. Approximating the value of *e*.

   a. Approximate *e* using two terms.

$$\begin{aligned} e &= 1 + \tfrac{1}{1} + \tfrac{1}{12} + \tfrac{1}{123} + \tfrac{1}{1234} + \cdots \\ &= ? \end{aligned}$$

```
1 ⠼⠛⠲ ⠠⠁⠏⠏⠗⠕⠭⠊⠍⠁⠞⠬ ⠮ ⠧⠁⠇⠥⠑ ⠷ ⠰⠑⠲
2   ⠁⠲ ⠠⠁⠏⠏⠗⠕⠭⠊⠍⠁⠞⠑ ⠰⠑ ⠥⠎⠬ ⠞⠺⠕ ⠞⠻⠍⠎⠲ ⠸⠩
3       ⠑
4         ⠨⠅ ⠼⠂⠬⠹⠂⠌⠂⠼⠬⠹⠂⠌⠂⠡⠆⠼⠬⠹⠂⠌⠂⠡⠆⠡⠒⠼
5           ⠬⠹⠂⠌⠂⠡⠆⠡⠒⠡⠲⠼⠬ ⠄⠄⠄
6         ⠨⠅ ⠿ ⠸⠱
```

Observation: The first link has a runover.
Line 1: Itemized text begins in cell 1.
Line 2: Subdivision begins in cell 3.
Lines 3-6: Nested linked expression.
Line 3: The anchor is in cell 7 (two cells to the right of the runover cell of the preceding material).
Lines 4 and 6: Each link begins in cell 9 (two cells to the right of the anchor).
Line 5: The first link requires a runover. The runover begins in cell 11 (four cells to the right of the anchor). The runover begins with a plus sign.

8.22.4 **Itemized Nested Linked Expression with No Narrative.** When a nested linked expression is itemized and immediately follow the identifier, transcribe the anchor on the same line as the identifier. Each link begins a new line, with the comparison sign two cells to the right of the cell in which the identifier begins. If the anchor or any link requires a runover, indent two cells further—that is, four cells to the right of the cell in which the identifier begins.

Example 8-42

Approximate the value of e.

1. $e = 1 + \frac{1}{1} + \frac{1}{12} + \frac{1}{123} + \frac{1}{1234} + \cdots$

   $= ?$

2. ...

```
1        ⠠⠁⠏⠏⠗⠕⠭⠊⠍⠁⠞⠑ ⠮ ⠧⠁⠇⠥⠑ ⠷ ⠰⠑⠲ ⠸⠩
2    ⠼⠂⠸⠲ ⠑
3      ⠨⠅ ⠼⠂⠬⠹⠂⠌⠂⠼⠬⠹⠂⠌⠂⠡⠆⠼⠬⠹⠂⠌⠂⠡⠆⠡⠒⠼
4        ⠬⠹⠂⠌⠂⠡⠆⠡⠒⠡⠲⠼⠬ ⠄⠄⠄
5      ⠨⠅ ⠿
6    ⠼⠆⠸⠲ ⠄⠄⠄ ⠸⠱
```

Line 1: Instructions begin in cell 5.
Line 2: The identifier begins in cell 1. Because there is no narrative, the anchor begins on this line.
Lines 3 and 5: Each link begins in cell 3 (two cells to the right of the cell in which the identifier begins).
Line 4: The first link requires a runover. The runover begins in cell 5 (four cells to the right of the cell in which the identifier begins). The runover begins with a plus sign.

PRACTICE 8J

Multiplying Mixed Numbers

A. $2\frac{1}{2} \cdot 1\frac{1}{4} = \left(2+\frac{1}{2}\right) \cdot \left(1+\frac{1}{4}\right)$

$= 2+\frac{2}{4}+\frac{1}{2}+\frac{1}{8}$

$= 2+\frac{1}{2}+\frac{1}{2}+\frac{1}{8}$

$= 2+1+\frac{1}{8} = 3\frac{1}{8}$

B. What will the remainder be in this problem?

$4\frac{1}{3} \times 3\frac{2}{5} \times 9\frac{11}{15} \times 2\frac{3}{4}$

$= \frac{13}{3} \times \frac{17}{5} \times \frac{146}{15} \times \frac{11}{4}$

$= \frac{13\times17\times146\times11}{3\times5\times15\times4}$

$= \frac{354{,}926}{900}$

= 394 with a remainder of ____.

For further practice, see Appendix A—Reading Practice.

EXERCISE 8

Prepare Exercise 8 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 8A

```
    ⠠⠓⠕⠗⠊⠵⠕⠝⠞⠁⠇ ⠠⠎⠊⠍⠏⠇⠑ ⠠⠋⠗⠁⠉⠰⠝ ⠠⠇⠔⠑
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
  ⠠⠐⠓ ⠜⠑ ⠐⠎ ⠑⠭⠁⠍⠏⠇⠑⠎ ⠷ ⠎⠊⠍⠏⠇⠑ ⠋⠗⠁⠉⠰⠝⠎⠲
⠸⠩ ⠹⠂⠌⠆⠼ ⠹⠂⠢⠌⠂⠖⠼ ⠹⠭⠌⠽⠼ ⠹⠁⠬⠃⠌⠉⠬⠙⠼
⠹⠨⠠⠙⠽⠌⠨⠠⠙⠭⠼ ⠹⠷⠭⠬⠽⠾⠌⠷⠭⠤⠽⠾⠼ ⠹⠔⠌⠂⠆⠼
⠷⠹⠒⠌⠆⠼⠁⠬⠹⠂⠌⠆⠼⠃⠾ ⠹⠒⠭⠌⠂⠶⠽⠼ ⠭⠤⠹⠂⠌⠲⠼⠷⠭⠤⠆⠭⠾
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠼⠂⠸⠲ ⠠⠧ ⠨⠅ ⠹⠂⠌⠒⠼⠨⠏⠗⠘⠆⠐⠓
⠼⠆⠸⠲ ⠹⠁⠌⠃⠼⠈⠡⠹⠉⠌⠙⠼ ⠨⠅ ⠹⠁⠉⠌⠃⠙⠼
⠼⠒⠸⠲ ⠱⠹⠁⠌⠃⠼⠱ ⠨⠅ ⠹⠱⠁⠱⠌⠱⠃⠱⠼ ⠸⠳
⠼⠙⠲ ⠠⠺⠗⠊⠞⠑ ⠁⠝ ⠑⠟⠥⠁⠰⠝ ⠞⠕ ⠩⠪ ⠞ ⠸⠩ ⠹⠒⠌⠲⠼
  ⠠⠄⠷ ⠹⠂⠌⠆⠼ ⠠⠄⠊⠎ ⠹⠒⠌⠦⠼⠸⠲
⠼⠢⠸⠲
  ⠭⠘⠆⠐⠹⠙⠭⠌⠙⠭⠼ ⠨⠅ ⠹⠲⠭⠘⠆⠐⠤⠭⠤⠆⠌⠷⠭⠬⠂⠾⠷⠽⠤⠂⠾⠼
  ⠸⠳
⠼⠋⠲ ⠠⠎⠕⠇⠧⠑ ⠹ ⠙⠊⠖⠻⠢⠞⠊⠁⠇ ⠑⠟⠥⠁⠰⠝⠒
    ⠸⠩ ⠭⠹⠙⠽⠌⠙⠭⠼⠬⠆⠽ ⠨⠅ ⠑⠘⠭⠘⠘⠆ ⠸⠳
⠼⠛⠲ ⠠⠮ ⠝⠥⠍⠃⠻ ⠸⠩ ⠨⠏ ⠸⠳ ⠊⠎ ⠮ ⠗⠁⠞⠊⠕ ⠷ ⠮
  ⠉⠊⠗⠉⠥⠍⠋⠻⠰⠑ ⠷ ⠁ ⠉⠊⠗⠉⠇⠑ ⠞⠕ ⠭⠎ ⠙⠊⠁⠍⠑⠞⠻⠲
  ⠠⠞ ⠊⠎⠂
    ⠸⠩ ⠨⠏ ⠨⠅ ⠹⠉⠊⠗⠉⠥⠍⠋⠑⠗⠑⠝⠉⠑⠌⠙⠊⠁⠍⠑⠞⠑⠗⠼⠸⠲
⠼⠦⠸⠲ ⠹⠂⠆⠌⠒⠒⠼ ⠨⠅ ⠹⠍⠌⠂⠂⠼
⠼⠔⠸⠲ ⠹⠲⠌⠒⠆⠼ ⠨⠅ ⠹⠂⠴⠨⠢⠌⠭⠼
⠼⠂⠴⠸⠲ ⠹⠂⠌⠲⠼⠬⠹⠒⠌⠲⠼⠤⠹⠂⠌⠆⠼ ⠨⠅ ⠹⠂⠌⠆⠼ ⠸⠳
```

PRACTICE 8B

```
1         ⠠⠙⠊⠁⠛⠕⠝⠁⠇ ⠠⠎⠊⠍⠏⠇⠑ ⠠⠋⠗⠁⠉⠰⠝ ⠠⠇⠔⠑
2
3   ⠠⠁⠐⠜ ⠠⠓⠪ ⠸⠍ ⠸⠩ ⠹⠆⠸⠌⠒⠼⠸⠄⠎ ⠸⠱ ⠜⠑ ⠐⠮ ⠔
4     ⠸⠩ ⠹⠢⠸⠌⠖⠼ ⠸⠱⠦
5   ⠰⠠⠃⠐⠜ ⠠⠢⠻⠛⠽ ⠊⠎ ⠁⠃⠎⠕⠗⠃⠫ ⠁⠞ ⠮ ⠗⠁⠞⠑ ⠷
6     ⠸⠩ ⠼⠦⠦⠴ ⠰⠠⠚⠸⠌⠰⠎ ⠸⠱ ⠿ ⠑⠁⠡ ⠎⠟⠥⠜⠑ ⠍⠑⠞⠻ ⠷
7     ⠮ ⠎⠥⠗⠋⠁⠉⠑⠲
8   ⠰⠠⠉⠐⠜ ⠸⠩ ⠽⠷⠴⠾ ⠨⠅ ⠨⠏⠸⠌⠲
9   ⠰⠠⠙⠾ ⠼⠂ ⠹⠋⠞⠸⠌⠎⠑⠉⠼ ⠈⠱⠈⠱ ⠼⠴⠨⠖⠦⠂⠦ ⠍⠏⠓
10  ⠰⠠⠑⠾ ⠠⠄⠠⠔ ⠽⠹⠂⠸⠌⠢⠼ ⠸⠱⠂ ⠰⠽ ⠊⠎ ⠮ ⠉⠕⠑⠖⠊⠉⠊⠢⠞
11    ⠷ ⠮ ⠋⠗⠁⠉⠰⠝ ⠸⠩ ⠼⠂⠸⠌⠢ ⠸⠱⠲
12  ⠰⠠⠋⠐⠜ ⠠⠞⠗⠥⠑⠸⠌⠠⠋⠁⠇⠎⠑⠒ ⠠⠮ ⠸⠩ ⠗⠊⠎⠑⠸⠌⠗⠥⠝ ⠸⠱
13    ⠗⠁⠞⠊⠕ ⠊⠎ ⠼⠑ ⠔ ⠛⠗⠁⠏⠓ ⠠⠁⠲
14  ⠰⠠⠛⠐⠜ ⠸⠩ ⠹⠁⠸⠌⠃⠼⠡⠹⠉⠸⠌⠙⠼ ⠨⠅ ⠹⠁⠉⠸⠌⠃⠙⠼ ⠸⠱
15  ⠰⠠⠓⠐⠜ ⠠⠁ ⠼⠑⠤⠽⠑⠜ ⠰⠠⠠⠉⠙ ⠺⠢⠞ ⠋ ⠑⠜⠝⠬ ⠔⠞⠻⠑⠌
16    ⠁⠞ ⠮ ⠗⠁⠞⠑ ⠷ ⠸⠩ ⠼⠂⠆⠨⠴⠖⠈⠴⠸⠌⠽⠑⠁⠗ ⠸⠱ ⠔
17    ⠼⠁⠊⠓⠙ ⠞⠕ ⠑⠜⠝⠬ ⠇⠑⠎⠎ ⠹⠁⠝
18    ⠸⠩ ⠼⠴⠨⠦⠶⠈⠴⠸⠌⠽⠑⠁⠗ ⠸⠱ ⠔ ⠼⠃⠚⠁⠑⠲
```

Line 3: The apostrophe-s is included inside the switch and so a punctuation indicator is needed for the apostrophe. Review 2.7 in Lesson 2.

Line 6: Because the slash means "per" ("Joules per second") a switch to Nemeth Code is required. The value ("880") is included in the switch. An English-letter indicator is required for each single-letter abbreviation.

Line 8: No fraction indicators are used because "pi" and "4" are of normal size and are printed on the baseline.

Line 12: The first slash is in literary context. A switch to Nemeth Code is required for the second slash because it means "over" ("rise over run") in a ratio.

PRACTICE 8C

```
              ⠠⠍⠊⠭⠫ ⠠⠝⠥⠍⠃⠻⠎
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠼⠁⠲ ⠠⠋⠔⠙ ⠮ ⠏⠗⠑⠍⠊⠥⠍ ⠿ ⠁
  ⠸⠩ ⠼⠂⠸⠹⠂⠸⠌⠆⠸⠼⠤⠽⠗⠲ ⠸⠱ ⠏⠕⠇⠊⠉⠽ ⠁⠞ ⠮ ⠽⠑⠜⠇⠽
  ⠗⠁⠞⠑ ⠷ ⠸⠩ ⠼⠆⠒⠈⠉ ⠠⠄⠏⠻ ⠈⠎⠂⠴⠴⠸⠲
⠼⠆⠸⠲ ⠼⠆⠸⠹⠂⠸⠌⠆⠸⠼ ⠋⠞ ⠬⠦ ⠊⠝ ⠨⠅ ⠿ ⠊⠝⠉⠓⠑⠎
⠼⠒⠸⠲ ⠷⠹⠂⠌⠆⠼⠈⠡⠒⠸⠹⠂⠌⠆⠸⠼⠾⠬⠷⠒⠸⠹⠂⠌⠆⠸⠼⠈⠡⠆⠾
⠼⠲⠸⠲
  ⠼⠂⠒⠸⠹⠂⠸⠌⠆⠸⠼⠬⠆⠸⠹⠆⠸⠌⠒⠸⠼ ⠨⠅ ⠼⠂⠖⠸⠹⠂⠸⠌⠖⠸⠼
⠼⠢⠸⠲ ⠼⠶⠸⠌⠲ ⠨⠅ ⠼⠂⠸⠹⠿⠸⠌⠲⠸⠼
⠼⠖⠸⠲ ⠹⠔⠌⠲⠼ ⠨⠅ ⠼⠆⠹⠭⠌⠲⠼ ⠸⠱
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
             ⠠⠋⠗⠁⠉⠰⠝ ⠠⠗⠑⠧⠊⠑⠺
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
    ⠠⠉⠕⠍⠏⠥⠞⠑ ⠑⠁⠡ ⠘⠂⠥⠝⠊⠞ ⠘⠂⠗⠁⠞⠑
  ⠸⠩ ⠷⠏⠗⠊⠉⠑⠸⠌⠏⠕⠥⠝⠙⠾⠸⠲
⠰⠁⠸⠲ ⠈⠎⠂⠨⠢⠴ ⠠⠄⠿ ⠼⠆⠸⠌⠒ ⠸⠱ ⠏⠨⠙ ⠷ ⠏⠕⠞⠁⠞⠕⠑⠎
⠰⠃⠲ ⠸⠩ ⠈⠎⠒⠨⠆⠴ ⠠⠄⠿ ⠹⠂⠸⠌⠆⠼ ⠸⠱ ⠏⠨⠙ ⠷ ⠠⠫⠁⠍
  ⠡⠑⠑⠎⠑
⠰⠉⠲ ⠸⠩ ⠈⠎⠖⠨⠴⠴ ⠠⠄⠿ ⠹⠒⠌⠲⠼ ⠸⠱ ⠏⠨⠙ ⠷ ⠙⠑⠇⠊
  ⠎⠍⠕⠅⠫ ⠞⠥⠗⠅⠑⠽
⠰⠙⠲ ⠸⠩ ⠈⠎⠂⠆⠨⠢⠴ ⠠⠄⠿ ⠼⠂⠸⠹⠂⠸⠌⠆⠸⠼ ⠸⠱ ⠏⠨⠙ ⠷
  ⠎⠇⠊⠉⠫ ⠓⠁⠍
```

PRACTICE 8D

⠠⠉⠕⠍⠏⠇⠑⠭ ⠠⠋⠗⠁⠉⠰⠝⠎

⠀⠀

⠸⠩ ⠠⠹⠹⠂⠌⠦⠼⠬⠹⠒⠌⠲⠼⠠⠌⠶⠠⠼

⠠⠹⠹⠂⠌⠆⠼⠬⠹⠂⠌⠒⠼⠠⠌⠹⠒⠌⠲⠼⠤⠹⠶⠌⠔⠼⠠⠼

⠠⠹⠂⠸⠌⠒⠬⠂⠸⠌⠲⠠⠌⠲⠸⠌⠢⠤⠂⠸⠌⠆⠠⠼

⠠⠹⠹⠨⠏⠌⠦⠼⠠⠌⠆⠨⠏⠠⠼

⠠⠹⠁⠠⠌⠹⠃⠌⠉⠼⠠⠼

⠠⠹⠒⠒⠸⠹⠂⠌⠒⠸⠼⠠⠌⠂⠴⠴⠠⠼

⠠⠹⠒⠸⠌⠢⠠⠌⠖⠠⠼

⠠⠹⠹⠒⠌⠢⠼⠠⠸⠌⠹⠶⠌⠔⠼⠠⠼ ⠸⠱

PRACTICE 8E

⠠⠘⠮ ⠜⠑ ⠎⠊⠍⠏⠇⠑ ⠋⠗⠁⠉⠰⠝⠎⠒ ⠸⠩

⠹⠂⠌⠆⠼

⠹⠁⠘⠆⠐⠌⠃⠘⠆⠐⠼

⠹⠽⠘⠹⠂⠌⠆⠼⠐⠌⠽⠘⠹⠂⠌⠲⠼⠐⠼ ⠸⠱

⠠⠹ ⠊⠎ ⠸⠂⠝ ⠁ ⠎⠊⠍⠏⠇⠑ ⠋⠗⠁⠉⠰⠝⠒

⠸⠩ ⠠⠹⠂⠸⠌⠒⠠⠌⠆⠸⠌⠒⠠⠼ ⠸⠱

⠠⠗⠑⠧⠊⠑⠺ ⠮ ⠗⠥⠇⠑⠎ ⠔ ⠼⠋⠲⠁⠃⠲⠑ ⠗⠑⠛⠜⠙⠬ ⠁⠝

⠑⠇⠇⠊⠏⠎⠊⠎ ⠕⠝ ⠮ ⠃⠁⠎⠑⠇⠔⠑ ⠷ ⠺⠗⠊⠞⠬ ⠱⠢ ⠭

⠋⠕⠇⠇⠪⠎ ⠁ ⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞⠲ ⠸⠩

⠭⠘⠹⠂⠌⠆⠼⠐⠄⠄⠄ ⠭⠘⠹⠂⠌⠆⠼⠐⠡⠽⠘⠤⠹⠂⠌⠆⠼⠐⠄⠄⠄

⠹⠭⠘⠹⠂⠌⠆⠼⠐⠬⠂⠌⠽⠘⠹⠂⠌⠆⠼⠐⠤⠂⠼ ⠸⠱

⠠⠏⠇⠕⠞ ⠮ ⠏⠕⠔⠞⠎ ⠸⠩ ⠷⠤⠹⠂⠌⠆⠼⠠ ⠲⠾⠠

⠷⠒⠠ ⠲⠸⠹⠂⠌⠲⠸⠼⠾⠠ ⠠⠄⠯ ⠷⠤⠔⠠ ⠹⠒⠌⠲⠼⠾ ⠸⠱⠲ ⠠⠮⠝

⠑⠭⠏⠗⠑⠎⠎ ⠸⠩ ⠹⠁⠘⠒⠸⠌⠲⠐⠌⠃⠘⠢⠸⠌⠲⠐⠼ ⠸⠱ ⠔

⠗⠁⠙⠊⠉⠁⠇ ⠿⠍⠲

PRACTICE 8F

⠠⠗⠁⠙⠊⠉⠁⠇ ⠠⠑⠭⠏⠗⠑⠎⠨⠝⠎

⠸⠩

⠼⠂⠸⠲ ⠜⠹⠃⠌⠁⠼⠬⠹⠁⠌⠃⠼⠻

⠼⠆⠸⠲ ⠜⠉⠸⠌⠙⠻

⠼⠒⠸⠲ ⠹⠂⠌⠲⠼⠜⠹⠂⠌⠆⠼⠻

⠼⠲⠸⠲ ⠜⠹⠂⠴⠌⠂⠖⠼⠻ ⠨⠅ ⠜⠂⠴⠻⠸⠌⠲

⠼⠢⠸⠲ ⠷⠜⠒⠻⠬⠜⠢⠻⠾⠷⠜⠒⠻⠤⠜⠢⠻⠾

⠼⠖⠸⠲ ⠼⠆⠜⠆⠻⠬⠶⠜⠆⠻ ⠨⠅ ⠷⠆⠬⠶⠾⠜⠆⠻ ⠨⠅ ⠼⠔⠜⠆⠻

⠼⠶⠸⠲ ⠹⠜⠒⠻⠌⠜⠆⠻⠼⠈⠡⠹⠜⠢⠻⠌⠜⠆⠻⠼ ⠨⠅ ⠹⠜⠂⠢⠻⠌⠆⠼

⠼⠦⠸⠲ ⠠⠹⠜⠆⠻⠤⠜⠹⠂⠌⠒⠼⠻⠠⠌⠜⠒⠻⠤⠜⠹⠂⠌⠆⠼⠻⠠⠼

⠼⠔⠸⠲ ⠜⠲⠦⠭⠘⠒⠐⠽⠻

⠼⠂⠴⠸⠲ ⠜⠷⠽⠤⠂⠾⠬⠜⠷⠆⠽⠾ ⠨⠅ ⠼⠂

⠼⠂⠂⠸⠲ ⠣⠝⠜⠙⠻

⠼⠂⠆⠸⠲ ⠣⠁⠬⠃⠜⠵⠤⠽⠻

⠼⠂⠒⠸⠲ ⠣⠲⠜⠶⠆⠔⠻⠬⠣⠖⠜⠆⠶⠻ ⠨⠅ ⠣⠿⠜⠿⠻

⠼⠂⠲⠸⠲ ⠼⠶⠣⠒⠜⠆⠆⠢⠻⠡⠶⠣⠢⠜⠆⠻

⠼⠂⠢⠸⠲ ⠣⠢⠜⠍⠻⠣⠢⠜⠝⠻ ⠨⠅ ⠣⠢⠜⠍⠝⠻ ⠸⠱

PRACTICE 8G

⠠⠝⠑⠌⠫ ⠠⠗⠁⠙⠊⠉⠁⠇ ⠠⠑⠭⠏⠗⠑⠎⠨⠝⠎
⠀⠀
⠸⠩
⠷⠂⠾ ⠜⠤⠹⠂⠌⠆⠼⠤⠊⠹⠨⠜⠒⠨⠻⠌⠆⠼⠻
⠷⠆⠾ ⠜⠨⠜⠂⠒⠨⠻⠬⠨⠜⠂⠢⠨⠻⠬⠨⠜⠂⠂⠶⠨⠻⠻
⠷⠒⠾ ⠜⠂⠤⠨⠜⠁⠤⠃⠨⠻⠻⠈⠡⠜⠂⠬⠨⠜⠁⠤⠃⠨⠻⠻
⠷⠲⠾ ⠜⠃⠨⠣⠒⠜⠃⠨⠨⠜⠃⠨⠨⠻⠨⠻⠻
⠷⠢⠾ ⠣⠁⠜⠨⠣⠃⠜⠨⠨⠣⠉⠜⠁⠃⠉⠨⠨⠻⠨⠻⠻
⠷⠖⠾ ⠜⠁⠘⠆⠐⠻⠜⠃⠘⠲⠐⠻⠜⠉⠻ ⠨⠅ ⠁⠃⠘⠆⠐⠜⠉⠻
⠷⠶⠾ ⠷⠎⠘⠆⠐⠣⠒⠜⠎⠘⠲⠐⠻⠾⠘⠆
⠷⠦⠾ ⠣⠒⠜⠭⠘⠆⠐⠨⠜⠖⠲⠭⠘⠖⠐⠨⠻⠻
⠷⠔⠾ ⠜⠨⠣⠒⠜⠨⠨⠣⠲⠜⠨⠨⠨⠣⠢⠜⠃⠘⠲⠦⠐⠨⠨⠨⠻⠨⠨⠻⠨⠻⠻
⠷⠂⠴⠾ ⠜⠭⠂⠬⠨⠜⠭⠆⠨⠻⠻
⠷⠂⠂⠾ ⠟⠘⠜⠗⠻⠐⠬⠎
⠷⠂⠆⠾ ⠜⠉⠤⠙⠤⠑⠬ ⠄⠄⠄⠻ ⠸⠱

PRACTICE 8H

⠠⠇⠔⠅⠫ ⠠⠑⠭⠏⠗⠑⠎⠨⠝⠎

⠼⠁⠲ ⠠⠊⠎ ⠮ ⠋⠕⠇⠇⠪⠬ ⠔⠑⠟⠥⠁⠇⠰⠽ ⠞⠗⠥⠑⠦
⠀⠀⠸⠩ ⠹⠒⠌⠢⠼⠷⠹⠆⠌⠒⠼⠭⠤⠹⠂⠌⠆⠼⠾
⠀⠀⠨⠂ ⠹⠆⠌⠢⠼⠷⠹⠂⠌⠲⠼⠭⠬⠹⠂⠌⠒⠼⠾
⠼⠆⠸⠲ ⠼⠒⠒⠸⠹⠂⠌⠒⠸⠼⠈⠴⠬⠲⠴⠈⠴⠬⠖⠂⠸⠹⠆⠌⠒⠸⠼⠈⠴
⠀⠀⠨⠅ ⠼⠂⠒⠲⠸⠹⠒⠌⠒⠸⠼⠈⠴ ⠨⠅ ⠼⠂⠒⠢⠈⠴ ⠸⠱
⠼⠉⠲ ⠠⠔ ⠍⠥⠇⠞⠊⠏⠇⠽⠬ ⠸⠩ ⠼⠢⠸⠹⠒⠌⠲⠸⠼⠈⠡⠲⠖ ⠸⠱⠂
⠀⠀⠗⠑⠉⠁⠇⠇ ⠞ ⠸⠩ ⠼⠢⠸⠹⠒⠌⠲⠸⠼ ⠨⠅ ⠼⠢⠬⠹⠒⠌⠲⠼⠸⠲
⠀⠀⠠⠄⠠⠐⠮⠿⠑⠂ ⠼⠲⠖⠈⠡⠷⠢⠬⠹⠒⠌⠲⠼⠾
⠀⠀⠨⠅ ⠷⠲⠖⠈⠡⠢⠾⠬⠷⠲⠖⠈⠡⠹⠒⠌⠲⠼⠾ ⠨⠅ ⠼⠆⠖⠲⠸⠹⠂⠌⠆⠸⠼
⠀⠀⠸⠱⠲

Lines 10-11: Notice that the print copy divided this equation after the equals sign, but the braille transcription follows Nemeth rules and divides before the comparison sign.

PRACTICE 8I

```
1      ⠠⠗⠑⠉⠕⠛⠝⠊⠵⠬ ⠁ ⠠⠝⠑⠌⠫ ⠠⠇⠔⠅⠫ ⠠⠑⠭⠏⠗⠑⠎⠨⠝
2   ⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
3     ⠠⠞⠕ ⠞⠑⠌ ⠮ ⠑⠟⠥⠁⠰⠝ ⠸⠩ ⠠⠗⠰⠞ ⠨⠅ ⠹⠠⠗⠌⠝⠼ ⠸⠱⠂
4   ⠥⠎⠑ ⠋⠳⠗ ⠸⠩ ⠼⠂⠴⠴⠴⠤⠨⠠⠺ ⠸⠱ ⠗⠑⠎⠊⠌⠕⠗⠎ ⠺⠊⠗⠫ ⠔
5   ⠎⠻⠊⠑⠎ ⠞⠕ ⠏⠗⠫⠊⠉⠞ ⠁ ⠞⠕⠞⠁⠇ ⠗⠑⠎⠊⠌⠨⠑ ⠷
6   ⠸⠩ ⠼⠆⠢⠴ ⠨⠠⠺⠸⠲
7     ⠠⠗⠰⠞ ⠨⠅ ⠹⠠⠗⠌⠝⠼ ⠨⠅ ⠹⠂⠴⠴⠴ ⠨⠠⠺⠌⠲⠼
8     ⠹⠂⠴⠴⠴ ⠨⠠⠺⠌⠲⠼ ⠨⠅ ⠼⠆⠢⠴ ⠨⠠⠺ ⠸⠱
9   ⠠⠮⠝⠂ ⠃⠽ ⠥⠎⠬ ⠮ ⠗⠑⠎⠊⠌⠨⠑ ⠮⠕⠗⠽ ⠑⠟⠥⠁⠰⠝ ⠸⠩
10    ⠹⠂⠌⠠⠗⠰⠞⠐⠼
11      ⠨⠅ ⠹⠂⠌⠠⠗⠂⠼⠬⠹⠂⠌⠠⠗⠆⠼⠬ ⠄⠄⠄ ⠬⠹⠂⠌⠠⠗⠰⠝⠐⠼
12      ⠸⠱⠂
13  ⠏⠇⠥⠛ ⠮ ⠸⠩ ⠼⠂⠢⠴⠤⠨⠠⠺ ⠠⠄⠯ ⠼⠂⠴⠴⠴⠤⠨⠠⠺ ⠸⠱
14  ⠗⠑⠎⠊⠌⠕⠗⠎ ⠔⠞⠕ ⠮ ⠑⠟⠥⠁⠰⠝ ⠵ ⠸⠩ ⠠⠗⠂ ⠠⠄⠯ ⠠⠗⠆⠸⠲
15    ⠹⠂⠌⠠⠗⠰⠞⠐⠼
16      ⠨⠅ ⠹⠂⠌⠂⠢⠴ ⠨⠠⠺⠼⠬⠹⠂⠌⠂⠴⠴⠴ ⠨⠠⠺⠼
17      ⠨⠅ ⠼⠴⠨⠴⠴⠶⠬⠴⠨⠴⠴⠂
18      ⠨⠅ ⠼⠴⠨⠴⠴⠦
19    ⠠⠗⠰⠞ ⠨⠅ ⠹⠂⠌⠴⠨⠴⠴⠦⠼ ⠨⠅ ⠼⠂⠆⠢ ⠨⠠⠺ ⠸⠱
```

Lines 7 and 8: Two sets of linked expressions are displayed to the first paragraph. Each linked expression begins in cell 3. Neither linked expression has a runover.

Line 9: The narrative paragraph continues in cell 1.

Line 10: The displayed expression begins in cell 3.

Line 11: The link will not fit on line 11 so it begins in the runover position, cell 3.

Line 12: The Nemeth Code terminator and comma do not fit on line 11. They are placed in the runover position, cell 3.

Lines 13-14: The narrative paragraph continues in cell 1.

Lines 15-18: The print layout makes this a nested linked expression, displayed to a 3-1 paragraph. The anchor begins in cell 3, and each link begins in cell 5,.

Line 19: The final linked expression is not part of the nested expression. It begins in in cell 3 (displayed to a 3-1 paragraph).

PRACTICE 8J

⠀⠀⠀⠀⠀⠀⠀⠀⠠⠍⠥⠇⠞⠊⠏⠇⠽⠬ ⠠⠍⠊⠭⠫ ⠠⠝⠥⠍⠃⠻⠎

⠀⠀

⠠⠁⠲ ⠸⠩ ⠼⠆⠸⠹⠂⠌⠆⠸⠼⠡⠂⠸⠹⠂⠌⠲⠸⠼

⠀⠀⠨⠅ ⠷⠆⠬⠹⠂⠌⠆⠼⠾⠡⠷⠂⠬⠹⠂⠌⠲⠼⠾

⠀⠀⠨⠅ ⠼⠆⠬⠹⠆⠌⠲⠼⠬⠹⠂⠌⠆⠼⠬⠹⠂⠌⠦⠼

⠀⠀⠨⠅ ⠼⠆⠬⠹⠂⠌⠆⠼⠬⠹⠂⠌⠆⠼⠬⠹⠂⠌⠦⠼

⠀⠀⠨⠅ ⠼⠆⠬⠂⠬⠹⠂⠌⠦⠼

⠀⠀⠨⠅ ⠼⠒⠸⠹⠂⠌⠦⠸⠼ ⠸⠱

⠰⠠⠃⠲ ⠠⠱⠁⠞ ⠺ ⠮ ⠗⠑⠍⠁⠔⠙⠻ ⠆ ⠔ ⠹ ⠏⠗⠕⠃⠇⠑⠍⠦ ⠸⠩

⠀⠀⠀⠀⠼⠲⠸⠹⠂⠌⠒⠸⠼⠈⠡⠒⠸⠹⠆⠌⠢⠸⠼⠈⠡⠔⠸⠹⠂⠂⠌⠂⠢⠸⠼

⠀⠀⠀⠀⠀⠀⠀⠀⠈⠡⠆⠸⠹⠒⠌⠲⠸⠼

⠀⠀⠀⠀⠀⠀⠨⠅ ⠹⠂⠒⠌⠒⠼⠈⠡⠹⠂⠶⠌⠢⠼⠈⠡⠹⠂⠲⠖⠌⠂⠢⠼

⠀⠀⠀⠀⠀⠀⠀⠀⠈⠡⠹⠂⠂⠌⠲⠼

⠀⠀⠀⠀⠀⠀⠨⠅ ⠹⠂⠒⠈⠡⠂⠶⠈⠡⠂⠲⠖⠈⠡⠂⠂⠌⠒⠈⠡⠢⠈⠡⠂⠢⠈⠡⠲⠼

⠀⠀⠀⠀⠀⠀⠨⠅ ⠹⠒⠢⠲⠠⠔⠆⠖⠌⠔⠴⠴⠼

⠀⠀⠀⠀⠀⠀⠨⠅ ⠼⠒⠔⠲ ⠸⠱ ⠾ ⠁ ⠗⠑⠍⠁⠔⠙⠻ ⠷ ⠨⠤⠲