LESSON 6

- LEVEL INDICATORS
    - Superscripts
    - Baseline Indicator
    - Subscripts
    - More about Superscripts and Subscripts

Format

- Itemized Material with Subdivisions
- Tabular Format
- Margins for Exercise Sets

Answers to Practice Material

LESSON PREVIEW

This lesson begins by looking at format. Itemized material with subdivisions is found throughout math textbooks, in exercise sets, and in answer sections. The rules differ somewhat from those followed in a nontechnical transcription. The topic of superscripts and subscripts is presented. Superscript, subscript, and baseline indicators are introduced. The lesson ends with another look at grouping signs as they relate to level indicators.

The Nemeth Code Book

The Nemeth Braille Code for Mathematics and Science Notation will be your primary source once you complete this course. This is a good time to begin supplementing the lesson material by reading about the topic in the Nemeth code book. Cross references are identified with a "NC" prefix. The Nemeth code book is available for downloading at www.brailleauthority.org.

Format

6.1 Margins for Itemized Material with Subdivisions (1-5; 3-5) [NC Rule 26]

To transcribe itemized material with lettered or numbered subdivisions, begin the main item designation in cell 1. Place any runovers in cell 5. Cell 3 is reserved for the beginning of each lettered or numbered subdivision. Any runovers to subdivisions also begin in cell 5. In the Nemeth Code, "subdivision" applies to all itemized sublevels regardless of the print indentation layout. In other words, for an exercise with any number of subdivisions, use margins 1-5 for the first level, and 3-5 for all sublevels.

- The first example shows two indentation levels in the print copy. In braille, all subdivisions begin in cell 3. All runovers are in cell 5.

Example 6-1

3. Consider the theory with primitive notions and operations as stated above. The axioms are the following:
   a. $\times$ is an associative operation.
   b. $(x \times y)' = y' \times x'$.
   c. If $x \times y = z \times z'$ for some z, then $x = y'$.
      (1) Show that the theory is *consistent*.
      (2) Show that this set of axioms is *dependent*.

```
⠼⠉⠲ ⠠⠒⠎⠊⠙⠻ ⠮ ⠮⠕⠗⠽ ⠾ ⠏⠗⠊⠍⠊⠞⠊⠧⠑ ⠝⠕⠰⠝⠎ ⠯
    ⠕⠏⠻⠁⠰⠝⠎ ⠵ ⠌⠁⠞⠫ ⠁⠃⠧⠲ ⠠⠮ ⠁⠭⠊⠕⠍⠎ ⠜⠑ ⠮
    ⠋⠕⠇⠇⠪⠬⠒
  ⠁⠲ ⠸⠩ ⠈⠡ ⠸⠱ ⠊⠎ ⠁⠝ ⠁⠎⠎⠕⠉⠊⠁⠞⠊⠧⠑ ⠕⠏⠻⠁⠰⠝⠲
  ⠰⠃⠲ ⠸⠩ ⠷⠭⠈⠡⠽⠾⠄ ⠨⠅ ⠽⠄⠈⠡⠭⠄⠸⠲
  ⠰⠉⠸⠲ ⠠⠄⠠⠊⠋ ⠭⠈⠡⠽ ⠨⠅ ⠵⠈⠡⠵⠄ ⠸⠱ ⠿ ⠐⠎ ⠰⠵⠂
    ⠮⠝ ⠸⠩ ⠭ ⠨⠅ ⠽⠄⠸⠱⠲
  ⠐⠣⠼⠁⠐⠜ ⠠⠩⠪ ⠞ ⠮ ⠮⠕⠗⠽ ⠊⠎ ⠨⠂⠒⠎⠊⠌⠢⠞⠲
  ⠐⠣⠼⠃⠐⠜ ⠠⠩⠪ ⠞ ⠹ ⠎⠑⠞ ⠷ ⠁⠭⠊⠕⠍⠎ ⠊⠎
    ⠨⠂⠙⠑⠏⠢⠙⠢⠞⠲
```

- The next example shows no text following the main item number, with subdivision a. beginning on the same line. In braille, subdivisions must begin in cell 3 on a new line.

Example 6-2

2. a. Write each of the following as a fraction or a mixed number. .12, .88, 3.49
b. Tell what digit is in tenth's place.

⠼⠃⠲
⠁⠲ ⠠⠺⠗⠊⠞⠑ ⠑⠁⠡ ⠷ ⠖ ⠋⠕⠇⠇⠪⠬ ⠵ ⠁ ⠋⠗⠁⠉⠰⠝ ⠕⠗
⠁ ⠍⠊⠭⠫ ⠝⠥⠍⠃⠻⠲ ⠸⠩ ⠼⠨⠂⠆⠠ ⠼⠨⠦⠦⠠
⠼⠒⠨⠲⠔ ⠸⠱
⠰⠃⠲ ⠠⠞⠑⠇⠇ ⠱⠁⠞ ⠙⠊⠛⠊⠞ ⠊⠎ ⠔ ⠞⠢⠹⠄⠎ ⠏⠇⠁⠉⠑⠲

- In the next example, the subdivisions are printed without indentation. In braille, subdivisions must begin in cell 3.

Example 6-3

1. Copy and multiply.

a. 2 × 2 × 3 × 3 × 4 × 4 × 5 × 5

b. 9 · 9 · 8 · 8 · 7 · 7 · 6 · 6

⠼⠁⠲ ⠠⠉⠕⠏⠽ ⠯ ⠍⠥⠇⠞⠊⠏⠇⠽⠲ ⠸⠩
⠰⠁⠸⠲ ⠼⠆⠈⠡⠆⠈⠡⠒⠈⠡⠒⠈⠡⠲⠈⠡⠲⠈⠡⠢⠈⠡⠢
⠰⠃⠸⠲ ⠼⠔⠡⠔⠡⠦⠡⠦⠡⠶⠡⠶⠡⠖⠡⠖ ⠸⠱

Reminder: Each subdivision's identifier begins in cell 3. The opening Nemeth Code indicator is placed at the end of the line of text that precedes the identifier.

6.1.1 **Paragraphs Within Itemized Material with Subdivisions (7-5).** If a main item or a subdivision has more than one paragraph, each new paragraph begins in cell 7, and its runovers begin in cell 5.

- The next example shows narrative displayed to the main item. *Braille Formats* guidelines dictate the layout for that paragraph. The second paragraph in subdivision b. is recognized by the beginning of a new, indented line.

Example 6-4

5. Does the method of creating whole number x-terms work with decimals? Consider this example.

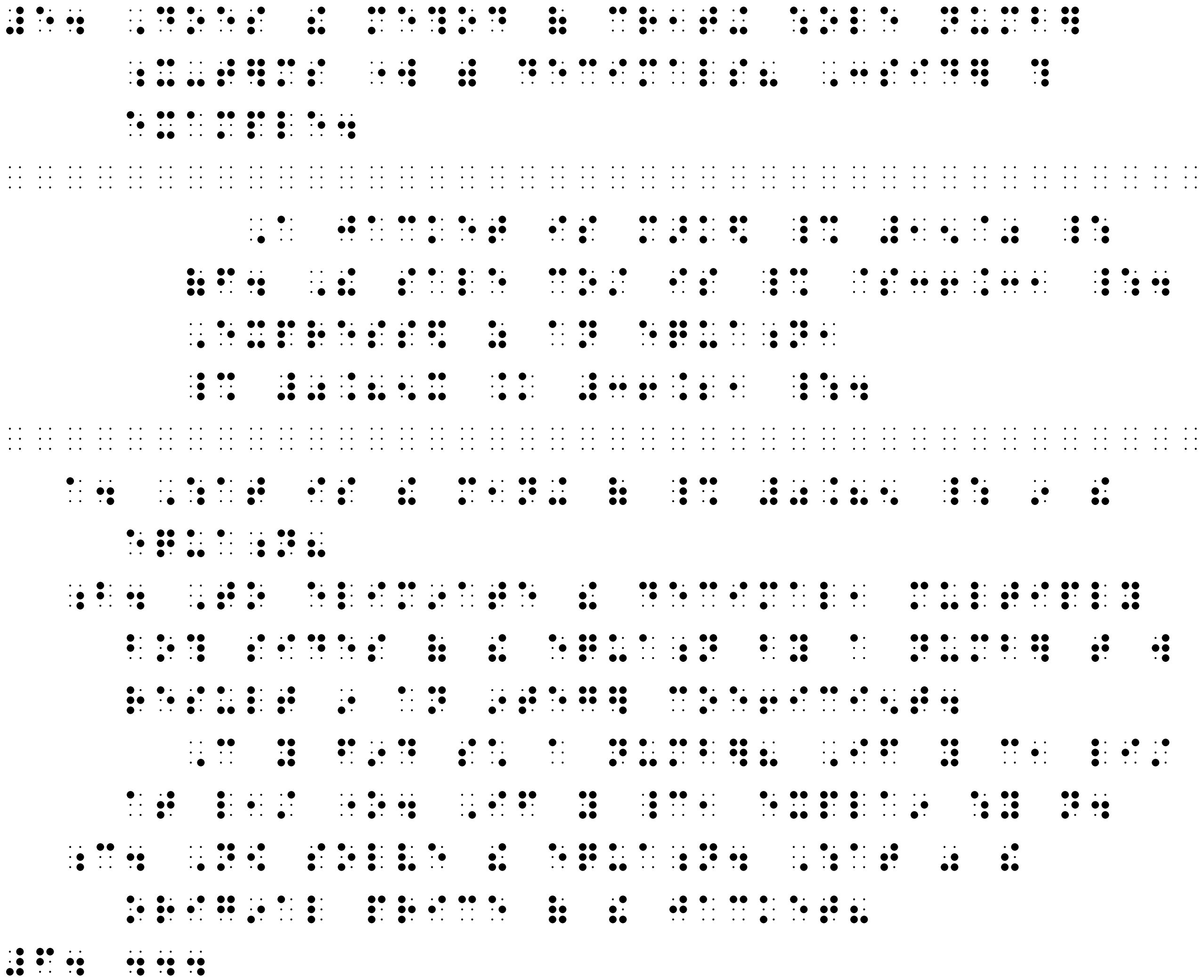
   A jacket is marked 15% off. The sale cost is \$36.31. Expressed as an equation, $0.85x = 36.21$.

   a. What is the meaning of 0.85 in the equation?

   b. To eliminate the decimal, multiply both sides of the equation by a number that will result in an integer coefficient.
      Can you find such a number? If you can, list at least one. If you cannot, explain why not.

   c. Now solve the equation. What was the original price of the jacket?

6. ...



In Nemeth, indentation of the first line of each new paragraph organizes the narrative. A blank line precedes and follows the displayed word problem, as directed by Braille Formats guidelines for displayed material, but the blocked *paragraph form is changed to indented form. Within the item/subdivision Nemeth format there are no blank lines between paragraphs or subdivisions.*

6.1.2 **Side-by-Side Layout in Print.** Even when subdivisions are printed on the same line across the page, in braille each identifier must begin on a new line. (This rule does not apply to spatial material, which will be studied later in this course.)

Example 6-5

1. Copy each problem below and perform the given operations on your TI-84 calculator.

   a. $170 \times 71 \div 8$ b. 1.25×12

```
⠼⠁⠲ ⠠⠉⠕⠏⠽ ⠑⠁⠡ ⠏⠗⠕⠃⠇⠑⠍ ⠆⠇ ⠯ ⠏⠻⠿⠍ ⠮ ⠛⠊⠧⠢
    ⠕⠏⠻⠁⠰⠝⠎ ⠕⠝ ⠽⠗ ⠠⠠⠞⠊⠤⠼⠓⠙ ⠉⠁⠇⠉⠥⠇⠁⠞⠕⠗⠲
    ⠸⠩
  ⠰⠁⠸⠲ ⠼⠂⠶⠴⠈⠡⠶⠂⠨⠌⠦
  ⠰⠃⠸⠲ ⠼⠂⠨⠆⠢⠈⠡⠂⠆ ⠸⠱
```

The opening Nemeth Code indicator falls in the runover cell (cell 5) of the main entry item. Each subdivision starts in cell 3 even though they are printed side by side.

Example 6-6

32. Which of the following numbers could not represent a probability?

    A. -0.1 B. 1 C. 1% D. 0.1

```
⠼⠉⠃⠲ ⠠⠱ ⠷ ⠮ ⠋⠕⠇⠇⠪⠬ ⠝⠥⠍⠃⠻⠎ ⠉⠙ ⠝ ⠗⠑⠏⠗⠑⠎⠢⠞
    ⠁ ⠏⠗⠕⠃⠁⠃⠊⠇⠰⠽⠦ ⠸⠩
  ⠰⠠⠁⠸⠲ ⠤⠼⠴⠨⠂
  ⠰⠠⠃⠸⠲ ⠼⠂
  ⠰⠠⠉⠸⠲ ⠼⠂⠈⠴
  ⠰⠠⠙⠸⠲ ⠼⠴⠨⠂ ⠸⠱
```

Notice that, although the numeral "1" (in subdivision B) can be transcribed in either code, code-switching clutter is avoided by staying in Nemeth.

PRACTICE 6A

23. Simplify and solve each equation below for x. Show your work and check your answer.

    a. $24 = 3x + 3$ b. $2(x - 6) = x - 14$

    c. $6 + 2.5x = 21$ d. $2(x + 4.5) = 32$

6.1.3 **Tabular Format.** When itemized material is arranged in tabular form so that items are numbered at the margin and subdivisions are aligned beneath lettered column headings, the material should be transcribed in one of the following ways, depending upon whether all of the columns can be accommodated across the braille page.

a. **When to Retain Column Format.** If all the columns can be accommodated across the braille page, the print columnar arrangement is followed. Each problem number begins in cell 1. The letter identifying each column is aligned with the first cell of the related column. A blank line is left above and below the lettered column headings. Two blank cells separate the columns. Guide dots are not used.

Example 6-7

	a	b	c
1.	$16 + 9$	$17 + 4$	$14 + 23$
2.	$46 + 15$	$87 + 12$	$95 + 54$
3.	$157 + 452$	$134 + 63$	$458 + 12$

```
⠸⠩

     ⠰⠁        ⠰⠃       ⠰⠉

⠼⠂⠸⠲ ⠼⠂⠖⠬⠔     ⠼⠂⠶⠬⠲    ⠼⠂⠲⠬⠆⠒
⠼⠆⠸⠲ ⠼⠲⠖⠬⠂⠢    ⠼⠦⠶⠬⠂⠆   ⠼⠔⠢⠬⠢⠲
⠼⠒⠸⠲ ⠼⠂⠢⠶⠬⠲⠢⠆  ⠼⠂⠒⠲⠬⠖⠒  ⠼⠲⠢⠦⠬⠂⠆

⠸⠱
```

b. **When Not To Retain Column Format.** If all the columns cannot be accommodated across the braille page, each subdivision in each problem must be lettered individually and the format in Section 6.1 must be followed.

Example 6-8

	a	b	c	d
1.	$16+9$	$17+4$	$14+23$	$37+18$
2.	$46+15$	$87+12$	$95+54$	$101+43$
3.	$157+452$	$134+63$	$458+12$	$935+298$

```
⠸⠩
⠼⠂⠸⠲
  ⠰⠁  ⠼⠂⠖⠬⠔
  ⠰⠃  ⠼⠂⠶⠬⠲
  ⠰⠉  ⠼⠂⠲⠬⠆⠒
  ⠰⠙  ⠼⠒⠶⠬⠂⠦
⠼⠆⠸⠲
  ⠰⠁  ⠼⠲⠖⠬⠂⠢
  ⠰⠃  ⠼⠦⠶⠬⠂⠆
  ⠰⠉  ⠼⠔⠢⠬⠢⠲
  ⠰⠙  ⠼⠂⠴⠂⠬⠲⠒
⠼⠒⠸⠲
  ⠰⠁  ⠼⠂⠢⠶⠬⠲⠢⠆
  ⠰⠃  ⠼⠂⠒⠲⠬⠖⠒
  ⠰⠉  ⠼⠲⠢⠦⠬⠂⠆
  ⠰⠙  ⠼⠔⠒⠢⠬⠆⠔⠦ ⠸⠱
```

6.1.4 **Varied Margins.** Runover margins for itemized material are determined individually for each item. That is, an item with no subdivisions will be (1-3); the next item may have subdivisions and so will be (1-5; 3-5), etc.

Example 6-9

1. What is the probability of rolling two 4's in a row with a standard 6-sided number cube?
2. Iggie designed two dartboards for the math meet. (See Figure 2.)
    a. What is the probability of landing in section A on Dartboard 1?
    b. What is the probability of landing in section A on Dartboard 2?

```
⠼⠁⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠏⠗⠕⠃⠁⠃⠊⠇⠰⠽ ⠷ ⠗⠕⠇⠇⠬ ⠞⠺⠕
  ⠼⠙⠄⠎ ⠔ ⠁ ⠗⠪ ⠾ ⠁ ⠌⠯⠜⠙ ⠼⠋⠤⠎⠊⠙⠫ ⠝⠥⠍⠃⠻
  ⠉⠥⠃⠑⠦
⠼⠃⠲ ⠠⠊⠶⠊⠑ ⠙⠑⠎⠊⠛⠝⠫ ⠞⠺⠕ ⠙⠜⠞⠃⠕⠜⠙⠎ ⠿ ⠮ ⠍⠁⠹
    ⠍⠑⠑⠞⠲ ⠐⠣⠠⠎⠑⠑ ⠠⠋⠊⠛⠥⠗⠑ ⠼⠃⠲⠐⠜
  ⠁⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠏⠗⠕⠃⠁⠃⠊⠇⠰⠽ ⠷ ⠇⠯⠬ ⠔ ⠎⠑⠉⠰⠝
    ⠠⠁ ⠕⠝ ⠠⠙⠜⠞⠃⠕⠜⠙ ⠼⠁⠦
  ⠰⠃⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠏⠗⠕⠃⠁⠃⠊⠇⠰⠽ ⠷ ⠇⠯⠬ ⠔ ⠎⠑⠉⠰⠝
    ⠠⠁ ⠕⠝ ⠠⠙⠜⠞⠃⠕⠜⠙ ⠼⠃⠦
```

LEVEL INDICATORS

[NC Rule 14]

6.2 Definition

A mathematical expression can contain symbols placed above or below the baseline level (the normal line of type in print). Superscripts appear above the baseline; subscripts appear below the baseline. Here are some expressions which contain superscripts and subscripts. The baseline is marked for orientation.

baseline 10^5 $\mathrm{H_2O}$ -4 °C πr^2 ${}^{235}_{92}\mathrm{U}$ _baseline_

The font size is increased for most examples in this lesson to help in determining the levels.

Superscripts

6.3 Superscript Level Indicator

In braille, indicators are used to identify the level of a superscript or subscript. The superscript level indicator is used to show that the symbols immediately following it appear on the first level above the baseline of writing.

⠘	Superscript Indicator

⫸ 10^3 ⠼⠂⠴⠘⠒

⫸ $y = [g(x)]^n$ ⠽ ⠨⠅ ⠈⠷⠛⠷⠭⠾⠈⠾⠘⠝

The characters in the superscript are spaced according to the rules of the Nemeth Code.

⫸ $3^{-0.05T}$ ⠼⠒⠘⠤⠴⠨⠴⠢⠠⠞

A letter or a Roman numeral with a superscript does not need an English-letter indicator because it is not followed by a space.

⫸ x^3 ⠭⠘⠒

Example 6-10

Statements $\mathrm{i}^2$and ii^2.

⠠⠌⠁⠞⠑⠰⠞⠎ ⠸⠩ ⠊⠘⠆ ⠠⠄⠯ ⠊⠊⠘⠆ ⠸⠱⠲

Example 6-11

Theorems I^2 and V^2.

⠠⠮⠕⠗⠑⠍⠎ ⠸⠩ ⠠⠊⠘⠆ ⠠⠄⠯ ⠠⠧⠘⠆ ⠸⠱⠲

6.3.1 Abbreviations with Superscripts

a. When a superscript applies to an abbreviation, the level indicator is unspaced from the abbreviation.

⋙ 19 in.^{3} ⠼⠂⠔ ⠊⠝⠲⠘⠒

b. The English-letter indicator is used for a single-letter abbreviation, even when unspaced from an indicator.

⋙ 5.5 m^2 ⠼⠢⠨⠢ ⠰⠍⠘⠆

c. Inside the switches, an abbreviation with a superscript is punctuated in mathematical mode.

⋙ 4 cm^3, ⠼⠲ ⠉⠍⠘⠒⠠

Instructions: Begin with the opening Nemeth Code indicator in cell 1. Start the three-column list on the next line, duplicating the side-by-side column format shown in print. Place the Nemeth Code terminator in cell 1 on the line following the list. A blank line is required before beginning the paragraph text.

PRACTICE 6B

$6x^2$	$24^?$	x^{2a}
π^2	$(y-k)^3$	8^{10}
n^m	$A^{(k+1)+m}$	$q^{b(w-1)}$
$k^{0.27}$	$e^{3,000}$	e^-

Measurements of **length** are measurements in **one dimension.** They are labeled as cm, ft, m, etc. Measurements of **area** are measurements in **two dimensions.** They are labeled as cm^2, ft^2, m^2, etc. Measurements of **volume** are measurements in **three dimensions.** They are labeled as cm^3, ft^3, m^3, etc.

6.4 Returning to the Baseline Level

When a superscript is the last item in the expression, the following circumstances bring the reader back to the baseline level: a space, a comma followed by a space, or a punctuation indicator. "A space" includes the space before a Nemeth Code terminator and/or transition to a new braille line.

a. Space. A space returns the reader to the baseline.

⫸ 10^2 10^3 10^4 ⠼⠂⠴⠘⠆ ⠼⠂⠴⠘⠒ ⠼⠂⠴⠘⠲

⫸ 10^2 and 10^3 ⠼⠂⠴⠘⠆ ⠠⠄⠯ ⠼⠂⠴⠘⠒

Example 6-12

Which term of $(a + b)^{12}$ has the factor b^5 ?

⠠⠱ ⠞⠻⠍ ⠷ ⠸⠩ ⠷⠁⠬⠃⠾⠘⠂⠆ ⠸⠱ ⠓⠁⠎ ⠮ ⠋⠁⠉⠞⠕⠗
⠸⠩ ⠃⠘⠢ ⠸⠱⠦

Example 6-13

5.3×10^{-3} 5.3×10^{-1} 5.3×10^{0} 5.3×10^{3}

⠸⠩ ⠼⠢⠨⠒⠈⠡⠂⠴⠘⠤⠒ ⠼⠢⠨⠒⠈⠡⠂⠴⠘⠤⠂ ⠼⠢⠨⠒⠈⠡⠂⠴⠘⠴
⠼⠢⠨⠒⠈⠡⠂⠴⠘⠒ ⠸⠱

Example 6-14

Adding cubic meters: $2\text{ m}^3 + 2\text{ m}^3 = 4\text{ m}^3$

⠠⠁⠙⠙⠬ ⠉⠥⠃⠊⠉ ⠍⠑⠞⠻⠎⠒
⠸⠩ ⠼⠆ ⠰⠍⠘⠒ ⠬⠆ ⠰⠍⠘⠒ ⠨⠅ ⠼⠲ ⠰⠍⠘⠒ ⠸⠱

The superscript indicator is unspaced from the abbreviation. The required space which follows the abbreviation (and its superscript) returns the reader to the baseline.

b. Comma and Space. A comma followed by a space returns the reader to the baseline.

Example 6-15

Which is the largest area? 3 cm^2, 7 m^2, 9 ft.^2

⠠⠱ ⠊⠎ ⠮ ⠇⠜⠛⠑⠌ ⠜⠑⠁⠦ ⠸⠩ ⠼⠒ ⠉⠍⠘⠆⠠
⠼⠶ ⠰⠍⠘⠆⠠ ⠼⠔ ⠋⠞⠲⠘⠆ ⠸⠱

Example 6-16

Several ways to notate the number nine: $5+4,\ 3^2,\ 17-8,\ 9\times 1$.

⠀⠀⠠⠎⠐⠑⠁⠇ ⠺⠁⠽⠎ ⠞⠕ ⠝⠕⠞⠁⠞⠑ ⠮ ⠝⠥⠍⠃⠻ ⠝⠔⠑⠒
⠸⠩ ⠼⠢⠬⠲⠠ ⠼⠒⠘⠆⠠ ⠼⠂⠶⠤⠦⠠ ⠼⠔⠈⠡⠂ ⠸⠱⠲

c. Punctuation Indicator. A punctuation indicator returns the reader to the baseline.

Example 6-17

5×5 can also be expressed as 5^2. 5^3 is another way to write ____.

⠀⠀⠸⠩ ⠼⠢⠈⠡⠢ ⠸⠱ ⠉ ⠁⠇ ⠆ ⠑⠭⠏⠗⠑⠎⠎⠫ ⠵
⠸⠩ ⠼⠢⠘⠆⠸⠲ ⠼⠢⠘⠒ ⠸⠱ ⠊⠎ ⠁⠝⠕⠮⠗ ⠺⠁⠽ ⠞⠕ ⠺⠗⠊⠞⠑
⠨⠤⠲

- Notice how the baseline level is re-established after each of the six superscripts in the next example.

Example 6-18

Chemistry The Avogadro constant is $6.022\ 140\ 857\times 10^{23}\ \mathrm{mol}^{-1}$;
$2.73159734(12)\times 10^{26}(\mathrm{lb\text{-}mol})^{-1}$, or $1.707248434(77)\times 10^{25}(\mathrm{oz\text{-}mol})^{-1}$,
expressed as pound-mole or ounce-mole.

1 ⠀⠀⠘⠂⠠⠡⠑⠍⠊⠌⠗⠽ ⠠⠮ ⠠⠁⠧⠕⠛⠁⠙⠗⠕ ⠒⠌⠁⠝⠞ ⠊⠎
2 ⠸⠩ ⠼⠖⠨⠴⠆⠆ ⠂⠲⠴ ⠦⠢⠶ ⠈⠡⠂⠴⠘⠆⠒ ⠍⠕⠇⠘⠤⠂ ⠸⠆
3 ⠼⠆⠨⠶⠒⠂⠢⠔⠶⠒⠲⠷⠂⠆⠾⠈⠡⠂⠴⠘⠆⠖ ⠷⠇⠃⠤⠍⠕⠇⠾⠘⠤⠂⠠ ⠠⠄⠕⠗
4 ⠼⠂⠨⠶⠴⠶⠆⠲⠦⠲⠒⠲⠷⠶⠶⠾⠈⠡⠂⠴⠘⠆⠢ ⠷⠕⠵⠤⠍⠕⠇⠾⠘⠤⠂ ⠸⠱⠂
5 ⠑⠭⠏⠗⠑⠎⠎⠫ ⠵ ⠏⠨⠙⠤⠍⠕⠇⠑ ⠕⠗ ⠳⠝⠉⠑⠤⠍⠕⠇⠑⠲

The baseline is re-established as follows:

Lines 2, 3, and 4: the space before each abbreviation (mol, (lb-mol), and (oz-mol));
Line 2: the punctuation indicator before the semicolon;
Line 3: the math comma and the following space ;
Line 4: the space before the Nemeth Code terminator.

6.5 Raised Hollow Dot

The hollow dot in the superscript position may be used to represent degrees of angle or degrees of temperature. Transcription of the "degrees" symbol requires a switch to Nemeth.

⠨⠡	Hollow Dot	∘
⠘⠨⠡	Raised Hollow Dot (representing degrees)	

Example 6-19

A right triangle contains one 90° angle.

⠠⠁ ⠐⠗ ⠞⠗⠊⠁⠝⠛⠇⠑ ⠒⠞⠁⠔⠎ ⠐⠕ ⠸⠩ ⠼⠔⠴⠘⠨⠡ ⠸⠱
⠁⠝⠛⠇⠑⠲

Example 6-20

Average normal body temperature is 98.6° F.

⠠⠁⠧⠻⠁⠛⠑ ⠝⠕⠗⠍⠁⠇ ⠃⠕⠙⠽ ⠞⠑⠍⠏⠻⠁⠞⠥⠗⠑ ⠊⠎
⠸⠩ ⠼⠔⠦⠨⠖⠘⠨⠡ ⠰⠠⠋⠲ ⠸⠱

> *Recall from Lesson 3, Section 3.4, that an abbreviation is transcribed in the same code as its associated number, and that, in an isolated problem where there are no context clues to determine whether a period applies to the abbreviation or merely ends the sentence, to assume that it applies to the abbreviation.*

PRACTICE 6C

1. Tell what number each of the following represents: 6^2, 5^{-3}, and $(3.15)^4$.
2. Which expression denotes the area of a circle?

   a. 360° b. πr^2 c. πd
3. The symbol for "feet squared" is ft^2. $1\text{ ft} \times 1\text{ ft} = 1\text{ ft}^2$
4. 6.02×10^{23} is an approximation of Avogadro's number.
5. $144\text{ ft}^2 + 144\text{ ft}^2 = \ldots$
6. Convert 100° F to degrees Celsius.

Introduction to the Baseline Indicator

6.6 Function of the Baseline Indicator

In an unspaced expression, a return to the baseline level is brought about by the use of the baseline indicator. Notice that the baseline indicator is the same symbol as the multipurpose indicator—dot 5. The indicator's function is understood in context.

⠐	`Baseline Indicator`

⫸ $a^2 + b^2 + c^2$ ⠁⠘⠆⠐⠬⠃⠘⠆⠐⠬⠉⠘⠆

Example 6-21

The Pythagorean equation relates the side lengths of a right triangle as $a^2 + b^2 = c^2$.

⠠⠮ ⠠⠏⠽⠹⠁⠛⠕⠗⠂⠝ ⠑⠟⠥⠁⠰⠝ ⠗⠑⠇⠁⠞⠑⠎ ⠮ ⠎⠊⠙⠑
⠇⠢⠛⠹⠎ ⠷ ⠁ ⠐⠗ ⠞⠗⠊⠁⠝⠛⠇⠑ ⠵
⠸⠩ ⠁⠘⠆⠐⠬⠃⠘⠆ ⠨⠅ ⠉⠘⠆ ⠸⠱⠲

The baseline is re-established before the plus sign.

Example 6-22

Draw a 30°-60°-90° triangle.

⠠⠙⠗⠁⠺ ⠁ ⠸⠩ ⠼⠒⠴⠘⠨⠡⠐⠤⠖⠴⠘⠨⠡⠐⠤⠔⠴⠘⠨⠡ ⠸⠱
⠞⠗⠊⠁⠝⠛⠇⠑⠲

The baseline is re-established before each hyphen.

Example 6-23

Solve. $(x^2 + y^2) - (x^2 + y^2)$

⠠⠎⠕⠇⠧⠑⠲ ⠸⠩ ⠷⠭⠘⠆⠐⠬⠽⠘⠆⠐⠾⠤⠷⠭⠘⠆⠐⠬⠽⠘⠆⠐⠾ ⠸⠱

The baseline is re-established before each plus sign and before each right parenthesis.

6.6.1 **Degrees Fahrenheit and Degrees Celsius.** The abbreviations F (Fahrenheit) and C (Celsius) are given special consideration. Follow print spacing when these abbreviations are printed with a degree sign (raised hollow dot). A baseline indicator will be required when the degree sign is unspaced from the letter. Note that, when the letter stands alone, an English-letter indicator is required, but when the letter is unspaced from the raised hollow dot, an English-letter indicator is no longer needed. F and C are punctuated in literary mode when spaced away from the degree sign. When the symbols are unspaced, the combination is mathematical and so mathematical punctuation is applied.

- 98.6° F, 99° F

  ⠼⠔⠦⠨⠖⠘⠨⠡ ⠰⠠⠋⠂ ⠼⠔⠔⠘⠨⠡ ⠰⠠⠋

- 20 °C, 30 °C

  ⠼⠆⠴ ⠘⠨⠡⠐⠠⠉⠠ ⠼⠒⠴ ⠘⠨⠡⠐⠠⠉

Example 6-24

Expressed in degrees Celsius, normal human body temperature is 37 °C.

⠠⠑⠭⠏⠗⠑⠎⠎⠩ ⠔ ⠙⠑⠛⠗⠑⠑⠎ ⠠⠉⠑⠇⠎⠊⠥⠎⠂ ⠝⠕⠗⠍⠁⠇
⠓⠥⠍⠁⠝ ⠃⠕⠙⠽ ⠞⠑⠍⠏⠻⠁⠞⠥⠗⠑ ⠊⠎
⠸⠩ ⠼⠒⠶ ⠘⠨⠡⠐⠠⠉ ⠸⠱⠲

Example 6-25

Express normal human body temperature in °C; in °F.

⠠⠑⠭⠏⠗⠑⠎⠎ ⠝⠕⠗⠍⠁⠇ ⠓⠥⠍⠁⠝ ⠃⠕⠙⠽ ⠞⠑⠍⠏⠻⠁⠞⠥⠗⠑
⠔ ⠸⠩ ⠘⠨⠡⠐⠠⠉⠸⠆ ⠠⠄⠔ ⠘⠨⠡⠐⠠⠋ ⠸⠱⠲

PRACTICE 6D

(1) $4(x-y)^3-2(x-y)^3$

(2) $3a^3b+6a^6b^2+9a^9b^3$

(3) Convert 10° F to °C. Express 10 °C in °F.

(4) $V=2\pi^2Rr^2$

(5) x^n-dimensional system

(6) $6^2\times 6^3=6^{2+3}=6^5$

(7) $2(36x^2-1)=2(6x)^2-1^2)$, which factors into $2(6x+1)(6x-1)$.

6.7 Higher Levels of Writing

Superscripts may carry superscripts of their own. In such cases, the superscript indicator is doubled, tripled, etc. to indicate superscripts on the second, third, or higher levels of writing.

⠘⠘	Superscript with Superscript (two levels above the baseline)
⠘⠘⠘	Superscript with Superscript with Superscript (three levels above the baseline)

⫸ $n^{a^{b}}$ ⠝⠘⠁⠘⠘⠃

⫸ $n^{a+1^{b+1^{c+1}}}$ ⠝⠘⠁⠬⠂⠘⠘⠃⠬⠂⠘⠘⠘⠉⠬⠂

If the print copy is difficult to analyze, a straightedge and a magnifier will help to determine the levels. In general, the print font gets smaller as the level gets higher.

6.7.1 **Combinations.** Each level indicator is read as it relates to the baseline level. The effect of one level indicator is terminated by another level indicator. Keep this in mind as you read the following examples.

⫸ $a^{(m^{n})}$ ⠁⠘⠷⠍⠘⠘⠝⠘⠾

The left and right parenthesis and the letter "m" are one level above the baseline; the letter "n" is two levels above the baseline. Notice the superscript indicator before the right parenthesis.

⫸ $10^{(^{-}4)}$ ⠼⠂⠴⠘⠷⠘⠘⠤⠘⠲⠾

The left and right parenthesis and the numeral "4" are one level above the baseline; the minus sign is two levels above the baseline. Notice the superscript indicator before the number 4.

Example 6-26

Will $E[x^2] = e^{2(a+b^2)}$?

⠠⠺ ⠸⠩ ⠠⠑⠈⠷⠭⠘⠆⠐⠈⠾ ⠨⠅ ⠑⠘⠆⠷⠁⠬⠃⠘⠘⠆⠘⠾ ⠸⠱ ⠦

The brackets are on the baseline level. The second superscript is $2(a+b^2)$ *which means that the final "2" is at the supersuperscript level. Notice the baseline indicator before the right bracket and the superscript indicator before the right parenthesis.*

Example 6-27

True or false? $e^{1,000} \times e^{2,000} = e^{3,000}$

⠠⠞⠗⠥⠑ ⠕⠗ ⠋⠁⠇⠎⠑⠦
⠸⠩ ⠑⠘⠂⠠⠴⠴⠴⠐⠈⠡⠑⠘⠆⠠⠴⠴⠴ ⠨⠅ ⠑⠘⠒⠠⠴⠴⠴ ⠸⠱

When a space brings the reader back to the baseline, a baseline indicator is not needed.

6.8 Certain Raised Signs

Some signs are printed in a raised position but are not considered to be superscripts.

6.8.1 **Raised Ordinal.** *Braille Formats* states that raised ordinal endings are not considered to be superscript. This guideline is also followed in a technical transcription.

Example 6-28

Name the 3rd and 4th item in the series.

⠠⠐⠝ ⠮ ⠼⠉⠗⠙ ⠯ ⠼⠙⠞⠓ ⠊⠞⠑⠍ ⠔ ⠮ ⠎⠻⠊⠑⠎⠲

The ordinal endings are printed in the raised position.

Example 6-29

What is the $3n^{th}$ degree?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠸⠩ ⠼⠒⠝⠞⠓ ⠸⠱ ⠙⠑⠛⠗⠑⠑⠦

"th" is printed in the raised position.

6.8.2 **Prime Sign.** The prime sign appears to be raised in print but it assumes the same level as the number or letter to which it applies.

`⠄    Prime Sign   '`

⋙ $m'^{\,2}$ ⠍⠄⠘⠆

The prime sign belongs with the letter m. Only the number 2 is in the superscript position.

⋙ $A^{2''}$ ⠠⠁⠘⠆⠄⠄

The prime signs belong with the number 2 in the superscript.

Example 6-30

plane angle $\alpha = 30°2'8''$

⠏⠇⠁⠝⠑ ⠁⠝⠛⠇⠑ ⠸⠩ ⠨⠁ ⠨⠅ ⠼⠒⠴⠘⠨⠡⠐⠆⠄⠦⠄⠄ ⠸⠱

The prime signs are not superscripts. They apply to numbers at the baseline of writing.

6.8.3 **Apostrophe-s.** In an apostrophe-s ending, the apostrophe is at the same level as the "s". Because a punctuation indicator returns the reader to the baseline, a level indicator is inserted to maintain the level of the apostrophe-s.

⫸ $A^{m+m+m's}$ ⠠⠁⠘⠍⠬⠍⠬⠍⠘⠸⠄⠎

The apostrophe-s applies to m+m+m.

Compare to this example where the "s" and its apostrophe are printed on the baseline level.

⫸ A^{m+m+m}'s ⠠⠁⠘⠍⠬⠍⠬⠍⠸⠄⠎

The apostrophe-s applies to the entire expression A^{m+m+m}.

PRACTICE 6E

1. Use a calculator to find 9^{9^9}.
2. Find the r^{th} term of $(x + y)^n$.
3. Label the x^2's and x^3's.
4. What is the meaning of x''^3?
5. Simplify: $(x^3 - y^3)^2 - (x^3 + y^3)^2$.
6. $x^{y^n z}$ or $x^{y^2 z}$

Subscripts

6.9 Subscript Level Indicators

Except as stated in Section 6.11 below, the subscript level indicator is used to show that the symbols immediately following it appear on the first level below the baseline of writing. Note that the subscript indicator is the same symbol as the English-letter indicator—dots 56. The indicator's function is understood in context.

⠰ Subscript Indicator

⫸ f_n ⠋⠰⠝

⫸ $a_{(k+1)}$ ⠁⠰⠷⠅⠬⠂⠾

Subscripts may carry subscripts of their own. In such cases, the subscript level indicator is doubled, tripled, etc. to indicate subscripts on the second, third, or lower levels.

⠰⠰ Subscript with Subscript
(two levels below the baseline)

⠰⠰⠰ Subscript with Subscript with Subscript
(three levels below the baseline)

⫸ n_{x_y} ⠝⠰⠭⠰⠰⠽

⫸ $P_{x+1_{y+1_{z+1}}}$ ⠠⠏⠰⠭⠬⠂⠰⠰⠽⠬⠂⠰⠰⠰⠵⠬⠂

Note: The -?- in the second column is in the subscript position. The subscript of the second item in the fourth column is "minus two".

PRACTICE 6F

3_c	10_8	$?_3$	x_{2+k}
3_{five}	$10_{\text{-?-}}$	$Ca(OH)_2$	y_{-2}
y_{n_k}	P_{3n}	a_{m1}	a_{m_1}

6.10 Returning to the Baseline Level

The same circumstances discussed with superscripts bring the reader back to the baseline level from a subscript: a space, a comma followed by a space, a punctuation indicator, or a baseline indicator. "A space" includes the space before a Nemeth Code terminator and/or transition to a new braille line.

Example 6-31

Add in base twelve: $27\mathrm{TE}_{12}$ and $\mathrm{E5T}_{12}$; $4\mathrm{E}9_{12}$, $8\mathrm{T}2_{12}$, and $\mathrm{T0E}_{12}$.

⠠⠁⠙⠙ ⠔ ⠃⠁⠎⠑ ⠞⠺⠑⠇⠧⠑⠒ ⠸⠩ ⠼⠆⠶⠞⠑⠰⠂⠆ ⠠⠄⠯
⠼⠑⠢⠞⠰⠂⠆⠸⠆ ⠼⠲⠑⠔⠰⠂⠆⠠ ⠼⠦⠞⠆⠰⠂⠆⠠ ⠠⠄⠯
⠼⠞⠴⠑⠰⠂⠆ ⠸⠱⠲ ⠈⠨⠣⠰⠞ ⠯ ⠰⠑ ⠜⠑ ⠉⠁⠏⠊⠞⠁⠇⠊⠵⠫ ⠔
⠏⠗⠔⠞⠲⠈⠨⠜

Example 6-32

$27\mathrm{TE}_{12} + \mathrm{E5T}_{12} \ = \underline{\hspace{2em}}_{12}$

⠸⠩ ⠼⠆⠶⠞⠑⠰⠂⠆⠐⠬⠑⠢⠞⠰⠂⠆ ⠨⠅ ⠤⠤⠤⠤⠰⠂⠆ ⠸⠱
⠈⠨⠣⠰⠞ ⠯ ⠰⠑ ⠜⠑ ⠉⠁⠏⠊⠞⠁⠇⠊⠵⠫ ⠔ ⠏⠗⠔⠞⠲⠈⠨⠜

Note that the level indicator is unspaced from the long dash to which it applies.

Example 6-33

$(R_H T_H) + (R_S T_S)$

⠸⠩ ⠷⠠⠗⠰⠠⠓⠐⠠⠞⠰⠠⠓⠐⠾⠬⠷⠠⠗⠰⠠⠎⠐⠠⠞⠰⠠⠎⠐⠾ ⠸⠱

Letters H and S are subscripts. Because H and S are not "single letters" (they are unspaced), an English-letter indicator is not needed. Dots (56) is functioning as a subscript indicator throughout this expression.

PRACTICE 6G

1. Name the numeral in base ten equal to:

   a. 47_8 b. 34_6 c. 1101_2

2. $\mathrm{C}_{\mathrm{hex}} = 12_{\mathrm{dec}} = 14_{\mathrm{oct}}$
3. What do we know if $P_{n_r} = \left(x_{n_k}, y_{n_k}\right)$?
4. $7_8 - 4_8$

6.11 Special Case—Nonuse of the Subscript Level Indicator

A special rule applies to certain subscripts. The subscript level indicator is not used when all of the following apply:

i. the subscript is *numeric*;

ii. the numeral is a *right subscript to a letter;*

iii. the subscript is on the *first level* below the baseline of writing.

Example 6-34

Apply Definitions i_2 and I_2 .

⠠⠁⠏⠏⠇⠽ ⠠⠙⠑⠋⠔⠊⠰⠝⠎ ⠸⠩ ⠊⠆ ⠠⠄⠯ ⠠⠊⠆ ⠸⠱⠲

Each numeral 2 is printed at the subscript level.

a. When a subscript does not require a subscript indicator, a return to the baseline is implied following the numeric subscript. A baseline indicator is not needed.

⫸ $\mathrm{H_2O}$ ⠠⠓⠆⠠⠕

Letters H and O are on the baseline of writing. Numeral 2 is a subscript.

⫸ $a_1a_2 + b_1b_2 + c_1c_2$ ⠁⠂⠁⠆⠬⠃⠂⠃⠆⠬⠉⠂⠉⠆

All numerals are subscripts. A baseline indicator is not needed to show that the letters and the plus signs are on the baseline level because no subscript indicators are used.

⋙ $Cd(NO_3)_2$ ⠠⠉⠙⠷⠠⠝⠠⠕⠒⠾⠰⠆

The subscripts are numerals 3 and 2. Everything else is on the baseline of writing, including the parentheses. The second subscript requires a level indicator because it is not a right subscript to a letter.

6.11.1 **Further Conditions.** The definition of "numeric" and "letter" include the following.

a. **Numeric Subscript.** The numeric subscript may contain a segmenting comma or a decimal point.

⋙ $x_{1,000}$ ⠭⠂⠠⠴⠴⠴

The numeral 1,000 is a right subscript to the letter x.

⋙ $x_{5.3}$ ⠭⠢⠨⠒

The numeral 5.3 is a right subscript to the letter x.

b. **"Letter"**

(1) The letter may carry one or more primes.

⋙ x'_2 ⠭⠄⠆

The numeral 2 is a right subscript to x'.

(2) The letter may be taken from any alphabet.

⋙ $\aleph_0$ ⠠⠠⠁⠴

Hebrew aleph, subscript zero.

(3) The letter may also carry a superscript. (See 6.17, *Simultaneous Superscripts and Subscripts*)

(4) The letter may be part of a two-letter chemical symbol.

⋙ Cl_4 ⠠⠉⠇⠲

Cl (the two-letter chemical symbol for chlorine), subscript 4.

c. **Not a Subscript.** When a number is printed on the baseline to the right of a letter, unspaced, a multipurpose indicator (dot 5) is inserted between two signs to indicate that they are printed horizontally, and that the number is not a subscript.

⋙ F1 ⠠⠋⠐⠂

6.11.2 **Restrictions.** Just because a numeral is a right subscript to a letter does not mean that the special rule can be applied. The subscript level indicator must be used if any of the following conditions apply:

(1) If the letter is functioning as a numeral in a nondecimal numeration system, a subscript indicator is required.

⫸ TE_{12} ⠼⠞⠑⠰⠂⠆

The subscript 12 means base-12 numeration.

(2) If the letter is part of a word or abbreviation, a subscript indicator is required.

⫸ five_3 ⠋⠊⠧⠑⠰⠒

(3) If the subscript contains any symbol other than a numeral with its comma or decimal point, a subscript indicator is required.

⫸ x_{2k} ⠭⠰⠆⠅

⫸ $x_{2'}$ ⠭⠰⠆⠄

⫸ x_{-2} ⠭⠰⠤⠆

(4) If the subscript carries a superscript or subscript of its own, a subscript indicator is required.

⫸ x_{2_n} ⠭⠰⠆⠰⠰⠝

(5) Numeric subscripts on the second or lower levels always require their appropriate subscript level indicators.

⫸ P_{n_1} ⠠⠏⠰⠝⠰⠰⠂

6.11.3 **Summary.** The rules regarding use/nonuse of the subscript indicator are summarized below.

a. A subscript level indicator is not used before a numeric subscript on the first level below the baseline of writing when the numeral is a right subscript to a letter. Furthermore, the following may also be true.

- The numeric subscript may contain a segmenting comma or a decimal point.
- The baseline letter may carry one or more primes or a superscript.
- The baseline letter may be from any alphabet.
- The baseline "letter" may be a two-letter chemical element.

b. A subscript level indicator must be used in the following circumstances.

- A numeral on the first level below the baseline requires a subscript indicator if the subscript contains any symbol other than a numeral with its comma or decimal point.
- A numeral on the first level below the baseline requires a subscript indicator if the subscript carries a superscript or subscript of its own.
- A subscript on the second or lower level always requires the appropriate subscript level indicator.
- If the subscript is a letter that is part of a word or an abbreviation, a subscript indicator is required.
- If the subscript is a letter functioning as a numeral in a nondecimal numeration system, a subscript indicator is required.

PRACTICE 6H

1) These expressions need subscript indicators: y_{-2}, x_{2+k}, a_{m1}, x_{3_n}, x_{y_2}.
2) These expressions do not need a subscript indicator: x_1, ax_2, CO_2, $z_{.4}$, β_2.
3) Decide whether these expressions require a subscript indicator and transcribe them correctly: shape$_4$, Q'_2, $\mathrm{C}_6\mathrm{Fe}_2\mathrm{O}_{12}$, n_k, x_{2k}, $P_{r_{s_t}}$, D_{56}, $\mathrm{G}_{9,999}$, and the hexadecimal number $2\mathrm{E}6\mathrm{B}_{16}$.
4) **Chemistry.** While $\mathrm{Na}_2\mathrm{ZnCl}_4$ could be cooled in the normal way, $\mathrm{Na}_2[\mathrm{CoCl}_4]$ had to be quenched in the liquid N_2.
5) $f_1(x) = g(x) \cdot q_2(x) + f_2(x)$

6.12 Spaces Within Superscripts and Subscripts

A space usually returns the reader to the baseline. Various strategies are used to retain the level in effect when a space occurs within a superscript or a subscript.

6.12.1 **Commas.** *Review:* A comma followed by a space re-establishes the baseline. The return to the baseline starts at the comma. No baseline indicator is necessary.

⫸ x^2, y^2, z^2 ⠭⠘⠆⠠ ⠽⠘⠆⠠ ⠵⠘⠆

⫸ (a_1, a_{1i}, a_{2i}) ⠷⠁⠂⠠ ⠁⠰⠂⠊⠠ ⠁⠰⠆⠊⠐⠾

Example 6-35

Add: 2_{five}, 3_{five}, and 4_{five}.

⠠⠁⠙⠙⠒ ⠸⠩ ⠼⠆⠰⠋⠊⠧⠑⠠ ⠼⠒⠰⠋⠊⠧⠑⠠ ⠠⠄⠯
⠼⠲⠰⠋⠊⠧⠑ ⠸⠱⠲

The commas are on the baseline of writing. The mathematical comma is transcribed even though these subscripts are words.

a. **The Contracted Comma.** When a comma separates individual items <u>within</u> a superscript or subscript, the contracted comma symbol is used. This symbol represents the comma or the comma and a space. The level in effect continues through this symbol.

⠪	`Contracted Comma plus Optional Space`	`,`

⋙ $x^{1,2}$ ⠭⠘⠂⠪⠆

⋙ $x_{i,j,k}$ ⠭⠰⠊⠪⠚⠪⠅

The contracted comma symbol is not to be used to replace a comma on the baseline of writing. A dot 2 or dot 6 comma is used for the comma that is part of the sentence structure.

Example 6-36

If ______, then $x_{i,j,k}$, as shown below.

⠠⠊⠋ ⠨⠤⠂ ⠮⠝ ⠸⠩ ⠭⠰⠊⠪⠚⠪⠅ ⠸⠱⠂ ⠵ ⠩⠪⠝ ⠆⠇⠲

Contracted commas are used within the subscript; a dot 2 comma is used for the comma that is part of the sentence structure (on the baseline level) outside of the Nemeth switches.

Example 6-37

Consider $x_{i,j}$, $x_{j,k}$, and $x_{i,k}$.

⠠⠒⠎⠊⠙⠻ ⠸⠩ ⠭⠰⠊⠪⠚⠠ ⠭⠰⠚⠪⠅⠠ ⠠⠄⠯ ⠭⠰⠊⠪⠅ ⠸⠱⠲

Contracted commas are used within each subscript; a dot 6 comma is used for the commas that are part of the sentence structure (on the baseline level) within the Nemeth switches.

In the next example, the commas separate three terms at the subscript level, but each "n" has a subscript as well. The correct level of each comma is shown by shrewd use of level indicators.

⫸ P_{n_1,n_2,n_3} ⠠⠏⠰⠝⠰⠰⠂⠰⠪⠝⠰⠰⠆⠰⠪⠝⠰⠰⠒

6.12.2 **Words, Abbreviations, and Letters.** If a subscript or a superscript contains a space between words, abbreviations, or letters, the level must be restated after the space. The level indicator is unspaced from the following word, abbreviation, or letter.

⫸ $n_{\text{obtuse angles}}$ ⠝⠰⠕⠃⠞⠥⠎⠑⠀⠰⠁⠝⠛⠇⠑⠎

The subscript indicator before the word "angles" shows that the word is in the subscript position.

⫸ $n_{\text{st. angles}}$ ⠝⠰⠎⠞⠲⠀⠰⠁⠝⠛⠇⠑⠎

The abbreviation st. (for "straight") has a related period. No punctuation indicator is used with abbreviations.

Note that an English letter will not need an English-letter indicator in such a case because it is no longer a "single letter" as defined by the Nemeth Code. (See Section 3.10.1 in Lesson 3.) Using "surface S" as an example, as a superscript it is transcribed ⠘⠎⠥⠗⠋⠁⠉⠑⠀⠘⠠⠎. As a subscript it is transcribed ⠰⠎⠥⠗⠋⠁⠉⠑⠀⠰⠠⠎.

6.12.3 **Comparison Signs.** The space before a comparison sign returns the reader to the baseline, as illustrated in the next example.

⫸ $a^2 = a \cdot a$ ⠁⠘⠆⠀⠨⠅⠀⠁⠡⠁

The superscript is 2. The equals sign is on the baseline.

If the comparison sign is within a superscript or a subscript, the level is reinstated before the comparison sign. The indicator is unspaced from the comparison sign. The space after a comparison symbol preserves the level that is already in effect.

⫸ $S_{u\,=\,a}$ ⠠⠎⠰⠥⠀⠰⠨⠅⠀⠁

The subscript is u = a. The level in effect extends through the space following the comparison sign.

6.12.4 **Ellipsis or Long Dash in Superscripts and Subscripts**

a. **New Level.** When an ellipsis or a long dash is located at a different level from the material preceding it, the appropriate level indicator is used before the ellipsis or long dash. The level indicator is unspaced from the symbol.

Example 6-38

Fill in the blank with a simplified superscript. $10^{6+3} = 10^{—}$

⠠⠋⠊⠇⠇ ⠔ ⠮ ⠃⠇⠁⠝⠅ ⠾ ⠁ ⠎⠊⠍⠏⠇⠊⠋⠊⠫
⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞⠲ ⠸⠩ ⠼⠂⠴⠘⠖⠬⠒ ⠨⠅ ⠼⠂⠴⠘⠤⠤⠤⠤ ⠸⠱

b. **Same Level.** An ellipsis or a long dash within a superscript or a subscript assumes the level in effect. The level extends through the required spaces before and after the ellipsis or long dash.

⋙ $P_{n_1, n_2, \ldots}$ ⠠⠏⠰⠝⠰⠰⠂⠰⠪⠝⠰⠰⠆⠰⠪ ⠄⠄⠄

The letter n, the commas, and the ellipsis are at the subscript level. Even though a contracted comma includes the following space, there must be a space before an ellipsis unless it is immediately preceded by an indicator. The subscript level applied to the second comma extends through the space before the ellipsis.

⋙ $x^{1\,+\,3\,+\,5\,+\,\ldots\,+\,(2n-1)}$

⠭⠘⠂⠬⠒⠬⠢⠬ ⠄⠄⠄ ⠬⠷⠆⠝⠤⠂⠾

The superscript is $1 + 3 + 5 + \ \ldots + (2n - 1)$. *The superscript level extends through the spaces before and after the ellipsis.*

c. **Exception.** If an ellipsis or long dash in a superscript or subscript is followed by a sign of comparison or by literary text, reading returns to the baseline level without the need for a baseline indicator.

⋙ $10^{7+—} = 10^{21}$

⠼⠂⠴⠘⠶⠬ ⠤⠤⠤⠤ ⠨⠅ ⠼⠂⠴⠘⠆⠂

The first superscript is 7+___. The superscript level extends through the required space before the long dash. The sign of comparison after the dash is at the baseline level.

⋙ $10^{7+—}$ equals 10^{21}

⠼⠂⠴⠘⠶⠬ ⠤⠤⠤⠤ ⠠⠄⠑⠟⠥⠁⠇⠎ ⠼⠂⠴⠘⠆⠂

The first superscript is 7+___. The superscript level extends through the required space before the long dash. The word after the dash is at the baseline level.

6.12.5 **Ellipsis or Long Dash on the Baseline of Writing.** Because the space before an ellipsis or long dash maintains the level in effect, an indicator is required to return to the baseline when the symbol is printed there. The baseline indicator takes the place of the required space.

⋙ $a^1b^2c^3d^4 \ldots z^n$

⠁⠘⠂⠐⠃⠘⠆⠐⠉⠘⠒⠐⠙⠘⠲⠐⠄⠄⠄ ⠵⠘⠝

There is no need to indicate a return to the baseline after a numeric subscript that does not require a subscript indicator.

⋙ $r_1 \ldots r_n$ ⠗⠂ ⠄⠄⠄ ⠗⠰⠝

The ellipsis is printed on the baseline. The subscripts are 1 and n.

6.12.6 **Segmented Numbers.** The effect of a level indicator extends through the space inserted in a numeral for the purpose of dividing it into short regular groups of digits.

⋙ $e^{3.14159\ 26535}$ ⠑⠘⠒⠨⠂⠲⠂⠢⠔ ⠆⠖⠢⠒⠢

The superscript is 3.14159 26535

Instructions: Translate the simulated braille by writing the mathematical characters in the blank space below each line. Show spaces and levels clearly. Compare your translation to the print copy in the answer section.

PRACTICE 6I

⠸⠩

⠼⠂⠸⠲ ⠭⠰⠂⠪⠆ ⠌⠨⠅ ⠭⠘⠊⠪⠚

⠼⠆⠸⠲ ⠭⠰⠝⠤⠂⠪⠝⠤⠂⠠ ⠭⠰⠝⠤⠂⠪⠝⠠ ⠭⠰⠝⠪⠝⠤⠂

⠼⠒⠸⠲ ⠠⠁⠘⠝⠬⠝⠬⠝ ⠘⠁⠇⠇ ⠘⠝⠘⠸⠄⠎ ⠸⠱ ⠜⠑ ⠑⠟⠥⠁⠇⠲

⠼⠙⠲ ⠸⠩ ⠎⠈⠾⠰⠞ ⠰⠨⠅ ⠁

⠼⠢⠸⠲ ⠑⠘⠂⠠⠴⠴⠴

⠼⠖⠸⠲ ⠁⠘⠍⠬⠅⠐⠨⠌⠁⠘⠍ ⠨⠅ ⠁⠘⠅

⠼⠶⠸⠲ ⠠⠏⠰⠎⠰⠰⠂⠰⠄⠄⠄ ⠎⠰⠰⠆ ⠠⠄⠯ ⠠⠏⠰⠟⠰⠰⠗⠪⠎

⠼⠦⠸⠲ ⠼⠂⠴⠰⠿ ⠨⠅ ⠼⠖⠰⠦

⠼⠔⠸⠲ ⠁⠄⠂⠠ ⠁⠄⠆⠠ ⠄⠄⠄⠠ ⠁⠄⠰⠝ ⠸⠱ ⠜⠑ ⠮ ⠔⠧⠻⠎⠑⠎⠲

⠼⠁⠚⠲ ⠸⠩ ⠠⠠⠁⠴ ⠸⠱ ⠗⠑⠏⠗⠑⠎⠢⠞⠎ ⠮ ⠉⠜⠙⠔⠁⠇⠰⠽ ⠷ ⠮
    ⠎⠑⠞ ⠷ ⠁⠇⠇ ⠝⠁⠞⠥⠗⠁⠇ ⠝⠥⠍⠃⠻⠎⠲

More about Superscripts and Subscripts

6.13 Superscript and Subscript Combinations

Combinations of subscripts to superscripts or of superscripts to subscripts require level indicators composed of two or more braille symbols. Keeping in mind that level indicators relate to the baseline, transcribing these expressions is an exercise in logical thinking.

Superscripts with Subscripts

⠘⠰	`Superscript With Subscript`

⋙ x^{n_1} ⠭⠘⠝⠘⠰⠂

"n, subscript one" is in the superscript position. "1" is a supersubscript.

⋙ $2^{\mathrm{Y}_0} = \mathrm{Y}_1$ ⠼⠆⠘⠠⠽⠘⠰⠴ ⠨⠅ ⠠⠽⠂

"Y, subscript zero" is in the superscript position. "0" is a supersubscript.

Reminder: The subscript indicator is omitted for a numeric subscript to a letter only for subscripts that are located on the first level below the baseline of writing. The super/subscript indicator is needed to show a numeric subscript in the superscript position.

Subscripts with Superscripts

⠰⠘	`Subscript With Superscript`

⋙ x_{n^2} ⠭⠰⠝⠰⠘⠆

"n, superscript two" is in the subscript position. "2" is a subsuperscript.

⋙ P_{3^n} ⠠⠏⠰⠒⠰⠘⠝

"3, superscript n" is in the subscript position. "n" is a subsuperscript.

Reminder: The subscript indicator is required when a numeric subscript to a letter carries a superscript or subscript of its own.

Instructions: Descriptions are included in a right-hand column for your benefit—do not transcribe the descriptions. *Formatting Guidelines:* Transcribe the bold type as cell-5 headings. Place the opening Nemeth Code indicator on the next line and begin the list on the following line. Place the Nemeth Code terminator on the line following each list, in cell 1. Analyze the levels carefully as you proofread.

PRACTICE 6J

Superscripts with Subscripts

$2^{\aleph_0} = \aleph_1$	$\aleph_0$ *is in the superscript position.*
$a = 2^{k_1}$ and $b = 2^{k_2}$	k_1 *and* k_2 *are in the superscript position.*
$(ab)^x = 2^{k_1 x} \cdot 2^{k_2 x}$	x, $k_1 x$, *and* $k_2 x$ *are superscripts.*
$\mathrm{e}^{\mathrm{i}\theta_1}$ times $\mathrm{e}^{\mathrm{i}\theta_2}$ equals $\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}$	$\mathrm{i}\theta_1$, $\mathrm{i}\theta_2$, *and* $\mathrm{i}(\theta_1 + \theta_2)$ *are superscripts.*

Subscripts with Superscripts

Z_{5^n}	5^n *is in the subscript position.*
$7t_{s^4}$	s^4 *is in the subscript position.*

6.14 Left Subscripts and Superscripts

The appropriate level indicator is transcribed before a subscript or superscript printed to the left of its related sign.

- ⋙ $^{14}\mathrm{C}$ ⠘⠂⠲⠐⠠⠉
- ⋙ ${}_3\mathrm{x}_1$ ⠰⠒⠐⠭⠂
- ⋙ ${}_{48}\mathrm{C}_9 \times {}_4\mathrm{C}_4$ ⠰⠲⠦⠐⠠⠉⠔⠈⠡⠰⠲⠐⠠⠉⠲

The "9" and the second "4" are numeric subscripts to the right of a letter on the baseline of writing, so no subscript indicator is needed.

6.14.1 **Raised Negative Sign.** In some texts, negative numbers are shown with a raised negative sign. The raised position of the negative sign must be shown in braille. A numeric indicator is not required when the negative sign is raised.

- ⋙ $^{-}4$ ⠘⠤⠐⠲

Compare: A numeric indicator is required when the negative sign is not raised.

⫸ -4 ⠤⠼⠲

Example 6-39

Explain why $4 + {}^{-}4 = 0$.

⠠⠑⠭⠏⠇⠁⠔ ⠱⠽ ⠸⠩ ⠼⠲⠬⠘⠤⠐⠲ ⠨⠅ ⠼⠴ ⠸⠱⠲

Example 6-40

A and B are points with coordinates $(5, 2)$ and $({}^{-}1, 4)$ respectively.

⠠⠁ ⠯ ⠰⠠⠃ ⠜⠑ ⠏⠕⠔⠞⠎ ⠾ ⠉⠕⠕⠗⠙⠔⠁⠞⠑⠎
⠸⠩ ⠷⠢⠠ ⠆⠾ ⠠⠄⠯ ⠷⠘⠤⠐⠂⠠ ⠲⠾ ⠸⠱ ⠗⠑⠎⠏⠑⠉⠞⠊⠧⠑⠇⠽⠲

Compare the subscripts in the next two examples. The first subscript is -2. A subscript indicator is required because the subscript contains a symbol other than a numeral. In the second subscript, the negative sign is raised It is a left superscript to the subscript "2".

⫸ x_{-2} ⠭⠰⠤⠆

⫸ $x_{{}^{-}2}$ ⠭⠰⠘⠤⠰⠆

6.15 Further Combinations

The Nemeth code book illustrates additional combinations of superscripts and subscripts. Due to the obscurity of such complex combinations, only a few examples are shown in this lesson manual. Proper interpretation of these characters will require reading the surrounding text in order to apply the correct indicators. (Notice that the letter on the baseline of writing is often a larger font.)

⫸ ${}_{y}x^{n}$

n is on the baseline of writing and has a left subscript. Question: Is the subscript y^{x} *or* ${}_{y}x$ *?*

This transcription shows left subscript "y" with a subsuperscript "x".

⠰⠽⠰⠘⠭⠐⠝

This transcription shows left subscript "x" with a left subsubscript "y".

⠰⠰⠽⠰⠭⠐⠝

⋙ ${}_{x_y}n$

n is on the baseline of writing and has a left subscript. Question: Is the subscript x_y *or* ${}^{x}y$ *?*

This transcription shows left subscript "x" with a subsubscript "y".

⠰⠭⠰⠰⠽⠐⠝

This transcription shows left subscript "y" with a left subsuperscript "x".

⠰⠘⠭⠰⠽⠐⠝

6.16 Consecutive Superscripts and Subscripts

When a character with right superscript or subscript is adjacent to a character with a left superscript or subscript, each superscript or subscript has its own level indicator. A multipurpose indicator is inserted between the consecutive symbols. A space may appear in print to help visually differentiate the two super/subscripts, but no space comes between the symbols in braille.

⋙ $p^{b}\,{}^{c}q$ ⠏⠘⠃⠐⠘⠉⠐⠟

(There is a small space between b and c in print.)

⋙ $P_{x}\,{}_{y}Q$ ⠠⠏⠰⠭⠐⠰⠽⠐⠠⠟

(There is a small space between x and y in print.)

⋙ $P_{1}\,{}_{2}Q$ ⠠⠏⠂⠐⠰⠆⠐⠠⠟

(There is a small space between 1 and 2 in print.) The first numeric subscript does not require a subscript indicator because it is a right subscript to a letter.

6.17 Simultaneous Superscripts and Subscripts

When a superscript and a subscript are printed directly above and below each other, the subscript is transcribed first.

⋙ 10_{8}^{3} ⠼⠂⠴⠰⠦⠘⠒

⋙ ${}^{238}_{92}\mathrm{U}$ ⠰⠔⠆⠘⠆⠒⠦⠐⠠⠥

Example 6-41

An example of an I^- iodine anion: ${}^{127}_{53}I^-$.

⠠⠁⠝ ⠑⠭⠁⠍⠏⠇⠑ ⠷ ⠁⠝ ⠸⠩ ⠠⠊⠘⠤ ⠸⠱ ⠊⠕⠙⠔⠑
⠁⠝⠊⠕⠝⠒ ⠸⠩ ⠰⠢⠒⠘⠂⠆⠶⠐⠠⠊⠘⠤ ⠸⠱⠲

The subscript indicator is or is not transcribed, according to the rules presented in Section 6.11. Notice that simultaneous superscripts and subscripts printed next to a letter in italics may not align exactly because of the slanted aspect of the italic typeform.

⋙ x_1^2 ⠭⠂⠘⠆

The numeric subscript does not require a subscript indicator because it is a right subscript to a letter.

⋙ ${}_m^n x$ ⠰⠍⠘⠝⠐⠭

The subscript is transcribed first.

⋙ $n_{a_{b_c}}^{x^{y^z}}$ ⠝⠰⠁⠰⠰⠃⠰⠰⠰⠉⠘⠭⠘⠘⠽⠘⠘⠘⠵

The entire subscript is transcribed before transcribing the superscript.

6.18 Nonsimultaneous Superscripts and Subscripts

When a mathematical expression carries both a superscript and a subscript which are not printed directly above and below each other, the superscript and subscript are transcribed in the same order as in print, and the baseline indicator is inserted between them.

⋙ ${x_1}^2$ ⠭⠂⠐⠘⠆

⋙ ${a^k}_m$ ⠁⠘⠅⠐⠰⠍

⋙ ${}_y{}^x n$ ⠰⠽⠐⠘⠭⠐⠝

⋙ ${}^b{}_a n$ ⠘⠃⠐⠰⠁⠐⠝

A magnifier and a straightedge can help determine whether superscripts or subscripts are simultaneous or nonsimultaneous. If in doubt whether the expression shows nonsimultaneous super/subscripts or if, instead, the super/subscripts have super/subscripts of their own, compare to the surrounding text for clues.

6.19 Detached Superscripts and Subscripts

When an entire superscript or subscript stands alone, it is transcribed without a level indicator. A transcriber's note explains its print position.

Sample transcriber's note:

The 2 is raised in print.

Example 6-42

In x^2, the ^{2} is the exponent.

⠠⠔ ⠸⠩ ⠭⠘⠆ ⠸⠱⠂ ⠮ ⠼⠃ ⠊⠎ ⠮ ⠑⠭⠏⠕⠝⠢⠞⠲ ⠈⠨⠣⠠⠮
⠼⠃ ⠊⠎ ⠗⠁⠊⠎⠫ ⠔ ⠏⠗⠔⠞⠲⠈⠨⠜

6.20 Literary Symbols and Level Indicators

If an ampersand, asterisk, crosshatch, dagger, double dagger, paragraph mark, or section mark is used mathematically and appears in a superscript or a subscript, its level must be indicated. The Nemeth symbols for these literary characters are listed in Lesson 5.

Example 6-43

A dagger may be used as a superscript in quantum mechanics, for example, $A^\dagger$.

⠠⠁ ⠙⠁⠶⠻ ⠍⠁⠽ ⠆ ⠥⠎⠫ ⠵ ⠁ ⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞ ⠔
⠟⠥⠁⠝⠞⠥⠍ ⠍⠑⠡⠁⠝⠊⠉⠎⠂ ⠿ ⠑⠭⠁⠍⠏⠇⠑⠂
⠸⠩ ⠠⠁⠘⠸⠻ ⠸⠱⠲

Summary

Here is a summary of the difference in print among four types of superscripts and subscripts as defined in the Nemeth Code.

Consecutive A right super/subscript belongs to the preceding character and a left super/subscript belongs to the next character.

Simultaneous A character has both a superscript and a subscript. The superscript is printed directly above the subscript.

Nonsimultaneous A character has both a superscript and a subscript. The two scripts are not printed directly above and below each other.

Detached A super/subscript stands alone without being associated with a letter or number. It is printed slightly above or below the baseline and is smaller than the rest of the text.

Note: There is no space between the subscript and the superscript in item #3.

PRACTICE 6K

1. Here are some expressions with left superscripts: ${}^{3}x,\ {}^{n}x,$ ${}^{-}2 + {}^{-}4 = {}^{-}6,\ ({}^{-}3)^{{}^{-}2\,+{}^{+}2}$.
2. ${}^{12}_{6}\mathrm{C}$ and ${}^{12}\mathrm{C}$ represent the same carbon isotope.
3. $\mathrm{D}_2{}^{18}\mathrm{O}$ is the doubly labeled water isotopologue!
4. In CO_2, the subscript ${}_2$ means "two oxygen atoms".
5. ${}_{n}P_{r} = K({}_{n-1}P_{r-1})$
6. $a_1^2 + b_1^2 + c_1^2$
7. $[t]_0^4$
8. $2 \times 10_6{}^2 + 3 \times 10_6{}^1 + 2$
9. $P_{x\,y}Q$
10. $\mathrm{NH}_4^+ + \mathrm{Cl}^- + \mathrm{H}_2\mathrm{O}$

6.21 More About Grouping Symbols and Level Indicators

As noted throughout this lesson, when a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed before the sign of grouping.

⋙ $(R_H T_H) + (R_S T_S)$

⠷⠠⠗⠰⠠⠓⠐⠠⠞⠰⠠⠓⠐⠾⠬⠷⠠⠗⠰⠠⠎⠐⠠⠞⠰⠠⠎⠐⠾

If no subscript indicator is used, a return to the baseline is implied.

⋙ (a_1, a_2, a_3) ⠷⠁⠂⠠ ⠁⠆⠠ ⠁⠒⠾

This right parenthesis does not need a baseline indicator.

Be watchful when determining the level of a grouping sign. Because the top and bottom margins of grouping signs may extend beyond the level in effect, it is better to compare the center of the symbol to the surrounding text if its printed level is in question.

a. When a grouping sign starts at a different level, the appropriate level indicator is placed before the grouping symbol to properly identify its location. Review the examples in Section 6.7.1 with this in mind.

Example 6-44

In this case, $x_{(a,\,b)} + y_a$.

⠠⠔ ⠹ ⠉⠁⠎⠑⠂ ⠸⠩ ⠭⠰⠷⠁⠪⠃⠾⠐⠬⠽⠰⠁ ⠸⠱⠲

The left grouping symbol begins the subscript level.

b. When a grouping symbol is printed on the baseline of writing, an associated exponent is at the superscript level, even though it may be raised to a higher level in print. This is a visual aid, not to be mistaken for a higher-degree exponent.

Example 6-45

$(a^2)^8$ means "a-squared to the 8th power," or a^{16}, whereas a^{2^8} means "a to the 2^8 power," or a^{256}.

⠸⠩ ⠷⠁⠘⠆⠐⠾⠘⠦ ⠸⠱ ⠍⠂⠝⠎ ⠦⠁⠤⠎⠟⠥⠜⠫ ⠞⠕ ⠮ ⠼⠓⠞⠓
⠏⠪⠻⠂⠴ ⠕⠗ ⠸⠩ ⠁⠘⠂⠖⠠ ⠠⠄⠐⠱⠁⠎ ⠁⠘⠆⠘⠘⠦ ⠸⠱ ⠍⠂⠝⠎
⠦⠁ ⠞⠕ ⠮ ⠸⠩ ⠼⠆⠘⠦ ⠸⠱ ⠏⠪⠻⠂⠴ ⠕⠗ ⠸⠩ ⠁⠘⠆⠢⠖ ⠸⠱⠲

The first exponent 8 is a superscript to (a^2). The second exponent 8 is a supersuperscript.

c. Lesson 4 explained that, when a letter touches only one grouping symbol, the English-letter indicator is applied (or is not applied) as though the grouping sign were not present. This rule is illustrated below in the context of a grouping sign that has a simultaneous subscript and superscript.

⫸ $s]_a^b$ ⠎⠈⠾⠰⠁⠘⠃

Think: s_a^b ⠎⠰⠁⠘⠃

Instructions: Treat each vertical bar in sentence (6) as an operation sign.

PRACTICE 6L

(1) $\{f_n\}$

(2) $|a_m - a_n|$

(3) $(x_1y_1 + x_2y_2)$

(4) $([\mathrm{CH}_3]_2\mathrm{CH})$

(5) $\mathrm{I}_{\mathrm{ue}}^{2''} = (\mathrm{H}'_{44}x'_{\mathrm{ve}})^{+'}$

(6) The dagger and the asterisk are used as superscripts in quantum mechanics: $A^\dagger, (x^\dagger)^\dagger = x,\ \langle\phi|\psi\rangle^* = \langle\psi|\phi\rangle$.

For further practice, see Appendix A—Reading Practice.

EXERCISE 6

Prepare Exercise 6 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 6A

```
⠼⠃⠉⠲ ⠠⠎⠊⠍⠏⠇⠊⠋⠽ ⠯ ⠎⠕⠇⠧⠑ ⠑⠁⠡ ⠑⠟⠥⠁⠰⠝ ⠆⠇ ⠿
    ⠰⠭⠲ ⠠⠩⠪ ⠽⠗ ⠐⠺ ⠯ ⠡⠑⠉⠅ ⠽⠗ ⠁⠝⠎⠺⠻⠲ ⠸⠩
  ⠰⠁⠸⠲ ⠼⠆⠲ ⠨⠅ ⠼⠒⠭⠬⠒
  ⠰⠃⠸⠲ ⠼⠆⠷⠭⠤⠖⠾ ⠨⠅ ⠭⠤⠂⠲
  ⠰⠉⠸⠲ ⠼⠖⠬⠆⠨⠢⠭ ⠨⠅ ⠼⠆⠂
  ⠰⠙⠸⠲ ⠼⠆⠷⠭⠬⠲⠨⠢⠾ ⠨⠅ ⠼⠒⠆ ⠸⠱
```

PRACTICE 6B

```
⠸⠩
⠼⠖⠭⠘⠆       ⠼⠆⠲⠘⠿           ⠭⠘⠆⠁
⠨⠏⠘⠆        ⠷⠽⠤⠅⠾⠘⠒         ⠼⠦⠘⠂⠴
⠝⠘⠍         ⠠⠁⠘⠷⠅⠬⠂⠾        ⠟⠘⠃⠷⠺⠤⠂⠾
⠅⠘⠴⠨⠆⠶      ⠑⠘⠒⠠⠴⠴⠴         ⠑⠘⠤
⠸⠱
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
  ⠠⠍⠂⠎⠥⠗⠑⠰⠞⠎ ⠷ ⠘⠂⠇⠢⠛⠹ ⠜⠑ ⠍⠂⠎⠥⠗⠑⠰⠞⠎ ⠔
⠘⠂⠐⠕ ⠘⠂⠙⠊⠍⠢⠨⠝⠲ ⠠⠮⠽ ⠜⠑ ⠇⠁⠃⠑⠇⠫ ⠵ ⠉⠍⠂ ⠋⠞⠂
⠰⠍⠂ ⠑⠞⠉⠲ ⠠⠍⠂⠎⠥⠗⠑⠰⠞⠎ ⠷ ⠘⠂⠜⠑⠁ ⠜⠑ ⠍⠂⠎⠥⠗⠑⠰⠞⠎
⠔ ⠘⠂⠞⠺⠕ ⠘⠂⠙⠊⠍⠢⠨⠝⠎⠲ ⠠⠮⠽ ⠜⠑ ⠇⠁⠃⠑⠇⠫ ⠵
⠸⠩ ⠉⠍⠘⠆⠠ ⠋⠞⠘⠆⠠ ⠰⠍⠘⠆ ⠸⠱⠂ ⠑⠞⠉⠲ ⠠⠍⠂⠎⠥⠗⠑⠰⠞⠎
⠷ ⠘⠂⠧⠕⠇⠥⠍⠑ ⠜⠑ ⠍⠂⠎⠥⠗⠑⠰⠞⠎ ⠔ ⠘⠂⠹⠗⠑⠑
⠘⠂⠙⠊⠍⠢⠨⠝⠎⠲ ⠠⠮⠽ ⠜⠑ ⠇⠁⠃⠑⠇⠫ ⠵ ⠸⠩ ⠉⠍⠘⠒⠠
⠋⠞⠘⠒⠠ ⠰⠍⠘⠒ ⠸⠱⠂ ⠑⠞⠉⠲
```

PRACTICE 6C

```
⠼⠁⠲ ⠠⠞⠑⠇⠇ ⠱⠁⠞ ⠝⠥⠍⠃⠻ ⠑⠁⠡ ⠷ ⠮ ⠋⠕⠇⠇⠪⠬
  ⠗⠑⠏⠗⠑⠎⠢⠞⠎⠒ ⠸⠩ ⠼⠖⠘⠆⠠ ⠼⠢⠘⠤⠒⠠ ⠠⠄⠯
  ⠷⠒⠨⠂⠢⠾⠘⠲ ⠸⠱⠲
⠼⠃⠲ ⠠⠱ ⠑⠭⠏⠗⠑⠎⠨⠝ ⠙⠢⠕⠞⠑⠎ ⠮ ⠜⠑⠁ ⠷ ⠁ ⠉⠊⠗⠉⠇⠑⠦
    ⠸⠩
  ⠰⠁⠸⠲ ⠼⠒⠖⠴⠘⠨⠡
  ⠰⠃⠸⠲ ⠨⠏⠗⠘⠆
  ⠰⠉⠸⠲ ⠨⠏⠙ ⠸⠱
⠼⠉⠲ ⠠⠮ ⠎⠽⠍⠃⠕⠇ ⠿ ⠦⠋⠑⠑⠞ ⠎⠟⠥⠜⠫⠴ ⠊⠎
  ⠸⠩ ⠋⠞⠘⠆⠸⠲ ⠼⠂ ⠋⠞ ⠈⠡⠂ ⠋⠞ ⠨⠅ ⠼⠂ ⠋⠞⠘⠆
⠼⠲⠸⠲ ⠼⠖⠨⠴⠆⠈⠡⠂⠴⠘⠆⠒ ⠸⠱ ⠊⠎ ⠁⠝ ⠁⠏⠏⠗⠕⠭⠊⠍⠁⠰⠝ ⠷
  ⠠⠁⠧⠕⠛⠁⠙⠗⠕⠄⠎ ⠝⠥⠍⠃⠻⠲
⠼⠑⠲ ⠸⠩ ⠼⠂⠲⠲ ⠋⠞⠘⠆ ⠬⠂⠲⠲ ⠋⠞⠘⠆ ⠨⠅ ⠄⠄⠄
⠼⠖⠸⠲ ⠠⠄⠠⠒⠧⠻⠞ ⠼⠂⠴⠴⠘⠨⠡ ⠰⠠⠋ ⠸⠱ ⠞⠕ ⠙⠑⠛⠗⠑⠑⠎
  ⠠⠉⠑⠇⠎⠊⠥⠎⠲
```

Runover margins for itemized material are determined individually for each question. Item 2 has subdivisions—its runover position (line 5) is cell 5.

PRACTICE 6D

```
⠸⠩
⠷⠂⠾ ⠼⠲⠷⠭⠤⠽⠾⠘⠒⠐⠤⠆⠷⠭⠤⠽⠾⠘⠒
⠷⠆⠾ ⠼⠒⠁⠘⠒⠐⠃⠬⠖⠁⠘⠖⠐⠃⠘⠆⠐⠬⠔⠁⠘⠔⠐⠃⠘⠒
⠷⠒⠾ ⠠⠄⠠⠒⠧⠻⠞ ⠼⠂⠴⠘⠨⠡ ⠰⠠⠋ ⠠⠄⠞⠕ ⠘⠨⠡⠐⠠⠉⠸⠲
  ⠠⠄⠠⠑⠭⠏⠗⠑⠎⠎ ⠼⠂⠴ ⠘⠨⠡⠐⠠⠉ ⠠⠄⠔ ⠘⠨⠡⠐⠠⠋⠸⠲
⠷⠲⠾ ⠠⠧ ⠨⠅ ⠼⠆⠨⠏⠘⠆⠐⠠⠗⠗⠘⠆
⠷⠢⠾ ⠭⠘⠝⠐⠤⠙⠊⠍⠑⠝⠎⠊⠕⠝⠁⠇ ⠠⠄⠎⠽⠌⠑⠍
⠷⠖⠾ ⠼⠖⠘⠆⠐⠈⠡⠖⠘⠒ ⠨⠅ ⠼⠖⠘⠆⠬⠒ ⠨⠅ ⠼⠖⠘⠢
⠷⠶⠾ ⠼⠆⠷⠒⠖⠭⠘⠆⠐⠤⠂⠾ ⠨⠅ ⠼⠆⠷⠖⠭⠾⠘⠆⠐⠤⠂⠘⠆⠐⠾ ⠸⠱⠂
  ⠱ ⠋⠁⠉⠞⠕⠗⠎ ⠔⠞⠕ ⠸⠩ ⠼⠆⠷⠖⠭⠬⠂⠾⠷⠖⠭⠤⠂⠾ ⠸⠱⠲
```

PRACTICE 6E

⠼⠁⠲ ⠠⠥⠎⠑ ⠁ ⠉⠁⠇⠉⠥⠇⠁⠞⠕⠗ ⠞⠕ ⠋⠔⠙
  ⠸⠩ ⠼⠔⠘⠔⠘⠘⠔ ⠸⠱
⠼⠃⠲ ⠠⠋⠔⠙ ⠮ ⠗⠹ ⠞⠻⠍ ⠷ ⠸⠩ ⠷⠭⠬⠽⠾⠘⠝ ⠸⠱⠲
⠼⠉⠲ ⠠⠇⠁⠃⠑⠇ ⠮ ⠸⠩ ⠭⠘⠆⠸⠄⠎ ⠠⠄⠯ ⠭⠘⠒⠸⠄⠎ ⠸⠱⠲
⠼⠙⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠍⠂⠝⠬ ⠷ ⠸⠩ ⠭⠄⠄⠘⠒⠸⠦
⠼⠢⠸⠲ ⠠⠄⠠⠎⠊⠍⠏⠇⠊⠋⠽⠒
  ⠷⠭⠘⠒⠐⠤⠽⠘⠒⠐⠾⠘⠆⠐⠤⠷⠭⠘⠒⠐⠬⠽⠘⠒⠐⠾⠘⠆⠸⠲
⠼⠖⠸⠲ ⠭⠘⠽⠘⠘⠝⠘⠵ ⠠⠄⠕⠗ ⠭⠘⠽⠘⠘⠆⠘⠵ ⠸⠱

PRACTICE 6F

⠸⠩
⠼⠒⠰⠉   ⠼⠂⠴⠰⠦   ⠿⠰⠒   ⠭⠰⠆⠬⠅
⠼⠒⠰⠋⠊⠧⠑   ⠼⠂⠴⠰⠿   ⠠⠉⠁⠷⠠⠕⠠⠓⠾⠰⠆   ⠽⠰⠤⠆
⠽⠰⠝⠰⠰⠅   ⠠⠏⠰⠒⠝   ⠁⠰⠍⠂   ⠁⠰⠍⠰⠰⠂
⠸⠱

PRACTICE 6G

⠼⠁⠲ ⠠⠐⠝ ⠮ ⠝⠥⠍⠻⠁⠇ ⠔ ⠃⠁⠎⠑ ⠞⠢ ⠑⠟⠥⠁⠇ ⠞⠕⠒ ⠸⠩
  ⠰⠁⠸⠲ ⠼⠲⠶⠰⠦
  ⠰⠃⠸⠲ ⠼⠒⠲⠰⠖
  ⠰⠉⠸⠲ ⠼⠂⠂⠴⠂⠰⠆
⠼⠆⠸⠲ ⠠⠉⠰⠓⠑⠭ ⠨⠅ ⠼⠂⠆⠰⠙⠑⠉ ⠨⠅ ⠼⠂⠲⠰⠕⠉⠞ ⠸⠱
⠼⠉⠲ ⠠⠱⠁⠞ ⠙ ⠺⠑ ⠐⠅ ⠊⠋
  ⠸⠩ ⠠⠏⠰⠝⠰⠰⠗ ⠨⠅ ⠷⠭⠰⠝⠰⠰⠅⠠ ⠽⠰⠝⠰⠰⠅⠐⠾⠸⠦
⠼⠲⠸⠲ ⠼⠶⠰⠦⠐⠤⠲⠰⠦ ⠸⠱

PRACTICE 6H

⠼⠁⠐⠜ ⠠⠘⠮ ⠑⠭⠏⠗⠑⠎⠨⠝⠎ ⠝⠑⠫ ⠎⠥⠃⠎⠉⠗⠊⠏⠞
⠔⠙⠊⠉⠁⠞⠕⠗⠎⠒ ⠸⠩ ⠽⠰⠤⠆⠠ ⠭⠰⠆⠬⠅⠠ ⠁⠰⠍⠂⠠
⠭⠰⠒⠰⠰⠝⠠ ⠭⠰⠽⠰⠰⠆ ⠸⠱⠲
⠼⠃⠐⠜ ⠠⠘⠮ ⠑⠭⠏⠗⠑⠎⠨⠝⠎ ⠙ ⠸⠂⠝ ⠝⠑⠫ ⠁ ⠎⠥⠃⠎⠉⠗⠊⠏⠞
⠔⠙⠊⠉⠁⠞⠕⠗⠒ ⠸⠩ ⠭⠂⠠ ⠁⠭⠆⠠ ⠠⠉⠠⠕⠆⠠ ⠵⠨⠲⠠
⠨⠃⠆ ⠸⠱⠲
⠼⠉⠐⠜ ⠠⠙⠑⠉⠊⠙⠑ ⠱⠑⠮⠗ ⠘⠮ ⠑⠭⠏⠗⠑⠎⠨⠝⠎ ⠗⠑⠟⠥⠊⠗⠑ ⠁
⠎⠥⠃⠎⠉⠗⠊⠏⠞ ⠔⠙⠊⠉⠁⠞⠕⠗ ⠯ ⠞⠗⠁⠝⠎⠉⠗⠊⠃⠑ ⠮⠍
⠉⠕⠗⠗⠑⠉⠞⠇⠽⠒ ⠸⠩ ⠎⠓⠁⠏⠑⠰⠲⠠ ⠠⠟⠄⠆⠠
⠠⠉⠖⠠⠋⠑⠆⠠⠕⠂⠆⠠ ⠝⠰⠅⠠ ⠭⠰⠆⠅⠠ ⠠⠏⠰⠗⠰⠰⠎⠰⠰⠰⠞⠠
⠠⠙⠢⠖⠠ ⠠⠛⠔⠠⠔⠔⠔ ⠸⠱⠂ ⠯ ⠮ ⠓⠑⠭⠁⠙⠑⠉⠊⠍⠁⠇
⠝⠥⠍⠃⠻ ⠸⠩ ⠼⠆⠑⠖⠃⠰⠂⠖ ⠸⠱⠲ ⠈⠨⠣⠰⠑ ⠯ ⠰⠃ ⠜⠑
⠉⠁⠏⠊⠞⠁⠇⠊⠵⠫ ⠔ ⠏⠗⠔⠞⠲⠈⠨⠜
⠼⠙⠐⠜ ⠘⠂⠠⠡⠑⠍⠊⠌⠗⠽⠲ ⠠⠱⠊⠇⠑ ⠸⠩ ⠠⠝⠁⠆⠠⠵⠝⠠⠉⠇⠲ ⠸⠱
⠉⠙ ⠆ ⠉⠕⠕⠇⠫ ⠔ ⠮ ⠝⠕⠗⠍⠁⠇ ⠺⠁⠽⠂
⠸⠩ ⠠⠝⠁⠆⠈⠷⠠⠉⠕⠠⠉⠇⠲⠨⠾ ⠸⠱ ⠸⠓ ⠞⠕ ⠆ ⠟⠥⠢⠡⠫ ⠔
⠮ ⠇⠊⠟⠥⠊⠙ ⠸⠩ ⠠⠝⠆⠸⠲
⠼⠢⠾ ⠋⠂⠷⠭⠾ ⠨⠅ ⠛⠷⠭⠾⠡⠟⠆⠷⠭⠾⠬⠋⠆⠷⠭⠾ ⠸⠱

PRACTICE 6I

1. $x_{1,\,2} \neq x^{i,\,j}$
2. $x_{n-1,\,n-1}, x_{n-1,\,n}, x_{n,\,n-1}$
3. $\mathrm{A}^{\mathrm{n+n+n\ all\ n's}}$ are equal.
4. $s]_{t\,=\,a}$
5. $e^{1{,}000}$
6. $a^{m+k} \div a^{m} = a^{k}$
7. $P_{s_1 \dots s_2}$ and $P_{q_{r,\,s}}$
8. $10_{-?-} = 6_8$
9. $a'_1, a'_2, \dots, a'_n$ are the inverses.
10. $\aleph_0$ represents the cardinality of the set of all natural numbers.

PRACTICE 6J

⠀⠀⠀⠀⠠⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞⠎ ⠾ ⠠⠎⠥⠃⠎⠉⠗⠊⠏⠞⠎
⠸⠩
⠼⠆⠘⠠⠠⠈⠘⠰⠴ ⠨⠅ ⠠⠠⠁⠂
⠁ ⠨⠅ ⠼⠆⠘⠅⠘⠰⠂ ⠠⠄⠯ ⠃ ⠨⠅ ⠼⠆⠘⠅⠘⠰⠆
⠷⠁⠃⠾⠘⠭ ⠨⠅ ⠼⠆⠘⠅⠘⠰⠂⠘⠭⠐⠡⠆⠘⠅⠘⠰⠆⠘⠭
⠑⠘⠊⠨⠹⠘⠰⠂ ⠠⠄⠐⠞⠎ ⠑⠘⠊⠸⠹⠘⠰⠆ ⠠⠄⠑⠟⠥⠁⠇⠎
⠀⠀⠑⠘⠊⠷⠨⠹⠘⠰⠂⠘⠬⠨⠹⠘⠰⠆⠘⠾
⠸⠱
⠀⠀⠀
⠀⠀⠀⠀⠠⠎⠥⠃⠎⠉⠗⠊⠏⠞⠎ ⠾ ⠠⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞⠎
⠸⠩
⠠⠵⠰⠢⠰⠘⠝
⠼⠶⠞⠰⠎⠰⠘⠲
⠸⠱

PRACTICE 6K

⠼⠁⠲ ⠠⠐⠓ ⠜⠑ ⠐⠎ ⠑⠭⠏⠗⠑⠎⠨⠝⠎ ⠾ ⠇⠑⠋⠞
  ⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞⠎⠒ ⠸⠩ ⠘⠒⠐⠭⠠ ⠘⠝⠐⠭⠠
  ⠘⠤⠐⠆⠬⠘⠤⠐⠲ ⠨⠅ ⠘⠤⠐⠖⠠ ⠷⠘⠤⠐⠒⠾⠘⠘⠤⠘⠆⠬⠘⠘⠬⠘⠆⠸⠲
⠼⠆⠸⠲ ⠰⠖⠘⠂⠆⠐⠠⠉ ⠠⠄⠯ ⠘⠂⠆⠐⠠⠉ ⠸⠱ ⠗⠑⠏⠗⠑⠎⠢⠞ ⠮
  ⠎⠁⠍⠑ ⠉⠜⠃⠕⠝ ⠊⠎⠕⠞⠕⠏⠑⠲
⠼⠉⠲ ⠸⠩ ⠠⠙⠆⠐⠘⠂⠦⠐⠠⠕ ⠸⠱ ⠊⠎ ⠮ ⠙⠳⠃⠇⠽ ⠇⠁⠃⠑⠇⠫
  ⠺⠁⠞⠻ ⠊⠎⠕⠞⠕⠏⠕⠇⠕⠛⠥⠑⠖
⠼⠙⠲ ⠠⠔ ⠸⠩ ⠠⠉⠠⠕⠆ ⠸⠱⠂ ⠮ ⠎⠥⠃⠎⠉⠗⠊⠏⠞ ⠼⠃ ⠨⠨⠣⠼⠃
  ⠊⠎ ⠇⠪⠻⠫ ⠔ ⠏⠗⠔⠞⠲⠈⠌⠜ ⠍⠂⠝⠎ ⠦⠞⠺⠕ ⠕⠭⠽⠛⠢
  ⠁⠞⠕⠍⠎⠴⠲
⠼⠑⠲ ⠸⠩ ⠰⠝⠐⠠⠏⠰⠗ ⠨⠅ ⠠⠅⠷⠰⠝⠤⠂⠐⠠⠏⠰⠗⠤⠂⠐⠾
⠼⠖⠸⠲ ⠁⠂⠘⠆⠐⠬⠃⠂⠘⠆⠐⠬⠉⠂⠘⠆
⠼⠶⠸⠲ ⠈⠷⠞⠈⠾⠰⠴⠘⠲
⠼⠦⠸⠲ ⠼⠆⠈⠡⠂⠴⠰⠖⠐⠘⠆⠘⠶⠒⠈⠡⠂⠴⠰⠖⠐⠘⠂⠐⠬⠆
⠼⠔⠸⠲ ⠠⠏⠰⠭⠐⠰⠽⠐⠠⠟
⠼⠂⠴⠸⠲ ⠠⠝⠠⠓⠲⠘⠬⠐⠬⠠⠉⠇⠘⠤⠐⠬⠠⠓⠆⠠⠕ ⠸⠱

PRACTICE 6L

⠸⠩
⠷⠂⠾ ⠨⠷⠋⠰⠝⠐⠨⠾
⠷⠆⠾ ⠳⠁⠰⠍⠐⠤⠁⠰⠝⠐⠳
⠷⠒⠾ ⠷⠭⠂⠽⠂⠬⠭⠆⠽⠆⠾
⠷⠲⠾ ⠷⠈⠷⠠⠉⠠⠓⠒⠈⠾⠰⠆⠐⠠⠉⠠⠓⠾
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