

LESSON 16

- SPATIAL ARRANGEMENTS OF FRACTIONS
- HYPERCOMPLEX FRACTIONS
- CONTINUED FRACTIONS
- INSTRUCTIONAL COMMENTARY
- STEM-AND-LEAF PLOTS

Answers to Practice Material

LESSON PREVIEW

The remaining lessons take a look at a variety of formatting topics. In this lesson, spatially arranged fractions are studied, a braille format for instructional commentary is given, and layouts for stem-and-leaf plots are introduced.

SPATIAL ARRANGEMENTS OF FRACTIONS

[NC 13.10]


Fractions are usually transcribed linearly, as discussed in Lesson 8. However, certain situations suggest or require a spatial arrangement.

Recall the terminology used for the parts of a fraction.

$$\begin{array}{l} 3 \quad \textit{numerator} \\ \hline 4 \quad \textit{denominator} \end{array}$$

16.1 Spatial Fraction Line

The spatial fraction line is comprised of a series of dots 25. An opening fraction indicator (1456) marks the beginning of the fraction line; a closing fraction indicator (3456) signals the end.



Spatial Fraction Line with opening and closing fraction indicators (varying length)

In a spatially arranged fraction the numerator is placed above the spatial fraction line and the denominator is placed below it. The fraction line is comprised of the same number of cells as the widest item above (numerator) or below (denominator), with the fraction indicators placed one cell beyond the width on either end, as illustrated in the next section.

Note: Code switch indicators are absent in many of the isolated examples in this lesson.

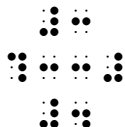
16.2 Numerator and Denominator

The numerator is centered above the fraction line; the denominator is centered below the fraction line. If exact centering is not possible, the item is moved to the left one cell. Since fractions do not contain material aligned for computation, the numeric indicator is used in the numerator and/or denominator according to the rules of the Nemeth Code. For the same reason, the English-letter indicator is used where necessary in the numerator and/or denominator.

Example 16-1

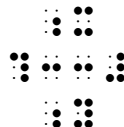
(a)

$$\frac{3}{4}$$



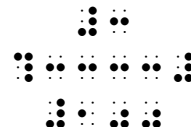
(b)

$$\frac{x}{y}$$



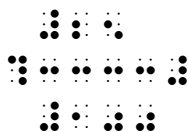
(c)

$$\frac{3}{100}$$



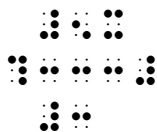
(d)

$$\frac{25}{100}$$



(e)

$$\frac{5x}{3}$$



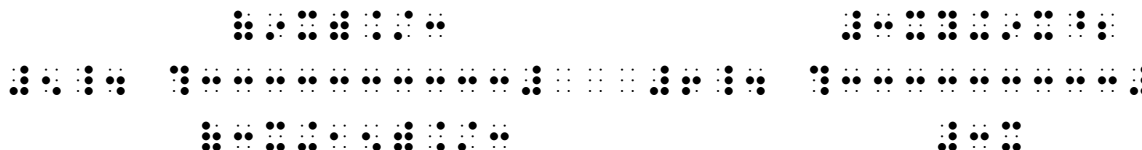
16.3 Placement of Identifiers with Spatially Arranged Fractions

An identifier, if present, is placed on the same braille line as the fraction line. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement.

Example 16-2

5. $\frac{(9x) \div 3}{(3x+15) \div 3}$

6. $\frac{3xy+9x^2}{3x}$



Recall from Lesson 9 that three blank cells come between the rightmost symbol of an arrangement and the next identifier.

PRACTICE 16A

1. Shandra invited seven friends to a pizza party. Two pizzas were ordered. Each pizza had eight slices. Which fraction shows how many slices of pizza each child can have if they share equally?

- a. $\frac{2}{8}$ number of pizzas
 number of children
 - b. $\frac{8}{8}$ number of slices
 number of children
 - c. $\frac{16}{8}$ number of slices
 number of children
-

16.5 Cancellation with Replacement Values [NC 12.1]

Recall that a spatial arrangement is required when numbers, letters, or abbreviations are canceled in print and replacement values are shown. *Reminder:* The print copy may show any type of stroke to indicate cancellation.

⋮	Opening Cancellation Indicator
⋮⋮	Closing Cancellation Indicator

When cancellation occurs within a fraction, replacement items are placed above canceled items in the numerator and below canceled items in the denominator. Each replacement item is centered with respect to the canceled term. If exact centering is not possible, the term is moved to the left one cell.

Example 16-5

$\frac{1}{25}$	1	⋮⋮
$\frac{25}{50}$	2	⋮⋮⋮⋮
$\frac{50}{2}$	3	⋮⋮⋮⋮⋮⋮
	4	⋮⋮⋮⋮
	5	⋮⋮

Line 1: Replacement "1" is centered above the canceled number. A numeric indicator is required.

Line 2: A numeric indicator is not required because the numeral is not preceded by a space.

Line 3: The fraction line is as long as the widest item in the fraction. In this case, both numerator and denominator have a 4-cell item.

Line 4: A numeric indicator is not required because the numeral is not preceded by a space.

Line 5: Replacement "2" is centered below the canceled number. A numeric indicator is required.

Example 16-6

$$\frac{\overset{1}{\cancel{25}}}{\underset{2}{\cancel{50}}} = \frac{1}{2}$$

Note that the second fraction ("one-half") is transcribed as a linear fraction because it doesn't contain cancellation. The equals sign and the linear fraction are placed on the same line as the spatial fraction line.

- 16.5.1 **Alignment Considerations with Cancellation.** Vertical alignment of place value is not a consideration in spatially arranged fractions when the material is not aligned for computation. Replacement items are centered above or below their canceled item. Placement of the replacement item is relative to the width of the canceled item, without respect to place value. Note the location of the replacement items in the next example.

Example 16-7

$$\frac{\overset{7}{\cancel{1449}}}{\underset{1}{\cancel{207}}} = 7$$

PRACTICE 16B

Simplify using cross canceling. Then multiply the numerators and multiply the denominators.

$$1. \quad \frac{\cancel{3}^1}{\cancel{15}_3} \times \frac{\cancel{5}^1}{\cancel{6}_2} = \frac{1}{6}$$

$$2. \quad \cancel{2}^1 \times \frac{8}{\cancel{2}_1} = \frac{1 \times 8}{1} = 8$$

16.5.3 Cancellation and Level Indicators. Care must be taken to place the opening cancellation indicator and its paired closing cancellation indicator on the same level.

$$\gg \quad \cancel{b^x} \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$$

Both the base and its superscript are canceled with one stroke. A baseline indicator places the cancellation terminator on the same level as the paired opening cancellation indicator.

$$\gg \quad b^{\cancel{x}} \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$$

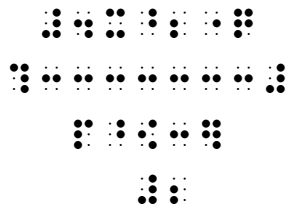
Only the superscript is canceled. The opening and closing cancellation indicators are at the superscript level.

Example 16-10

$$\frac{\cancel{a}^a}{\cancel{a}_1} = a \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$$

The denominator is centered under the fraction line.

Example 16-11

$$\frac{4x^2q}{p^{\cancel{2}^2}}$$


The replacement numeral "2" is placed below the canceled superscript in the denominator, even though it is printed beside the canceled superscript.

PRACTICE 16C


1.
$$\frac{2b+a}{b} \div \frac{4b^2-a^2}{b^2} = \frac{\cancel{(2b+a)}^1}{\cancel{1}} \cdot \frac{\cancel{b^2}^b}{\cancel{(2b+a)}_1(2b-a)} = \frac{b}{2b-a}$$

16.6 Cancellation Without Replacement Values [NC 12.1]

Material containing cancellation with no replacement may be transcribed linearly.


16.6.1 **Linear Presentation of Simple Fractions with Canceled Items.** When replacement items are not give for canceled material in the numerator and denominator, a linear fraction may be transcribed. The cancellation indicators must exactly represent what is canceled in print.

Example 16-12

$$\frac{\cancel{r} \cancel{s} \cancel{t}}{\cancel{r} \cancel{s} \cancel{t} v}$$


Each letter "r" "s" and "t" is printed with a slash through it.

Example 16-13


$$\frac{\cancel{(rst)}}{\cancel{(rst)}v}$$


Each letter combination "rst" as well as the parentheses are printed with a line through them.

16.8 Transcription of Hypercomplex Fractions

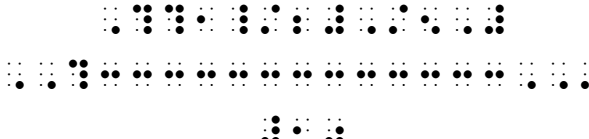
Although a linear transcription is possible, using a linear arrangement for the numerator and a linear arrangement for the denominator, within a spatial arrangement overall, gives a clearer representation of the hypercomplex fraction.

- 16.8.1 **Hypercomplex Fraction Indicators.** Opening and closing hypercomplex fraction indicators mark the beginning and end of a spatial hypercomplex fraction line.

	Spatial Hypercomplex Fraction Line with opening and closing fraction indicators (varying length)
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Here is the transcription of the hypercomplex fraction shown on the previous page. The complex fraction in the numerator is transcribed as a linear arrangement within the overall spatial arrangement.

Example 16-16 |

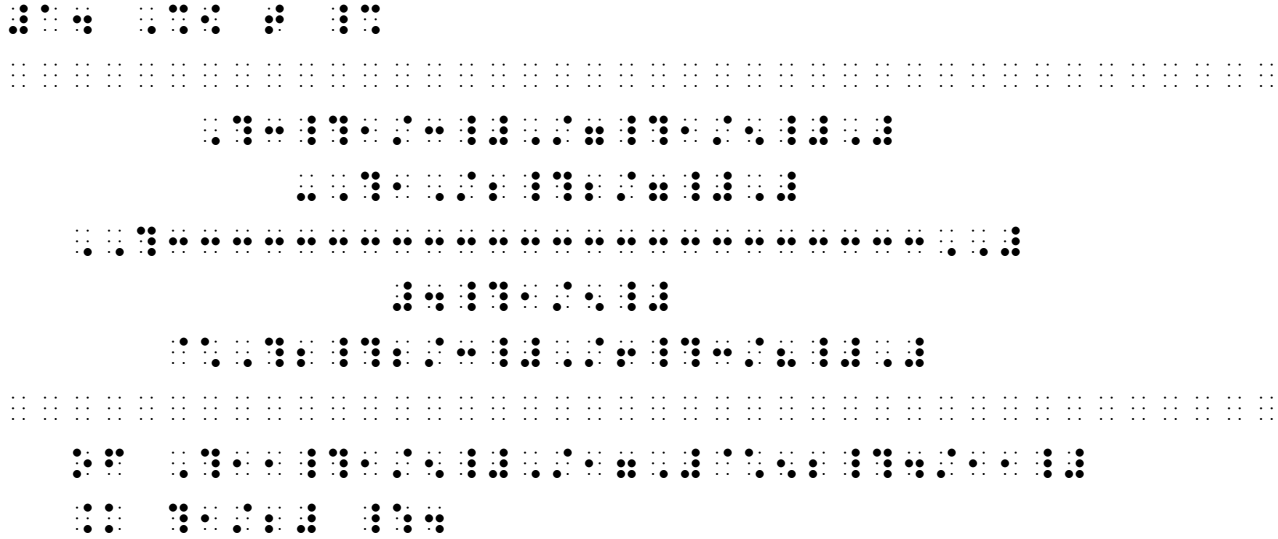
$\frac{\frac{1}{2}}{\frac{5}{10}}$	
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Terms of the fraction are centered above and below their fraction lines.

If the numerator and denominator are too long to fit on one line, follow the rules regarding division of mathematical expressions between braille lines. (Lesson 15) The next example shows an expression which will fit in 40 cells as first transcribed, but if it is displayed, division of the denominator is necessary.

Example 16-17

1. Show that $\frac{3\frac{1}{3} - \frac{1}{7\frac{1}{5}}}{4\frac{1}{5} \times \frac{2\frac{2}{3}}{6\frac{3}{8}}}$ of $\frac{11\frac{1}{5}}{17} \times 52\frac{4}{11} = \frac{1}{2}$.



Numerator and denominator are divided before an operation sign. The numerator is divided before the minus sign; the denominator is divided before the multiplication cross. Terms of the fraction are centered above and below the spatial fraction line. Note that the word "of" is part of the equation, so it is transcribed in mathematical context, without contractions.

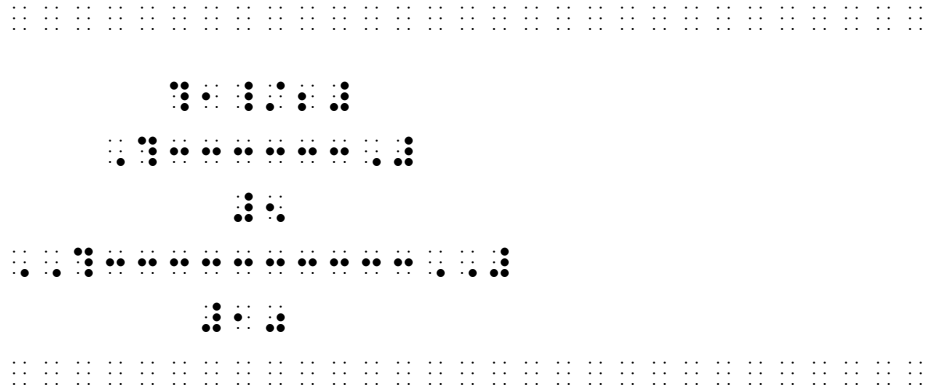
16.8.2 **Other Layouts.** The combined transcription method illustrated above is the preferred way of transcribing a hypercomplex fraction. However, it is permissible to use an entirely spatial arrangement or an entirely linear arrangement. In particular, linear hypercomplex fractions are difficult to assemble mentally. Example 16-16 is illustrated below using these two alternate methods. Symbols and indicators used in the linear method are as follows.

⠠⠠⠠⠠⠠	Opening Hypercomplex Fraction Indicator
⠠⠠⠠⠠⠠	Horizontal Hypercomplex Fraction Line
⠠⠠⠠⠠⠠	Diagonal Hypercomplex Fraction Line
⠠⠠⠠⠠⠠	Closing Hypercomplex Fraction Indicator

Example 16-18

$$\frac{\frac{1/2}{5}}{10}$$

(entirely spatial)



(entirely linear)



16.9 Higher Orders of Complexity

Hypercomplex fractions of higher order may be transcribed in the manner described above, with dot 6 added the appropriate number of times before the fraction indicators and the matching fraction lines. No examples are shown.

Instructions: Transcribe this hypercomplex fraction using the combined method—that is, transcribe each complex fraction as a linear arrangement within the overall spatial arrangement. Numerators and denominators can be determined by noting the length of each fraction line in print.

PRACTICE 16E

1. Solve for r in terms of s .

$$\frac{\frac{r^2 - 4s^2}{s^2}}{\frac{r + 2s}{s}} = \frac{4r - 2s^2}{\frac{3s}{2s^2 - 3r}} = \frac{4r}{4r}$$

CONTINUED FRACTIONS

[NC 13.9]

16.10 Definition and Recognition

In essence, a continued fraction is a fraction within a fraction. Each denominator is the sum of a whole number and a fraction. The digit in the denominator of one fraction becomes the digit in the numerator of the next fraction. You may find them in the study of irrational numbers such as pi and certain square roots.

The continued fraction shown below can be described as follows. The fraction begins with $1 - \frac{7}{3}$. The first denominator is $3 + \frac{3}{2}$. The next denominator is $2 + \frac{2}{2}$. The final denominator is $2 - \frac{2}{3}$.

$$n = 1 - \frac{7}{3 + \frac{3}{2 + \frac{2}{2 - \frac{2}{3}}}}$$

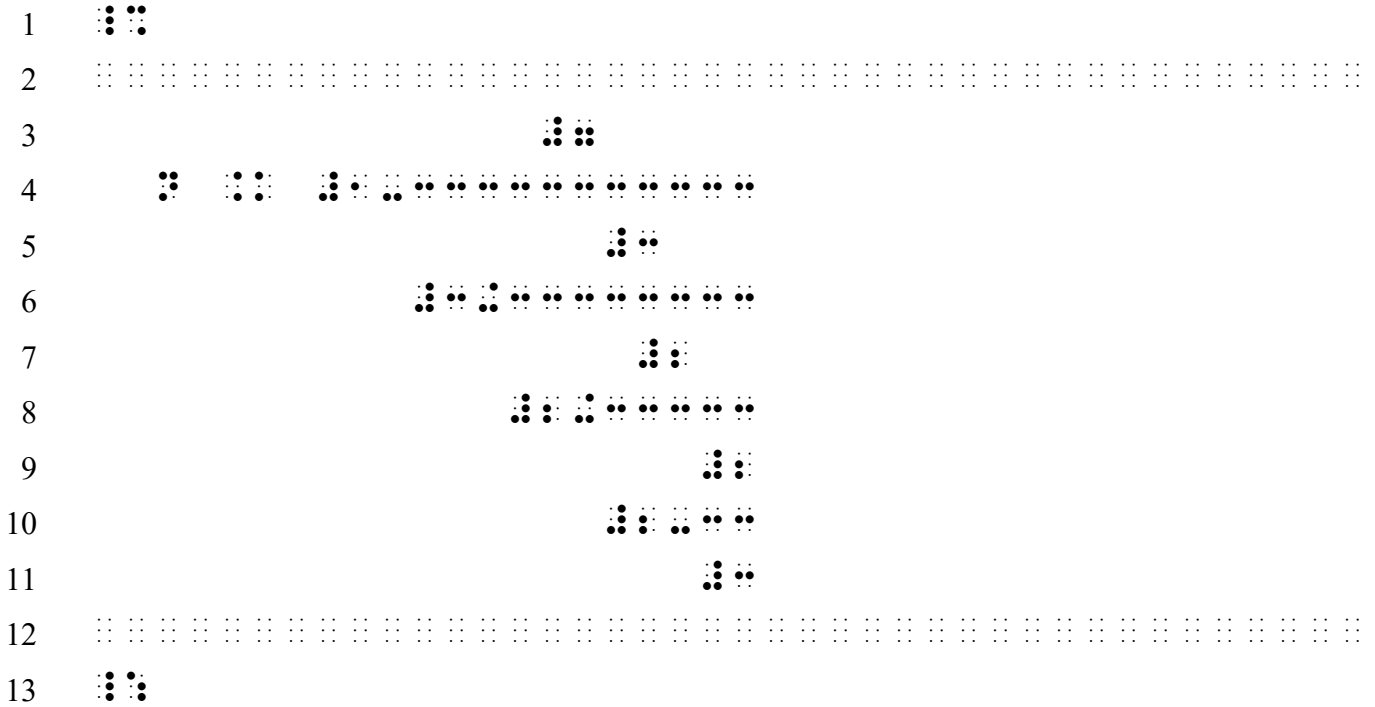
16.10.1 **Transcription Rules.** A continued fraction is transcribed as a spatial arrangement. The layout essentially mimics the print arrangement. Each fraction line begins in the cell after the plus or minus sign (unspaced). Opening and closing fraction indicators are not used. All fraction lines end in the same cell, aligned with the rightmost symbol in the arrangement.

Each new denominator's whole number is aligned directly below the first cell of the separation line above it. Its numerator is centered with respect to the fraction line below it. If that number cannot be exactly centered, it is placed one cell to the left of center.

The rules will make sense when you study the example below, which is the transcription of the continued fraction shown above.

Example 16-19

$$n = 1 - \frac{7}{3 + \frac{3}{2 + \frac{2}{2 - \frac{2}{3}}}}$$



Line 1: The opening Nemeth Code indicator is transcribed on the line preceding the required blank line.

Line 3: The numerator is centered over the fraction line below it.

Line 4: The length of this fraction line extends to the rightmost character in the arrangement, which is on lines 9 and 11.

Line 5: This numerator "3" is centered over the fraction line below it. Likewise for the numerators on lines 7 and 9.

Line 6: The numeric indicator is directly aligned with the first cell of the fraction line two lines above. Likewise for lines 8 and 10.

Line 13: The Nemeth Code terminator is in cell 1 on the line following the required blank line.

PRACTICE 16F

The square root of 3 can be expressed as a continued fraction.

$$\sqrt{3} = 1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \dots}}}$$

INSTRUCTIONAL COMMENTARY

[NC 4.8.9 and 26.4.5]

16.11 Format for Instructional Commentary

When instructional commentary appears alternated with math problems, place the comment on the line following the related expression, blocked 4 cells to the right of the runover position for the expression. Explain this format in a transcriber's note.

Sample transcriber's note: "Comments printed beside related math problems are placed on the line following the expression, blocked four cells to the right of the runover position of the expression."

When switching into or out of Nemeth before a change of margins, place the switch indicators after the last item of the line rather than at the beginning of the next line. This maintains clarity in the indented margin pattern. The single-word switch indicator is used, as needed.

Comments are often printed in a different color or typeface. (In the examples in this section, instructional commentary is printed in italics.) The variant typeform is disregarded in the braille transcription.

Notice in the examples how the comments are clearly set off by indentation.

Example 16-20

If y varies inversely as x , and $y = 3$ when $x = 4$, find y when $x = 18$.

$$\frac{x_1}{y_2} = \frac{x_2}{y_1} \quad \textit{Given.}$$

$$\frac{4}{y_2} = \frac{18}{3} \quad \textit{Substitute the known values.}$$

$$18y_2 = 12 \quad \textit{Now cross multiply.}$$

$$y_2 = \frac{12}{18} \textit{ or } \frac{2}{3} \quad \textit{Divide each side by 18 and simplify.}$$

The value of y when $x = 18$ is $\frac{2}{3}$.

Line 3: The anchor begins in cell 5

Lines 4, 9, 13, 17: Each link begins in cell 7. Runovers to the links are in cell 9 (lines 5 and 10).

Lines 6, 11, 14: Each comment begins in cell 13, which is four cells to the right of the runover position of the previous line, even if no runover is present.

Lines 7-8, 12, 15-16: Runovers to comments are blocked (cell 13, in this case).

Lines 9-10: Because the canceled numbers do not have replacements, the fraction is transcribed linearly.

Instructions: Transcribe this as one paragraph with four lines of displayed material between the two sentences.

PRACTICE 16G

What percent of 48 is 54?

$$n \cdot 48 = 54$$

Write an equation.

$$\frac{48n}{48} = \frac{54}{48}$$

Divide each side by 48.

$$n = 1.125$$

Simplify.

$$= 112.5\%$$

Change the decimal to a percent.

Answer: 54 is 112.5% of 48.

STEM-AND-LEAF PLOTS

16.12 Recognition

A stem-and-leaf plot is a method of showing data distribution in columns and rows. A vertical line partitions the information into data on the left, called the stem, and data on the right, called the leaf. Stem and leaf data may consist of numbers, letters, and/or blank entries and may be spaced or unspaced in the print copy. Stem data are right justified to the vertical line; leaf data are left justified to the vertical line. The columns may include a heading. Here is an example.

Stem	Leaf
10	0 4
9	3 4 5 7 8 9
8	0 2 2 9
7	5 8 9
6	3

A distinctive feature of a stem-and-leaf plot is the vertical line separating the stems from the leaves.

16.13 The Table

A stem-and-leaf plot is transcribed in Nemeth notation using the guidelines for tables outlined in *Braille Formats*. The entire table is transcribed in Nemeth, including the column headings (if present), even if the numbers or letters are unmodified. Column headings begin in cell 1 and no contractions are used. Column separation lines follow on the next line below the headings, covering the same number of cells as the widest data entry in each column. Column headings are not transcribed if they do not appear in print.

The vertical line between the stem column and the leaf column is transcribed as dots 456, preceded and followed by one blank cell. This symbol is transcribed in every row, including between column headings and between column separation lines.

<code>⠠⠨⠠</code> Vertical Line (preceded and followed by a space)

The following Nemeth rules apply to this example.

- The numeric indicator is omitted in the body of the plot.
- Spaces are not inserted between single-digit entries.
- The data are justified to the vertical line as shown in print.

16.16 Alphabetic Data

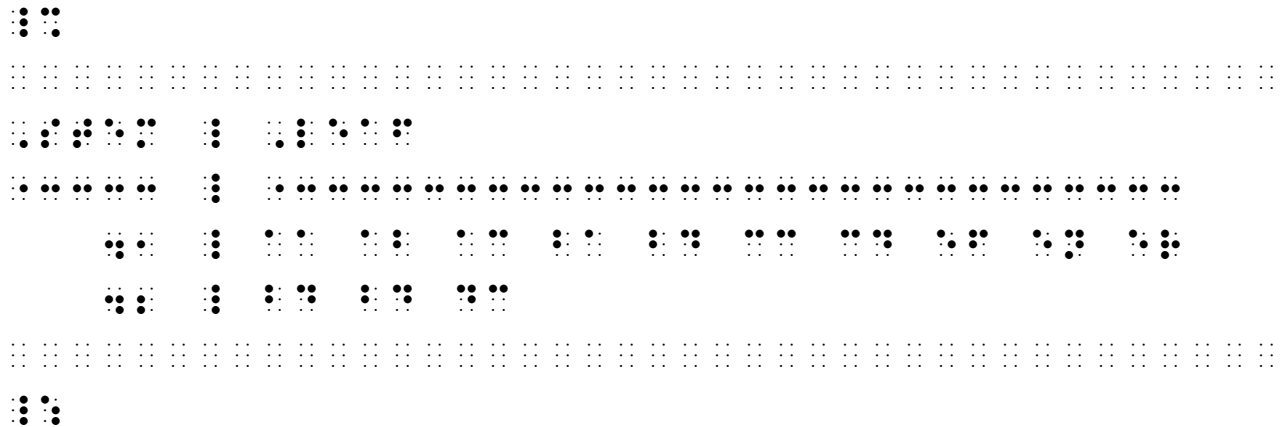
For alphabetic data, the following rules apply.

- The English-letter indicator is omitted in the body of the plot.
- Spaces are not inserted between single-letter data entries.
- Data consisting of groups of two or more letters require one blank cell between entries.

The leaves in the next example consist of pairs of lowercase letters. There is no key.

Example 16-27

Stem	Leaf
41	aa ab ac ba bd cc cd ef en er
42	bd bd dc



Reminder: The numeric indicator is omitted in the body of the plot (Stem column).

16.16.1 **Alphabetic Key.** In an alphabetic key, the English-letter indicator is not used for the portion of the key that replicates an entry in the plot. The value assigned to the key is transcribed using the English-letter indicator as required by the Nemeth Code.

⤵ c | d = cd ⠠⠉ ⠠⠣ ⠠⠉⠠⠣ ⠠⠉⠠⠣⠠⠉⠠⠣

16.17 Blank Entries

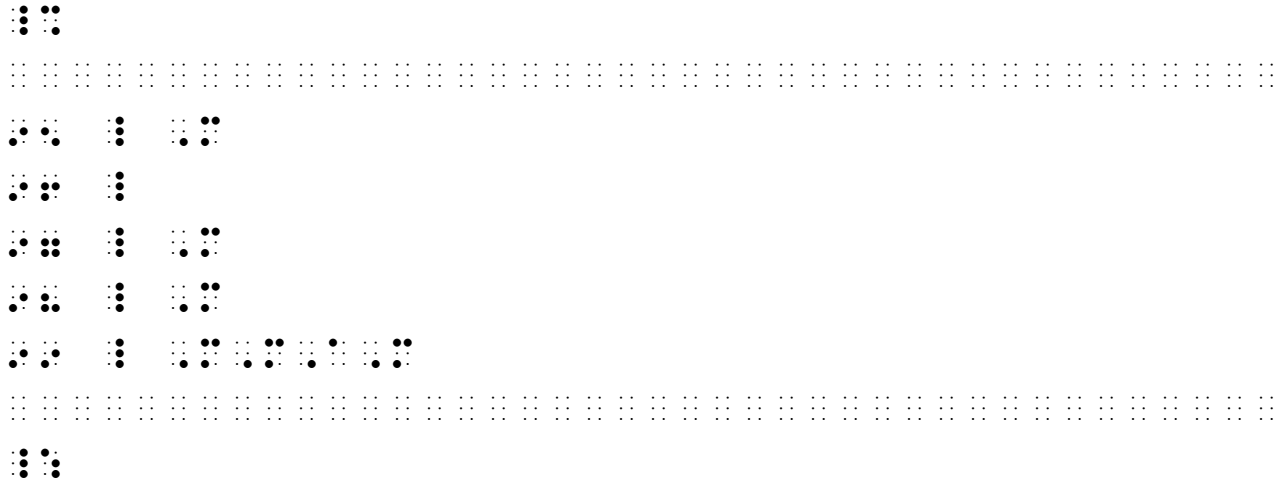
A blank entry in a stem-and-leaf plot is shown as blank space in braille. Do not transcribe a general omission symbol. Do not fill the width of the column with dot 5s.

The following rules also apply to the next example.

- The numeric indicator and the English-letter indicator are omitted in the body of the plot.
- Spaces are not inserted between single-letter data entries.
- Capital letters are capitalized individually.

Example 16-28

95	M
96	
97	M
98	M
99	M M A M



This plot has no key and no column headings—none are added in braille. The blank space in the second row indicates a blank entry in the leaf column. Single letters are unspaced and are capitalized individually.

16.18 Runovers Within the Table

16.18.1 **Print Runovers.** A runover of leaves shown in the print copy should be ignored if all leaves will fit on one row in braille. Use the full width of the available cells in the row.

Example 16-29

Stem	Leaf
1	2 2 2 3 4 4 5 5 5 5 6 6 7
	7 7 7 8 8 9 9 9 9 9
2	0 1 1 1 2 6 6 8
3	0

1 | 2 = 12

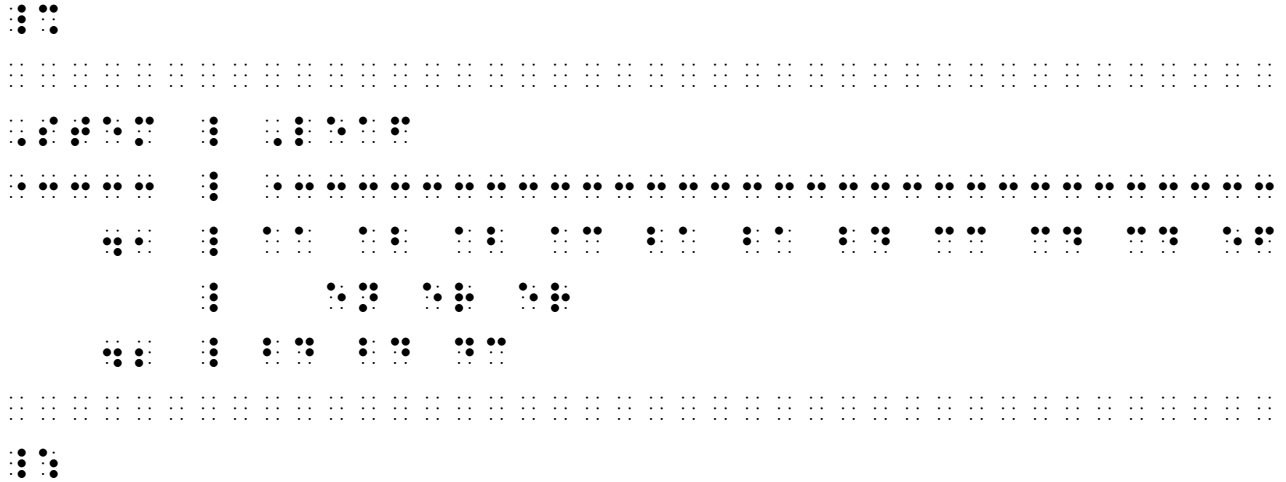
Braille representation of the table above. The first row of leaf data (2 2 2 3 4 4 5 5 5 5 6 6 7) is split across two lines in print but fits on one line in Braille. The second row of leaf data (7 7 7 8 8 9 9 9 9 9) also fits on one line in Braille. The third row (0 1 1 1 2 6 6 8) and fourth row (0) fit on one line each. The equation 1 | 2 = 12 is shown below the table.

The first row of leaf data requires two lines in print, but not in braille.

16.18.2 **Braille Runovers.** If leaf data will not fit on one line in the braille transcription, indent the runover line two cells to the right after using as much of the braille line as possible. Do not repeat the stem row heading. The next example illustrates, expanding the two-letter data example shown previously.

Example 16-30

Stem	Leaf
41	aa ab ab ac ba ba bd cc cd cd ef en er er
42	bd bd dc



The vertical line is transcribed on every line in the table.

16.19 Back-To-Back Plot

A back-to-back stem-and-leaf plot is used when two sets of data being compared. There are three columns. The stem is the middle column. Data is read from the stem outward, which means that data in the left leaf is read from right to left. There will be two keys. Here are the rules as they apply to the next example.

- *The Keys:* The left column's key is transcribed first, followed on the next line by the right column's key.
- *The Columns:* Column separation lines are transcribed below only the lines that contain column headings.
- *The Runovers:* Right leaf data are read from left to right; runovers to the right leaf data column are shifted two cells to the right. Left leaf data are read from right to left; runovers to the left leaf data column are shifted two cells to the left.

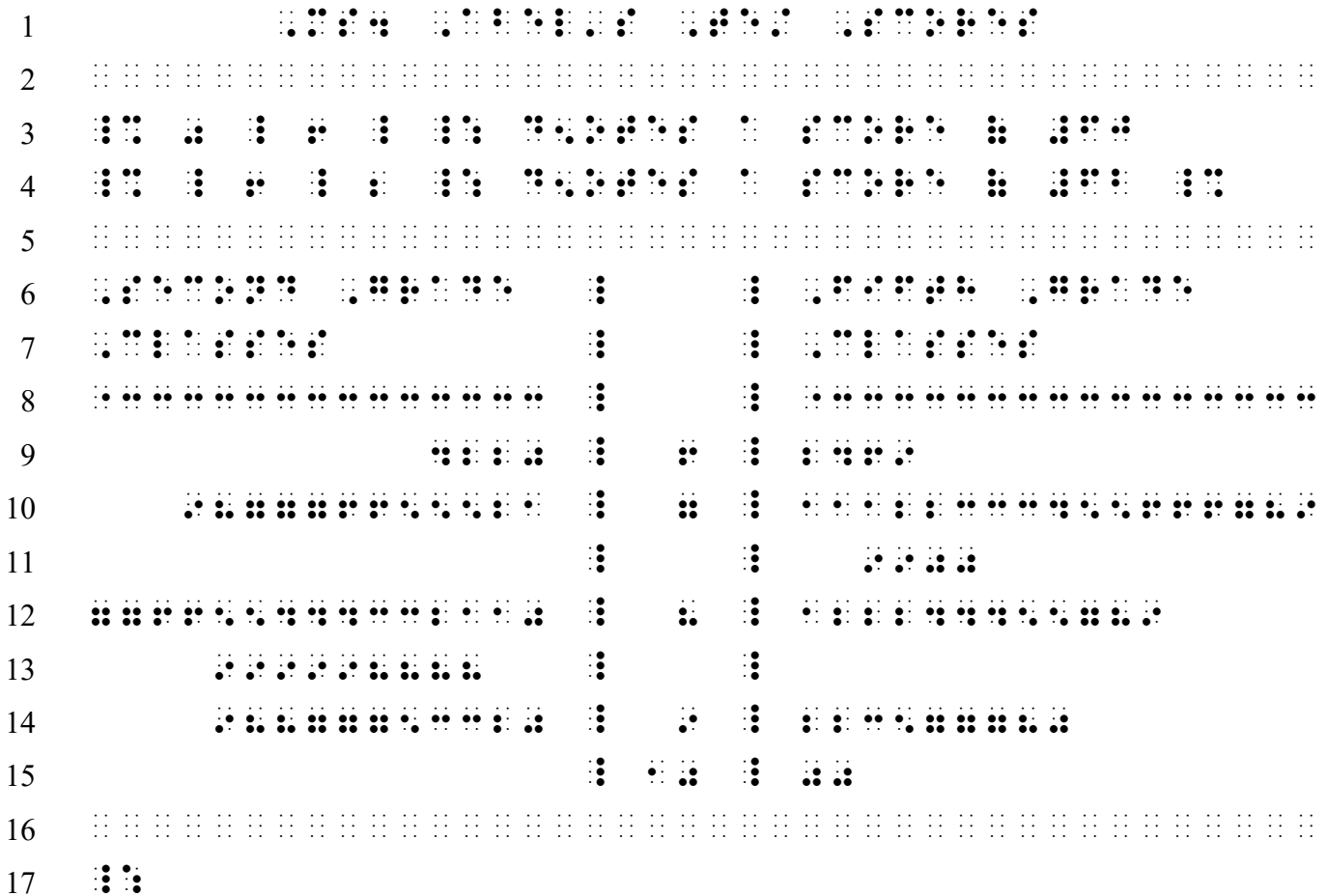
Example 16-31

Ms. Abel's Test Scores

Second Grade Classes		Fifth Grade Classes
4220	6	2469
987776655521	7	111223334556667899900
999998888776655444332110	8	122244455789
98877753320	9	223577780
	10	00

0 | 6 | denotes a score of 60

| 6 | 2 denotes a score of 62



- Line 3: Key to left column. *Italic typeform is disregarded.*
- Line 4: Key to right column. *Italic typeform is disregarded.*
- Line 5: A blank line precedes the plot.
- Lines 6-7: Column headings are uncontracted in a stem-and-leaf plot. These column headings require two lines. Runovers to column headings are blocked.
- Line 8: The stem column (middle column) has no heading, so no column separation line is transcribed.
- Line 11: Reading left to right, runovers to the right leaf data column are shifted two cells to the right.

Line 13: Reading right to left, runovers to the left leaf data column are shifted two cells to the left.

Line 15: The blank left-leaf entry in the last row is a blank space in braille.

Line 16: The blank line is required following the table.

Line 17: The Nemeth Code terminator is alone on this line, outside of the table arrangement.

PRACTICE 16H

A pet store owner constructed the following stem-and-leaf plot showing the number of guinea pigs at each of her seventeen stores.

Stem	Leaf
0	7 8
1	
2	0 6 8 8 8
3	0 2 6 6 7 8
4	1 2 6 6
5	

Key: 2 | 0 represents 20 guinea pigs

How many stores have fewer than 36 guinea pigs?

For further practice, see Appendix A—Reading Practice.

EXERCISE 16

Prepare Exercise 16 for your grader.

PRACTICE 16H

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
- 11
- 12
- 13
- 14
- 15
- 16
- 17
- 18
- 19

Braille practice content for lines 2 through 19. Line 2 contains a single row of Braille characters. Line 3 contains a single row of Braille characters. Line 4 contains a single row of Braille characters. Line 5 contains a single row of Braille characters. Line 6 contains a single row of Braille characters. Line 7 contains a single row of Braille characters. Line 8 contains a single row of Braille characters. Line 9 contains a single row of Braille characters. Line 10 contains a single row of Braille characters. Line 11 contains a single row of Braille characters. Line 12 contains a single row of Braille characters. Line 13 contains a single row of Braille characters. Line 14 contains a single row of Braille characters. Line 15 contains a single row of Braille characters. Line 16 contains a single row of Braille characters. Line 17 contains a single row of Braille characters. Line 18 contains a single row of Braille characters. Line 19 contains a single row of Braille characters.