

LESSON 15

Format

- MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS

Answers to Practice Material

LESSON PREVIEW

In this lesson, we look at the methods for transcribing a mathematical expression that is too long to fit on the current line. This often occurs when line length is restricted due to the indented margins in displayed mathematical material. However, even a 40-cell line may not provide enough room for a particularly long expression. The examples in this lesson serve as a good review of other aspects of the Nemeth Code.

MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS

[NC 26.2]

15.1 Review

A mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. When there is insufficient space on the remainder of a line, the entire expression is brought down to the next line. One or both switch indicators can be placed on a separate line if, by doing so, the math will fit, undivided, on one line. Likewise, an identifier can be placed alone on the line if, by doing so, the math will fit, undivided, on the next line. Keeping the mathematical expression intact on one line is the priority.

In this lesson we discuss what to do when a mathematical expression is too long to be contained within the current margins – that is, when a division is unavoidable. First, here is a summary of items that must not be divided, and a review of runover rules already covered.

15.1.1 Symbols to Keep Together. The components of the following symbols must not be divided between braille lines.

- a. A symbol of operation using plus and minus (Lesson 5)
- b. A symbol of comparison compounded vertically or horizontally (Lesson 5)
- c. A shape symbol with structural or interior modification (Lesson 11)
- d. A character within a keystroke construction (Lesson 11)
- e. Superposed symbols (Lesson 13)
- f. Tally marks belonging to the same group (Lesson 13)

15.1.2 Expressions to Keep Together. The following expressions must not be divided between braille lines, even if divided in print.

- a. A hyphenated expression of which one component is mathematical. (Lesson 2)
- b. An abbreviation and its related numeral or letter. (Lessons 3 and 4)
- c. An enclosed list. (Lesson 4)
- d. A fraction, a mixed number. (Lesson 8)
- e. A shape symbol and its name (numeral, letter, or sequence of letters). (Lesson 11)
- f. The components of an expression modified according to the five-step rule. (Lesson 12)
- g. A function name (or its abbreviated form) and its argument. (Lesson 14)
- h. A two-part function name. (Lesson 14)

15.1.3 Runover Rules Already Studied. When a long expression won't fit on the braille line within current margins, the following rules apply. The new line begins in the runover cell of the current format.

- a. **Long Numeral** (Lesson 1) Divide a long numeral after a comma if a comma is present. A hyphen is inserted at the point of division. If the numeral does not contain a comma, the hyphen may be inserted after any digit. The numeric indicator is restated before the first digit of the continuation of the numeral on the next line.
- b. **Enclosed List** (Lesson 4) Divide an enclosed list after a comma used to separate the items.
- c. **Linked Expression** (Lesson 8) Divide a linked expression before a comparison sign. It is not necessary to divide at every comparison sign unless it is a nested linked expression.
- d. **Keystroke** (Lesson 11) Division may be made after any item in the keystroke string, but not within the keystroke.

15.1.4 **Nemeth Code Switch Indicators.** (Lessons 1 and 3) If both switch indicators will not fit on the same line as the math expression, the opening Nemeth Code indicator may fall on the previous line. The Nemeth Code terminator and any related punctuation may be placed on the following line.

15.2 Mathematical Units

The braille transcriber has only 40 cells available on a line, at most – perhaps as few as 30 cells for the runover to a nested link displayed to a subdivision. We often encounter a mathematical expression that will not fit on the current line. The margins in place at the time should not be changed in order to accommodate a long expression.

Runover sites should be chosen carefully. Every attempt should be made to keep the following mathematical units intact.

- a fraction
- a numerator
- a denominator
- a mixed number
- a base and its exponent; a subscript and its related item
- a grouped expression
- a radical expression
- a modified expression

A long or complicated mathematical expression can be organized into a series of mathematical units by following the procedures presented in this section. When the transcriber applies these principles, the reader is able to mentally reassemble the expression. On the other hand, a poorly divided expression will hinder the reader's understanding of the mathematics. Shrewd application of these guidelines can be properly rendered even if the transcriber is unfamiliar with the particular mathematics.

- 15.2.1 Follow the list below when choosing division sites, starting with Step i, keeping in mind that items enclosed within grouping symbols should not be divided.
- i. Before a comparison sign on the baseline. (See [15.3](#))
 - ii. Before an operation sign on the baseline. (See [15.4](#))
 - iii. Before a mathematical unit. (See [15.5](#))
 - iv. After a termination indicator. (See [15.6](#))

Special considerations affect division of function notation, integral notation, and Sigma and Pi notation. (See [15.7](#))

In order to focus on the layout, the isolated examples with no narrative do not include code switch indicators. Unless otherwise noted, embedded material is assumed to be within a 3-1 paragraph and displayed material begins in cell 3 with runovers in cell 5.

15.3 Step i: Divide Before a Comparison Sign on the Baseline

Linked Expressions: As you learned in Lesson 8, when a linked expression will not fit on one braille line, a division is made at the link, before the sign of comparison. A few examples are shown here, as a review. You may wish to revisit the topic of linked expressions and nested linked expressions in Lesson 8. Key points are noted below.

- The comparison sign at which the division is made must be on the baseline of writing. A comparison sign within a modifier, superscript, subscript, fraction, radical expression, etc. is not a suitable division site. (Examples [15-7](#) and [15-8](#))
- The print copy may divide after a comparison sign, but the braille transcription follows Nemeth rules and divides before the comparison sign. (Example [15-35](#))
- The link begins in the appropriate cell according to the current format. (Examples [15-1](#) and [15-2](#))
- If the expression contains more than one link, it is not necessary to divide at every link unless it is a nested linked expression. (Example [15-2](#))
- Even if the anchor consists of only one letter or number, if the link will not fit on the line with the anchor, the line is divided after the anchor. (Example [15-3](#))
- When a line begins with a sign of comparison, the transition to a new braille line terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines. (Example [15-4](#))
- In itemized formats, if an anchor will not fit on the line with its identifier but it fits on the next line starting in the runover cell, put it there in order to keep the anchor intact. The identifier will then be the only item on the first line. (Examples [15-5](#) and [15-6](#))
- Material within mathematical grouping signs is a unit and should not be divided. (Example [15-8](#))

Example 15-1

(two layouts)

$$1,778 + 1,294 + 865 + 905 + 2574 + 485 + 100 > 8,000$$

Embedded:

$$1,778 + 1,294 + 865 + 905 + 2574 + 485 + 100 > 8,000$$

As an embedded expression, the link continues on the next line in the runover cell of the current format.

Displayed:

$$1,778 + 1,294 + 865 + 905 + 2574 + 485 + 100 > 8,000$$

As a displayed expression, the link begins on the next line, indented two cells from the anchor.

Example 15-2

(displayed)

To factor the expression $-2ab + a^2 + b^2$,

Jared wrote: $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$

Dom wrote: $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$

Both solutions are correct. Explain.

1 $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$

2 $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$

3 $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$

4 $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$

5 $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$

6 $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$

7 $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$

Lines 1-2: Paragraph begins in cell 3 with runovers in cell 1.

Lines 3-6: Two displayed expressions – each begins in cell 3, with runover in cell 5.

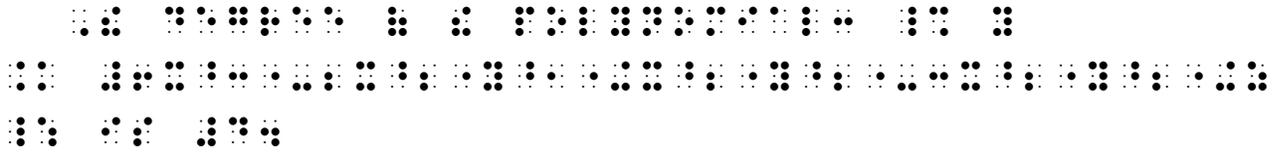
Lines 4 and 6: It is not necessary to divide the linked expression at every comparison sign.

Line 7: Paragraph continues in the runover cell (cell 1).

Example 15-3

(embedded)

The degree of the polynomial $y = 6x^3 - 2x^2y^1 + x^2y^2 - 3x^2y^2 + z$ is 4.



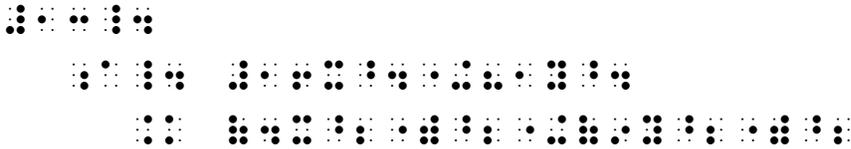
This anchor consists of the letter y and sits alone at the end of line 1 because it does not fit on the same line as its long link.

Example 15-4

(as a subdivision starting in cell 3)

13.

a. $16x^4 + 81y^4 = (4x^2)^2 + (9y^2)^2$



The link begins in the current runover cell—cell 5, in this case. Transition to a new line following a sign of comparison returns the reader to the baseline.

Example 15-5

(a main division starting in cell 1)

13. $16x^4 + 81y^4 = (4x^2)^2 + (9y^2)^2$



The expression begins in the current runover cell—cell 3, in this case—because it does not fit on the line above with its identifier.

Instructions: Keep the following points in mind as you select runover sites. (1) Keep mathematical units intact on one line, if possible; (2) before dividing an expression, try placing the switch indicators on a different line; (3) before dividing an expression, try placing the identifier on a different line; (4) if the entire expression will not fit on the line, divide before a comparison sign on the baseline.

PRACTICE 15A

A. Solve the linear inequalities.

i. $(x + \frac{10}{3})(x + \frac{19}{3}) > (3x + \frac{46}{3})(\frac{x}{3} + 1)$

ii. $\frac{2x}{3} - 3 > \frac{16x}{21} - \frac{13}{3} - \frac{2x}{15}$

iii. $(a - 1)^2 - (a - 7)(a - 3) < 2a + 0.8$

B. $2 \times 423 = (2 \times 400) + (2 \times 20) + (2 \times 3) = 800 + 40 + 6 = 846$

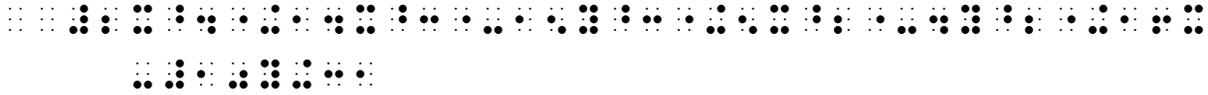
Binomial coefficients get their name because they are the *coefficients* in the expansion of a *binomial*:

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.$$

Example 15-10

(displayed)

$$2x^4 + 14x^3 - 15y^3 + 5x^2 - 4y^2 + 16x - 10y + 31$$

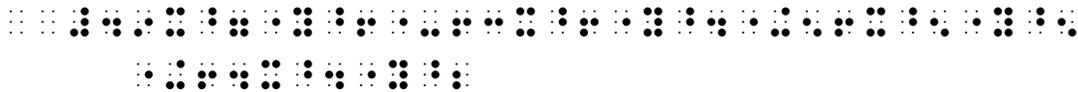


Runover line (line 2): A numeral preceded by a minus sign requires a numeric indicator when the minus sign begins a braille line.

Example 15-11

(displayed)

$$49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2$$



The baseline indicator is the first symbol in the runover line of this divided expression.

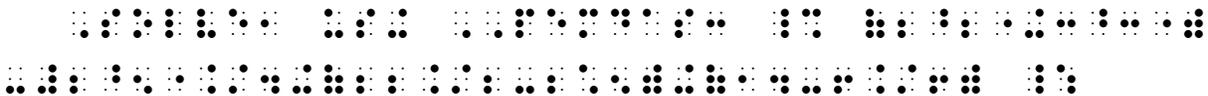
15.4.1 A Sign of Operation within a Mathematical Unit

- a. **Keep Together: Grouped Expressions.** Unless unavoidable, items enclosed within grouping signs should not be divided between lines. An operation sign within a grouped expression is not a suitable division site.

Example 15-12

(embedded)

Solve, using PEMDAS: $(2^2 + 3^3) - 2^5 \div 4 + (22 \div 2 - 2 \cdot 5) + (14 - 6 \div 6)$

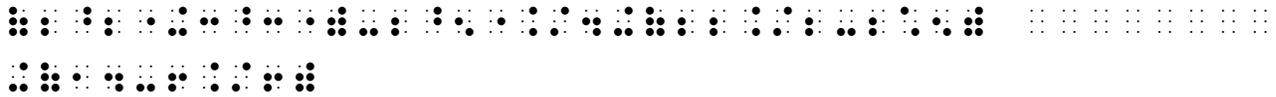


The first grouped expression will fit on the current line of this embedded expression.

Example 15-13

(embedded)

$$(2^2 + 3^3) - 2^5 \div 4 + (22 \div 2 - 2 \cdot 5) + (14 - 6 \div 6)$$



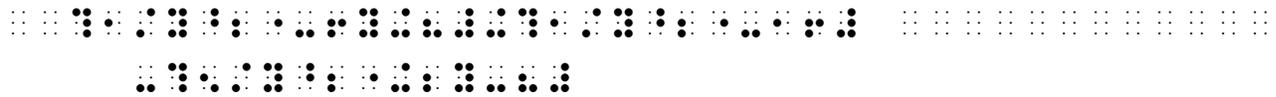
Division is made before an operation sign that is outside of the grouping symbols.

- b. **Keep Together: Fractions and Other Mathematical Units.** An operation sign within a fraction, modifier, superscript, subscript, radical expression, etc. is not a suitable division site.

Example 15-14

(displayed)

$$\frac{1}{y^2 - 6y + 8} + \frac{1}{y^2 - 16} - \frac{5}{y^2 + 2y - 8}$$



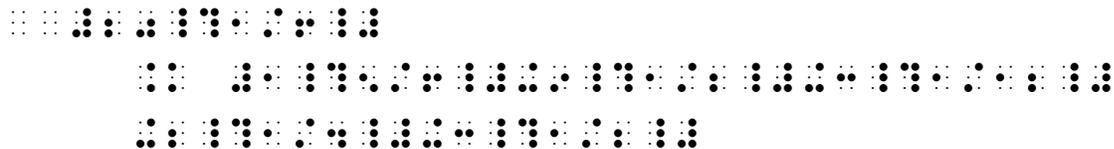
Only the operation signs on the baseline are suitable division sites, not those in the denominators of these fractions.

- 15.4.2 **Linked Expressions.** If an anchor or a link must be divided, further rules apply. ("Anchor" and "link" were defined in Lesson 8.) If a division occurs within the anchor or its link, a division must always be made before the link (Step i, divide before a comparison sign on the baseline). Furthermore, in a linked expression with more than one link, division must occur before each link if any link requires division. The resulting transcription maintains an orderly representation of mathematical units.

Example 15-15

(displayed)

$$20 \frac{1}{6} = 1 \frac{5}{6} + 9 \frac{1}{2} + 3 \frac{1}{12} + 2 \frac{1}{4} + 3 \frac{1}{2}$$

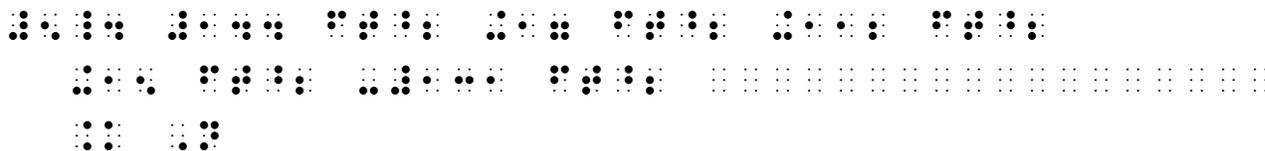


Step i: The linked expression is divided before the equals sign. Step ii: The long link is divided before a plus sign.

Example 15-16

(itemized)

5. $144 \text{ ft}^2 + 17 \text{ ft}^2 + 112 \text{ ft}^2 + 15 \text{ ft}^2 - 131 \text{ ft}^2 = N$

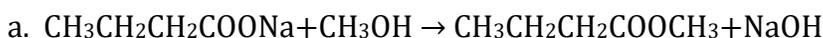


Working backwards, the linked expression is divided before the equals sign (Step i). The long anchor is divided before a plus sign (Step ii). The link must not be placed on line 2, even though it will fit, because Step ii has been applied.

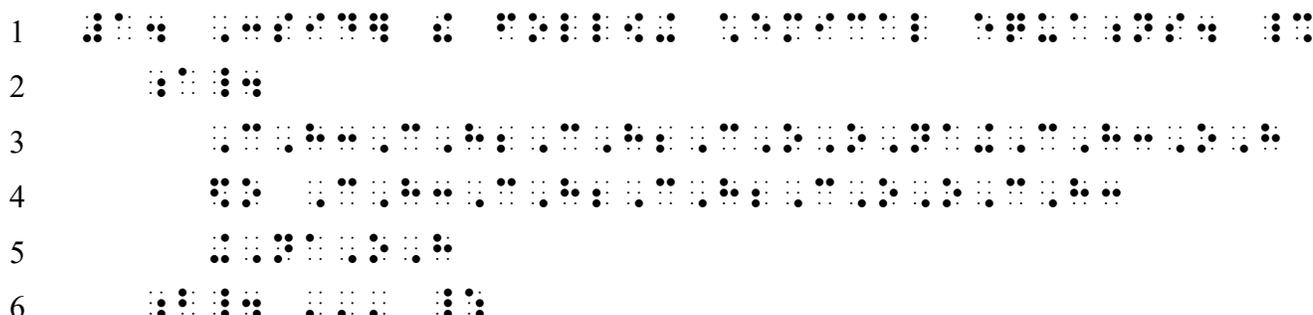
Example 15-17

(itemized)

1. Consider the following chemical equations.



b. ...



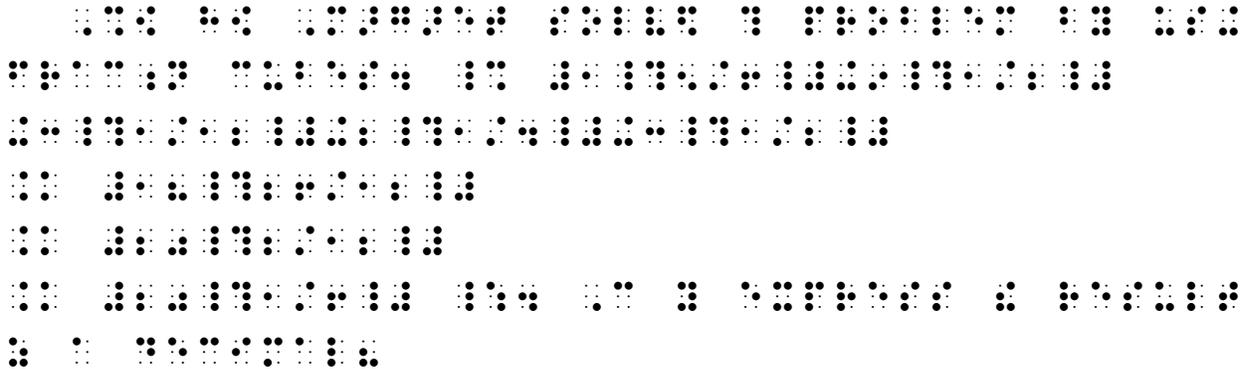
Lines 2-3: The anchor does not fit on the line with the identifier, but it will fit entirely on the next line. It begins in the runover position for subdivisions, which is cell 5.

Lines 4-5: Line 4 begins with the comparison sign (right-pointing arrow). The link will not fit on one line. It is divided before the plus sign. Because the subscript 3 does not require a subscript indicator, there is no baseline indicator needed before the plus sign on line 5.

Example 15-18

(embedded)

Show how Margaret solved this problem by using fraction cubes. $1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2} = 18\frac{26}{12} = 20\frac{2}{12} = 20\frac{1}{6}$. Can you express the result as a decimal?



Because the anchor is divided, the expression is also divided before each link.
The paragraph continues following the completion of the embedded expression.

Instructions: In addition to the tactics outlined with PRACTICE 15A, keep the following point in mind as you select runover sites. Apply Step i (divide before a comparison sign) before applying Step ii (divide before an operation sign).

PRACTICE 15B

- A. $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 = 490^\circ$
- B. $2\frac{3}{4} \text{ yd} + 1\frac{3}{4} \text{ yd} + \frac{3}{4} \text{ yd} = 5\frac{1}{4} \text{ yd}$
- C. $\sqrt{(x + a^2) + (y + a^2)} - \sqrt{(x - a^2) + (y - a^2)} = \pm 2a$

Sommer's Routine: Sommer's routine can be represented by the following expression.

$$\text{Routine S: } 5 + 3 + 4(1 + (-1)) + (-3) + (-5) + 3(5 + (-2)) + 1$$

Draw a simple diagram to represent *Routine S*.

Instructions: Apply the tactics outlined with PRACTICE 15A and PRACTICE 15B as you select runover sites. Note that the ellipsis in item c. is on the baseline of writing.

PRACTICE 15C

1. Compute and/or simplify.

a.
$$\frac{\left(\frac{3}{2}\right) \times \left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{1 \times 2 \times 3}$$

b.
$$\frac{\frac{dx}{dt} \frac{d^2y}{dt^2} - \frac{d^2x}{dt^2} \frac{dy}{dt}}{(dx/dt)^3}$$

c.
$$(\pm)a_{1i_1} a_{2i_2} a_{3i_3} a_{4i_4} \dots a_{ni_n}$$

Note: In item 2, the expression is displayed.

PRACTICE 15E

1. $\sqrt[3]{x^1} \sqrt[6]{x^2} \sqrt[12]{x^3} \sqrt[24]{x^4} \sqrt[48]{x^5} \dots$

2. The general solution for one root of the cubic equation is

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}.$$

*Instructions: Make your division decision based on the mathematician's preference.
(The unusual letter in the last numerator is a lowercase Greek zeta.)*

PRACTICE 15G

The solutions involving the direct functions

$$\int \frac{\log^2(z+1)}{z} dz = \log(-z) \log^2(z+1) + 2\text{Li}_2(z+1) \log(z+1) - 2\text{Li}_3(z+1)$$

and

$$\int_0^\infty \frac{\log(t+1) \log\left(1 + \frac{1}{t^2}\right)}{t} dt = C\pi - \frac{3\zeta(3)}{8}$$

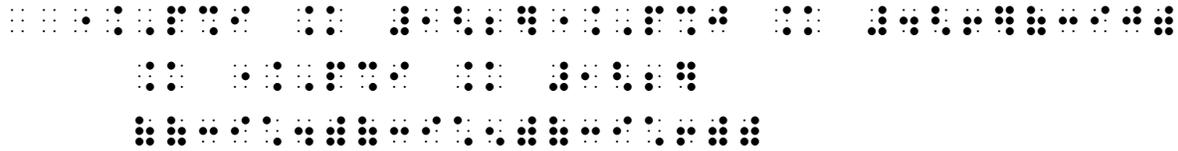
can be found in Chapter 7.

15.7.3 **Sigma and Pi Notation.** To a mathematician, it is important to keep the Sigma or Pi and its associated modifiers or superscript/subscript intact, and also to begin the associated expression which follows (the "argument") on the same line, if possible. When a choice must be made, it is advisable to apply division strategies to the argument. This may mean dividing within a mathematical unit such as a fraction or a grouped expression.

Example 15-34

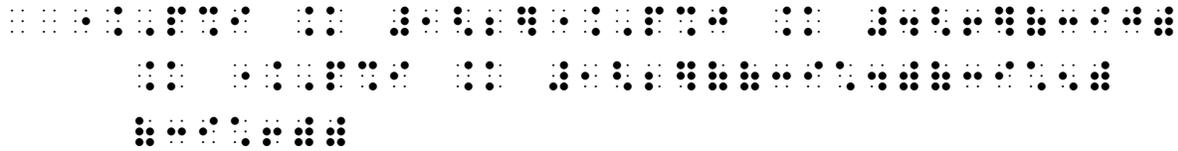
$$\prod_{i=1}^2 \prod_{j=4}^6 (3ij) = \prod_{i=1}^2 ((3i \cdot 4)(3i \cdot 5)(3i \cdot 6))$$

Displayed: Option 1, dividing after the termination indicator.



The link must be divided. The Nemeth Code recommends dividing after the termination indicator of the modification. Although this keeps the grouped expression intact, it separates the Pi notation from its related expression. The mathematicians we consulted recommend Option 2, below.

Displayed: Option 2, dividing between grouped factors.



The link must be divided. To keep the argument starting on the same line as the Pi notation, division is made between factors inside the larger grouping signs.

Instructions: Transcribe this example as if it were embedded within narrative, beginning with an opening Nemeth Code switch indicator in cell 1. Make your division decision based on the mathematician's preference.

PRACTICE 15H

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}$$

SUMMARY

If a mathematical expression must be divided, the following strategies and rules were explored in this lesson.

- When a linked expression will not fit on one line, division is made before the comparison sign. A comparison sign within a grouped expression, fraction, modifier, superscript, subscript, radical expression, etc. is not a suitable division site.
- When a link will not fit on one line, division is made before an operation sign. An operation sign within a grouped expression, fraction, modifier, superscript, subscript, radical expression, etc. is not a suitable division site.
- In a linked expression, if either the anchor or any link must be divided, a division must also be made before each link.
- Fractions are kept intact by dividing before an opening fraction indicator. If a fraction must be divided, division is made before the fraction line. A mixed number should not be divided.
- Transition to a runover line does not take the place of a necessary baseline indicator. The baseline indicator will be the first symbol on the new line.
- When an item and its exponent or subscript are too long to fit on a single braille line, division is made before a change-of-level indicator.
- The space between a function and its argument is not a suitable division site. If the entire expression will not fit on the line, division strategies are applied to the argument.
- Items within grouping symbols should not be divided. If consecutive groupings do not fit on the line, a division may be made between groupings.
- If a grouped expression will not fit on one line, division strategies are applied within the grouping symbols. If the grouped expression is an enclosed list, division is made after a comma.
- Division may be made after a mathematical termination indicator such as termination of a radical expression or termination of a modified expression.
- When an embedded math expression must be divided, it may begin on the current line provided division is made in accordance with the principles defined in this section.
- When a displayed math expression must be divided, all runovers begin two cells to the right of the original display cell unless it is a nested linked expression in which case a second indent level is applied.

EXERCISE 15

Prepare Exercise 15 for your grader.

