

LESSON 8

- INTRODUCTION TO FRACTIONS
 - Simple Fractions
 - Mixed Numbers
 - Complex Fractions
 - More Fraction Rules
- RADICAL EXPRESSIONS

Format

- LINKED EXPRESSIONS
 - Division of Linked Expressions
 - Nested Linked Expression

Answers to Practice Material

LESSON PREVIEW

Fractions, mixed numbers, and radical expressions are studied. The "linked expression" is defined and its format is discussed in detail, including the special case of the nested linked expression.

Note: The font size in some of the examples is larger than standard, for clarity. Some of the larger symbols may appear to be bold. Do not consider the size difference or the darker image to be a variant typeform.

INTRODUCTION TO FRACTIONS

Two types of fractions are presented in this lesson: simple fractions (including mixed numbers) and complex fractions.

A fraction is composed of three parts: a numerator, a denominator, and a fraction line.

$$\begin{array}{ccc} \text{numerator} \swarrow & & \\ & \frac{3}{4} & \leftarrow \text{fraction line} \\ \nwarrow \text{denominator} & & \end{array}$$

Fractions are printed in a variety of ways. The numerator may be printed above the denominator or they may be printed on the same level. The fraction line may be horizontal or diagonal. Here are three examples of the fraction "three fourths" printed in different styles.

$$\frac{3}{4} \qquad 3/4 \qquad 3/4$$

The numerator and/or denominator may also consist of or contain words or abbreviations. Here are four examples.

$$\text{m/s} \qquad (\text{Spoken: meters per second})$$

$$\text{ft./sec.} \qquad (\text{Spoken: feet per second})$$

$$\frac{\text{rise}}{\text{run}} \qquad (\text{Spoken: rise over run})$$

$$3.5\%/year \qquad (\text{Spoken: 3.5 percent per year})$$

In a technical transcription, fractions are transcribed in Nemeth.

Simple Fractions

[NC 13.1]

8.1 Definition

For the purposes of the Nemeth Code, a simple fraction is one whose numerator and denominator contain no fractions, except possibly at the superscript or subscript level. These are simple fractions:

$$\frac{1}{2} \quad \frac{a^2}{b^2} \quad \frac{72 \text{ mi.}}{4 \text{ hr.}} \quad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}} \quad 5/12$$

This is not a simple fraction because the numerator contains a fraction: $\frac{1/3}{5}$

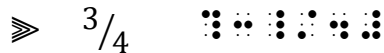
8.4 The Diagonal Simple Fraction Line

The type of fraction line used in the print copy (horizontal or diagonal) is replicated in the braille transcription. In a simple fraction, the diagonal fraction line is transcribed as the symbol shown below. Note that the diagonal simple fraction line consists of two braille cells.



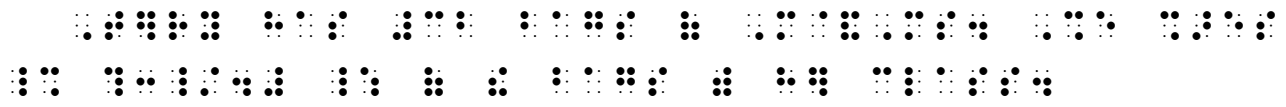
When a diagonal fraction line is printed, it may not be clear where the fraction begins and where it ends. The transcriber must not attempt to analyze the math. Instead, application of the following rules will prevent misinterpretation of the expression.

- 8.4.1 **Use of Simple Fraction Indicators with the Diagonal Simple Fraction Line.** When the numerator and denominator are printed at different levels of writing on either side of the diagonal line, the construction is a fraction and so simple fraction indicators are used. Do not confuse this type style with superscripts and subscripts. In this example, the numeral 3 is the numerator and the numeral 4 is the denominator.

➤ $3/4$ 

Example 8-7 |

Terry has 32 bags of M&Ms. She shares $3/4$ of the bags with her class.



- 8.4.2 **Type Size on the Baseline of Writing.** If the numerator and denominator are printed at the same level of writing on either side of the diagonal line, the transcriber must notice the type size.

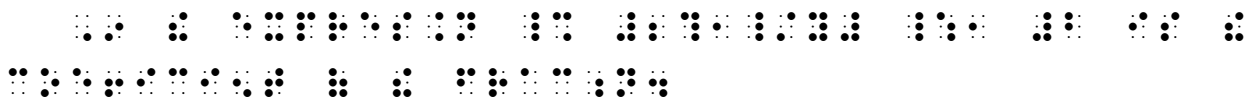
- a. **Fraction in Smaller Type.** If the type is in a different size from that normally used for similar expressions throughout the text, identify this as a fraction by using simple fraction indicators. In the example below, note that the fraction is printed on the baseline of writing—it is not a subscript.

➤ $2 \frac{1}{y}$ 

The numeral 1 is smaller than the numeral 2. The space between the coefficient and the fraction is not shown in braille.

Example 8-8 |

In the expression $2 \frac{1}{y}$, 2 is the coefficient of the fraction.




Instructions: Determine whether the slash is used mathematically, that is, does it require a switch to Nemeth? If it does, then determine if fraction indicators are required.

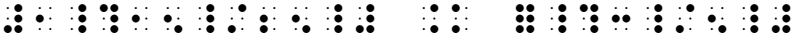
PRACTICE 8B

Diagonal Simple Fraction Line

- A) How many $\frac{2}{3}$'s are there in $\frac{5}{6}$?
 - B) Energy is absorbed at the rate of 880 J/s for each square meter of the surface.
 - C) $y(0) = \pi/4$
 - D) $1 \text{ ft/sec} \approx 0.6818 \text{ mph}$
 - E) In $y \frac{1}{5}$, y is the coefficient of the fraction $\frac{1}{5}$.
 - F) True/False: The rise/run ratio is 5 in graph A.
 - G) $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
 - H) A 5-year CD went from earning interest at the rate of 12.06%/year in 1984 to earning less than 0.87%/year in 2015.
-

8.5.2 **Mixed Numbers and Omissions.** If any part of a mixed number contains a sign of omission, the mixed number fraction indicators are used.

$\gg \frac{7}{4} = 1\frac{?}{4}$


$\gg 1^{15}/_{25} = ?^{3}/_{5}$


8.5.3 **Nonuse of Mixed Number Fraction Indicators.** If the fractional part of the expression contains a letter, it no longer fits the Nemeth Code's definition of "mixed number." Appropriate fraction indicators are used (or are not used) according to the rules.

$\gg 3\frac{1}{y}$


$\gg x\frac{3}{8}$


PRACTICE 8C

Mixed Numbers

- Find the premium for a $1\frac{1}{2}$ -yr. policy at the yearly rate of 24¢ per \$100.
- $2\frac{1}{2}$ ft + 8 in = ? inches
 3. $(\frac{1}{2} \times 3\frac{1}{2}) + (3\frac{1}{2} \times 2)$
- $13\frac{1}{2} + 2\frac{2}{3} = 16\frac{1}{6}$
5. $7/4 = 1\frac{?}{4}$
6. $\frac{9}{4} = 2\frac{x}{4}$

Fraction Review

Compute each **unit rate** (price/pound).

- \$1.50 for $\frac{2}{3}$ pound of potatoes
- \$4.20 for $\frac{1}{2}$ pound of Edam cheese
- \$6.00 for $\frac{3}{4}$ pound of deli smoked turkey
- \$12.50 for $1\frac{1}{2}$ pounds of sliced ham

Complex Fractions

[NC 13.5]

8.6 Definition of Complex Fraction

A complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers.

$$\begin{array}{l} \text{numerator} \\ \text{denominator} \end{array} \quad \frac{4 \frac{3}{4}}{5} \quad \leftarrow \text{complex fraction line}$$

Here are more examples of complex fractions.

$$\frac{\frac{4}{3}}{12} \quad \frac{\frac{a}{b} - \frac{c}{d}}{\frac{a}{b} + \frac{c}{d}} \quad \frac{1}{3/8} \quad \frac{1/3}{2/4}$$

Reminder: A fraction is not a complex fraction if the only fractions it contains are at the superscript or subscript level. Such a fraction is a simple fraction.

This is a simple fraction, not a complex fraction: $\frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

8.6.1 Use of Complex Fraction Indicators and Complex Fraction Lines. The opening and closing complex fraction indicators are used to enclose a complex fraction.

Complex Fraction Indicators		
⠠⠠	Opening	
⠨⠨	Closing	

The main complex fraction line is represented by its appropriate braille symbol—either horizontal or diagonal.

⠠⠠	Horizontal Complex Fraction Line	—
⠠⠠⠠	Diagonal Complex Fraction Line	/

The examples shown above are transcribed as follows. Simple fraction indicators enclose each simple fraction when required; complex fraction indicators enclose the entire complex fraction. We suggest that you underline the complex fraction indicators and the complex fraction line in each example in order to analyze each transcription.

$$\ggg \frac{4\frac{3}{4}}{5} \quad \dots \dots \dots$$

$$\ggg \frac{4}{\frac{3}{12}} \quad \dots \dots \dots$$

$$\ggg \frac{1/3}{2/4} \quad \dots \dots \dots$$

$$\ggg \frac{\frac{a}{b} - \frac{c}{d}}{\frac{a}{b} + \frac{c}{d}} \quad \dots \dots \dots$$

Although the denominator in the next example does not require simple fraction indicators (see 8.4.2.b), it is still a fraction and so the overall construction is a complex fraction.

$$\ggg \frac{1}{3/8} \quad \dots \dots \dots$$

Instructions: Begin each complex fraction on a new braille line, not side by side as printed. Read left to right.

PRACTICE 8D

Complex Fractions

$$\frac{\frac{1}{8} + \frac{3}{4}}{7}$$

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{7}{9}}$$

$$\frac{1/3+1/4}{4/5-1/2}$$

$$\frac{\pi}{2\pi}$$

$$\frac{a}{\frac{b}{c}}$$

$$\frac{33\frac{1}{3}}{100}$$

$$\frac{3/5}{6}$$

$$\frac{3}{5} / \frac{7}{9}$$

Instructions: Determine the formatting before beginning your transcription. Where does each paragraph begin? Which expressions are embedded and which are displayed? What is the proper cell placement for the displayed expressions?

PRACTICE 8E

These are simple fractions:

$$\frac{1}{2} \quad \frac{a^2}{b^2} \quad \frac{y^{\frac{1}{2}}}{y^4}$$

This is not a simple fraction: $\frac{1/3}{2/3}$

Review the rules in 6.12.5 regarding an ellipsis on the baseline of writing when it follows a superscript.

$$x^{\frac{1}{2}} \dots x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \dots \frac{x^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1}$$

Plot the points $\left(-\frac{1}{2}, 4\right)$, $\left(3, 4\frac{1}{4}\right)$, and $\left(-9, \frac{3}{4}\right)$. Then express $\frac{a^{3/4}}{b^{5/4}}$ in radical form.

8.13 Spacing

The spacing before and after a radical expression is subject to the spacing rules for the signs preceding or following the radical expression.

$$\gg \sqrt{9} - \sqrt{4} = 1 \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

No space is left between a radical expression and a letter, a numeral, a fraction, a sign of grouping, a braille indicator, or another radical expression when these items are unspaced in print and belong to the same expression.

$$\gg \sqrt{5}y \quad \dots \dots \dots$$

$$\gg \sqrt{x^2} \quad \dots \dots \dots$$

$$\gg 2a\sqrt{4ab} \quad \dots \dots \dots \dots \dots \dots \dots$$

$$\gg \sqrt{4}\sqrt{87} = 2\sqrt{87} \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$\gg \sqrt{y} dx + (1 + x) dy = 0, y(0) = 1$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

Reminder: In print, derivative notation dx, dy, etc. is often preceded and followed by a space within an expression, for clarity. In braille, the terms are not spaced unless a space is required with the item preceding or following them. (Review 4.15.1 in Lesson 4.)

Example 8-29 |

Simplify. $\frac{2 - \sqrt{\frac{1}{4}}}{3 - \sqrt{\frac{1}{2}}}$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

8.14 Index of Radical

A small number or letter that may appear next to the radical sign is the *index* of the radical. This print example shows an index "3".

$$\sqrt[3]{27}$$

In braille, the index-of-radical indicator and the index precede the radical sign.

PRACTICE 8G

Nested Radical Expressions

$$(1) \sqrt{-\frac{1}{2} - i\frac{\sqrt{3}}{2}}$$

$$(2) \sqrt{\sqrt{13} + \sqrt{15} + \sqrt{117}}$$

$$(3) \sqrt{1 - \sqrt{a-b}} \times \sqrt{1 + \sqrt{a-b}}$$

$$(4) \sqrt{b^3 \sqrt{b} \sqrt{b}}$$

$$(5) \sqrt[a]{\sqrt[b]{\sqrt[c]{\sqrt{abc}}}}$$

$$(6) \sqrt{a^2} \sqrt{b^4} \sqrt{c} = ab^2 \sqrt{c}$$

$$(7) (s^2 \sqrt[3]{s^4})^2$$

$$(8) \sqrt[3]{x^2 \sqrt{64x^6}}$$

$$(9) \sqrt{\sqrt[3]{\sqrt[4]{\sqrt[5]{\sqrt{b^{48}}}}}}$$

$$(10) \sqrt{x_1 + \sqrt{x_2}}$$

$$(11) q^{\sqrt{r}} + s$$

$$(12) \sqrt{c + d + e + \dots}$$

LINKED EXPRESSIONS

[NC 26.5]

8.20 Definition of Linked Expression

A linked expression contains at least one sign of comparison. The part preceding the first sign of comparison is called the *anchor*. Each remaining part, beginning with a sign of comparison and ending before the next sign of comparison, is called a *link*. In its simplest form, $x = y$ is a linked expression where x is the anchor and $= y$ is the link.

$$12.5\% > \frac{1}{10}$$

The anchor is 12.5% and the link is $> \frac{1}{10}$

$$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$$

The anchor is 6×245 , followed by three links each beginning with an equals sign.

8.22 Special Case—Nested Linked Expressions [NC 26.5.3.b]

An expression with two or more links may be subject to special Nemeth format rules if it appears in print in a particular arrangement as described in this section.

Print Layout

- The expression is displayed.
 - There is an exception regarding itemized problems – see b, below.
- The first line contains only the anchor or only the anchor and the first link.
- Each following link begins on a new line, and the comparison signs beginning each link are vertically aligned.
 - An exception applies to the last line – see a, below.

The following linked expression meets the three requirements.

To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c). \end{aligned}$$

- a. It is common for the last line of the expression to contain more than one link. As long as the other conditions are met, this layout meets the requirements for this nested format.

We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

$$\begin{aligned} 12\frac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \frac{125}{1000} = \frac{1}{8} \end{aligned}$$

- b. The next example shows an itemized problem with no narrative. Although the expression is not displayed, this arrangement follows all of the other print layout specifications: the comparison signs are vertically aligned, and – other than on the first and last line – no sign of comparison is preceded by any expression on its left. Rules regarding this layout are outlined in Section [8.22.4](#).

$$\begin{array}{l} 1. \quad 12\frac{1}{2}\% = 12.5\% \\ \quad \quad \quad = .125 \\ \quad \quad \quad = \frac{125}{1000} = \frac{1}{8} \end{array}$$

When the print layout meets the definition of a nested linked expression, one of the following Nemeth formats is applied.

8.22.1 Margin Requirements for a Nested Linked Expression. The margins which are applied to a nested linked expression follow a reliable pattern, which can be generalized as follows.

- The anchor begins two cells to the right of the runover margin of the material to which it is displayed.
- Each link that starts on a new line begins two cells to the right of the anchor cell.
- Runovers to anchor or links begin four cells to the right of the anchor cell.

Note: Rules regarding how to divide a link that will not fit on the line will be discussed in Lesson 15. In this lesson, in order to illustrate runovers within a nested linked expression, a runover line will begin with a sign of operation.

8.22.2 Nested Linked Expression Displayed to Narrative. When a nested linked expression occurs in unitemized explanatory portions of the text (3-1), the anchor begins in cell 3 and each link begins in cell 5. In braille, each link begins on a new line, even when the print copy shows more than one link on the last line.

Reminder: A line is not skipped above or below displayed mathematical material unless a blank line is required under other rules or guidelines.

PRACTICE 8I

Recognizing a Nested Linked Expression

To test the equation $R_t = \frac{R}{n}$, use four 1000- Ω resistors wired in series to predict a total resistance of 250 Ω .

$$R_t = \frac{R}{n} = \frac{1000 \Omega}{4}$$
$$\frac{1000 \Omega}{4} = 250 \Omega$$

Then, by using the resistance theory equation

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n},$$

plug the 150- Ω and 1000- Ω resistors into the equation as R_1 and R_2 .

$$\frac{1}{R_t} = \frac{1}{150 \Omega} + \frac{1}{1000 \Omega}$$
$$= 0.007 + 0.001$$
$$= 0.008$$
$$R_t = \frac{1}{0.008} = 125 \Omega$$

8.22.3 **Nested Linked Expression Displayed to Itemized Material.** Apply the general pattern when a nested linked expression is displayed to itemized material: begin the anchor two cells to the right of the current runover margin; begin each two cells to the right of the anchor cell; begin runovers four cells to the right of the anchor cell.

- a. **Itemized Text with No Subdivisions.** When a nested linked expression occurs in itemized text containing no subdivisions (1-3), the anchor begins in cell 5 and each link begins in cell 7. Runovers begin in cell 9.
- b. **Itemized Text with Subdivisions.** When a nested linked expression occurs in itemized text containing subdivisions (1-5; 3-5), the anchor begins in cell 7 and each link begins in cell 9. Runovers begin in cell 11.

PRACTICE 8J

Multiplying Mixed Numbers

A. $2\frac{1}{2} \cdot 1\frac{1}{4} = \left(2 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{4}\right)$

$$= 2 + \frac{2}{4} + \frac{1}{2} + \frac{1}{8}$$
$$= 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8}$$
$$= 2 + 1 + \frac{1}{8} = 3\frac{1}{8}$$

B. What will the remainder be in this problem?

$$4\frac{1}{3} \times 3\frac{2}{5} \times 9\frac{11}{15} \times 2\frac{3}{4}$$
$$= \frac{13}{3} \times \frac{17}{5} \times \frac{146}{15} \times \frac{11}{4}$$
$$= \frac{13 \times 17 \times 146 \times 11}{3 \times 5 \times 15 \times 4}$$
$$= \frac{354,926}{900}$$

= 394 with a remainder of ____.

For further practice, see Appendix A—Reading Practice.

EXERCISE 8

Prepare Exercise 8 for your grader.

PRACTICE 8H

1
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

2

3

4

5

6

7

8

9

10

11

12

Lines 10-11: Notice that the print copy divided this equation after the equals sign, but the braille transcription follows Nemeth rules and divides before the comparison sign.

