

LESSON 9

- **ARROWS**
 - Construction of Braille Arrows
 - Vertical, Slanted, and Curved Arrow Shafts
 - Boldface and Compounded Arrows
- **INTRODUCTION TO SPATIAL ARRANGEMENTS**
 - Spatial Arrangements with Addition and Subtraction
 - Introduction to Cancellation
 - Arrangement on the Page
 - Placement of Code Switch Indicators

Answers to Practice Material

LESSON PREVIEW

The construction of braille arrows is demonstrated, showing several styles of arrowheads and arrow shafts. Braille format for spatially arranged addition and subtraction problems is studied.

Note: Due to the amount of vertical space used with spatial arrangements, this lesson consumes more pages than most.

ARROWS

[NC Rule 22]

9.1 Arrows Used in Mathematics

Here are some examples of arrows which bear mathematical meaning.

$$\begin{array}{lll} A \rightarrow B & \overrightarrow{PQ} & 2\text{CO} \rightleftharpoons \text{CO}_2 + \text{C} \\ f: a \mapsto b & P \uparrow Q & A \rightarrow B \leftrightarrow C \leftarrow D \end{array}$$

Note that some chemical arrows are constructed differently from math arrows. In a chemistry transcription, the arrows of *Chemical Notation Using the Nemeth Braille Code* should be used.

9.2 Construction of Braille Arrows

The components of an arrow are transcribed in the following order.

- (1) The shape indicator. (See below.)
- (2) The arrow direction, if it must be indicated (vertical or slanted). (Sections [9.9.1](#) and [9.9.2](#).)
- (3) The typeface of the arrow, if it must be indicated. (Section [9.10](#).)
- (4) The left arrowhead or dot, if any. (Sections [9.5](#), [9.7](#), and [9.8](#).)
- (5) The arrow shaft, if required. (Sections [9.4](#) and [9.9](#).)
- (6) The right arrowhead or dot, if any. (Sections [9.4](#), [9.7](#), and [9.8](#).)

An explicit terminator is not necessary.

$\ddot{\cdot}$ Shape Indicator

9.3 Spacing and Punctuation with Arrows

When arrows function as a sign of comparison, the entire construction is spaced according to the rules governing comparison signs. When not functioning as a sign of comparison, an arrow is spaced according to its context. Spacing issues will be illustrated throughout this lesson.

Punctuation associated with an arrow is mathematical.

9.4 Horizontal Arrow Shafts

The length of a shaft is indicated by the number of times the arrow shaft symbol is used. Two braille symbols represent the ordinary shaft length. By comparison, one braille symbol represents a short shaft, and three or more symbols indicate a longer shaft.

Horizontal Arrow Shafts		
⠠⠠	Ordinary length, single	—
⠠	Short, single	—
⠠⠠⠠	Long, single	—
⠠⠠	Ordinary length, double	==
⠠	Short, double	=
⠠⠠⠠	Long, double	==
⠠⠠⠠	Dashed	- - -
⠠⠠⠠	Dotted	...
⠠⠠⠠	Wavy	~

9.5 Barbed and Solid Arrowheads

An arrowhead may be barbed, solid, blunted, straight, or curved, and may occur at the left, at the right, or at both ends of an arrow shaft. An arrowhead may also appear with only its upper or lower barb. First, we will look at barbed and solid arrowheads appearing at the end of a horizontal arrow shaft.

The short line in the barbed images represents the point where the arrow shaft meets the arrowhead.

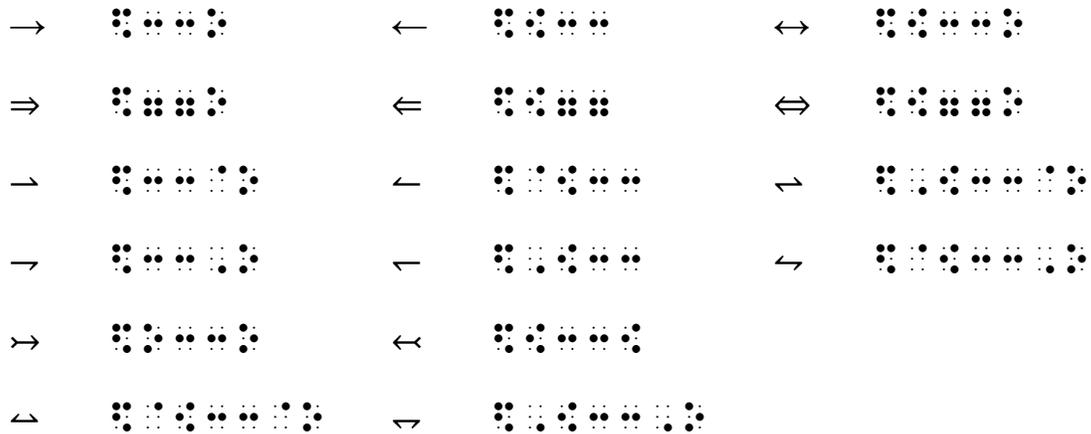
Barbed and Solid Arrowheads		
⠠	Barbed left full	←
⠠	Solid left	◀
⠠	Barbed right full	→
⠠	Solid right	▶
⠠⠠	Barbed left lower	↙
⠠⠠	Barbed right lower	↘
⠠⠠	Barbed left upper	↖
⠠⠠	Barbed right upper	↗

Notice that solid arrowheads are transcribed using the same symbol as full barbed arrowheads.

Many types of arrows can be constructed using the given shaft and arrowhead symbols. Some examples are shown on the next page.

Note: Code switch indicators are omitted in the isolated examples in this section.

Consider the shaft in the barbed arrows shown below to be the standard length for the book. A two-cell "ordinary length" shaft is transcribed. Notice the similarity between the print and braille symbols.



An additional rule may apply to the first arrow shown above. See 9.6.

By comparison, the shafts in the arrows shown below are longer. A three-cell shaft is transcribed.



And, by comparison, the shafts in the arrows shown next are shorter. A one-cell shaft is transcribed.



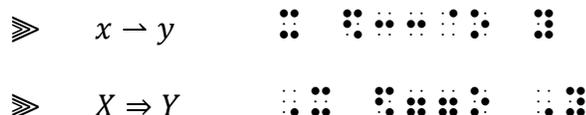
Other shaft styles are illustrated below.

Dashed and dotted

Wavy



a. **Spacing.** Note the spacing before and after these arrows which are functioning as comparison signs.



9.6.1 **Nonuse of the Contracted Form of the Right-Pointing Arrow.** All other right-pointing arrows require the use of the appropriate shaft symbol.

➤	$X \dashrightarrow Y$	⠠⠠	⠠⠠	⠠⠠	⠠⠠
➤	$X \rightsquigarrow Y$	⠠⠠	⠠⠠⠠⠠	⠠⠠	
➤	$X \longrightarrow Y$	⠠⠠	⠠⠠⠠⠠	⠠⠠	
➤	$X \rightarrow Y$	⠠⠠	⠠⠠	⠠⠠	

Instructions: Leave one space (one blank cell) between the arrows in A. and B. Assume shafts in items A, C, and D to be of regular length, and in item B to be long. In item D, divide the long expression before the arrow that comes between the double parentheses. Assume all to be in regular type even though they may appear darker.

PRACTICE 9A

Arrows with Barbed Ends

- A. \Leftrightarrow \rightarrow \leftrightarrow \longleftrightarrow \dashrightarrow \rightsquigarrow \rightsquigarrow \rightsquigarrow
- B. \longleftarrow \longleftrightarrow \rightleftarrows \rightleftarrows
- C. The principal operator of the left formula in Fig. 7.2 is \leftrightarrow , while the principal operator of the right formulas is \rightarrow .
- D. Construct a truth table for $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$.

9.7 Blunted, Straight, and Curved Arrowheads

Here are three other types of arrowheads identified in the Nemeth Code. Examples with a single shaft of ordinary length are shown. The longer horizontal line in each image represents the end of the arrow shaft.

Please disregard the differences in size and line weight in these images.

BLUNTED ARROWHEADS

Each blunted arrowhead can be left or right pointing.

<p>⠠ Full</p>	<p>⠠⠠⠠⠠</p> <p>⠠⠠⠠⠠</p> <p>⠠⠠⠠⠠⠠⠠</p>	<p>┌</p> <p>┐</p> <p>┌┐</p>
<p>⠠⠠ Lower only</p>	<p>⠠⠠⠠⠠</p> <p>⠠⠠⠠⠠⠠⠠</p>	<p>┌</p> <p>┐</p>
<p>⠠⠠ Upper only</p>	<p>⠠⠠⠠⠠</p> <p>⠠⠠⠠⠠⠠⠠</p>	<p>└</p> <p>┘</p>

STRAIGHT ARROWHEADS

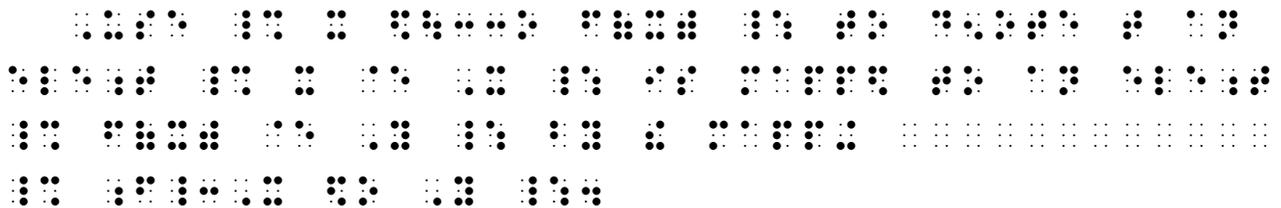
Each straight arrowhead can be left or right pointing.

<p>⠠ Full</p>	<p>⠠⠠⠠⠠</p> <p>⠠⠠⠠⠠</p> <p>⠠⠠⠠⠠⠠⠠</p>	<p>┆</p> <p>┆</p> <p>┆┆</p>
<p>⠠⠠ Lower only</p>	<p>⠠⠠⠠⠠</p> <p>⠠⠠⠠⠠⠠⠠</p>	<p>┆</p> <p>┆</p>
<p>⠠⠠ Upper only</p>	<p>⠠⠠⠠⠠</p> <p>⠠⠠⠠⠠⠠⠠</p>	<p>┆</p> <p>┆</p>

CURVED ARROWHEADS		
⠠⠠	Full, left pointing	⠠⠠⠠⠠ ⠠⠠ ←
⠠⠠	Full, right pointing	⠠⠠⠠⠠ ⠠⠠ →
		⠠⠠⠠⠠⠠⠠ ⠠⠠ ↔
⠠⠠⠠	Lower only, left pointing	⠠⠠⠠⠠⠠⠠ ⠠⠠ ↶
⠠⠠⠠	Lower only, right pointing	⠠⠠⠠⠠⠠⠠ ⠠⠠ ↷
⠠⠠⠠	Upper only, left pointing	⠠⠠⠠⠠⠠⠠ ⠠⠠ ↶
⠠⠠⠠	Upper only, right pointing	⠠⠠⠠⠠⠠⠠ ⠠⠠ ↷

Example 9-3

Use $x \mapsto f(x)$ to denote that an element $x \in X$ is mapped to an element $f(x) \in Y$ by the mapping $f: X \rightarrow Y$.



Review 5.7 in Lesson 5 regarding transcription of the colon used in mapping notation.

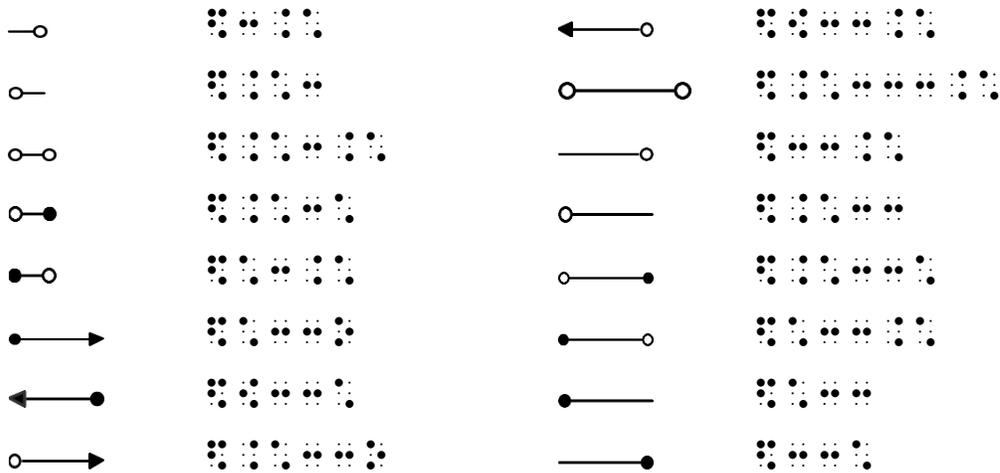
9.8 Arrows with Dotted Ends

Arrows may be represented by a shaft preceded or followed by a solid or hollow dot.

⠠⠠	Solid Dot	•
⠠⠠	Hollow Dot	◦

Dotted ends are most commonly encountered in geometry and will be illustrated in the lesson on modified expressions. *Note:* Arrow symbols used with graphic number lines are not represented by the braille symbols shown above. Refer to the most recent edition of *Guidelines and Standards for Tactile Graphics* for number line techniques.

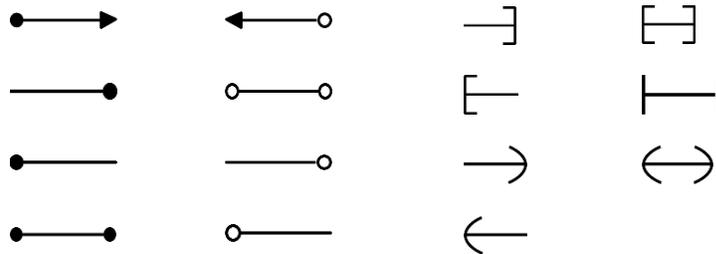
An sample list of arrows with dotted ends is shown on the next page.



Instructions: Transcribe in four columns, as printed. Assume all shafts are of regular length. Arrowheads printed as a solid triangle are considered to be barbed arrowheads.

PRACTICE 9B

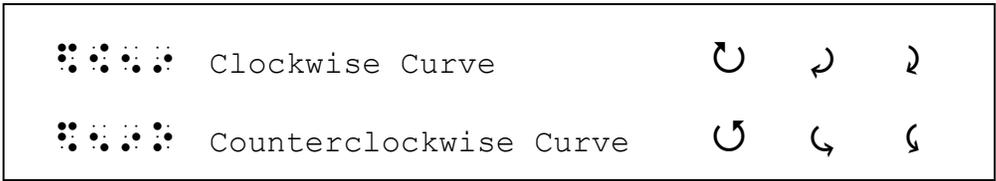
Other Types of Arrowheads



9.9.3 **Curved Arrow Shaft [NC 22.5.1]** Direction indicators are not used with curved arrows. Instead, the curved shaft symbol is used, and the type of arrowhead reveals the direction of curvature.



Rotation direction is indicated as follows. A curved shaft preceded by a *left*-pointing arrowhead represents a clockwise arrow. A curved shaft followed by a *right*-pointing arrowhead represents a counterclockwise arrow.



PRACTICE 9C

Other Types of Arrow Shafts

Vertical



Slanted



Curved



Can you figure out how to construct this spear with a northwest, blunted arrowhead?



Boldface and Compounded Arrows

9.10 Boldface Arrow [NC 22.6]

When an arrow is printed in boldface type and the typeform is determined to be significant and therefore retained, the boldface typeform indicator (dots 456) is incorporated into the structure of the arrow. The typeform indicator is placed at the beginning of the arrow symbol, before the arrowhead or arrow shaft. When a direction indicator is required, the direction indicator is transcribed first, followed by the boldface typeform indicator.

⠠⠠⠠⠠⠠⠠	Boldface Pointing Right	→
⠠⠠⠠⠠⠠⠠	Boldface Pointing Left	←
⠠⠠⠠⠠⠠⠠⠠⠠	Boldface Pointing Left and Right	↔
⠠⠠⠠⠠⠠⠠⠠⠠	Boldface Pointing Up	↑
⠠⠠⠠⠠⠠⠠⠠⠠	Boldface Pointing Down	↓
⠠⠠⠠⠠⠠⠠⠠⠠	Boldface Pointing Up and Down	↕

Reminder: A right-pointing arrow in nonregular type requires a shaft. The contracted form is not used.

9.11 Arrows Used as Signs of Comparison Compounded Vertically [NC 21.9]

Lesson 5 introduced signs of comparison compounded vertically—two or more simple signs of comparison arranged one under the other in print. When arrows are so arranged, the same principle applies. The combination of symbols becomes a single comparison sign "compounded vertically". The uppermost arrow symbol is transcribed first, immediately followed by and unspaced from the symbol for the lower arrow. Each arrow requires a shape indicator.

When the arrow is part of a sign of comparison compounded vertically, the uncontracted form of the right-pointing arrow is used. The contracted form of the right-pointing arrow is never used in the construction of arrows compounded vertically.

⠠⠠⠠⠠⠠⠠	Uncontracted Right-Pointing Arrow	→
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Arrows compounded vertically are most often encountered in the subject of chemistry. When the rules of *Chemical Notation Using the Nemeth Braille Code* are being followed, use the arrow constructions from that code, which differ from the Nemeth symbols.

When arrows compounded vertically are encountered in mathematics, use the constructions shown below. Mathematical arrows which are not shown in this list should be transcribed in accordance with the principle outlined in this section.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Pointing Right Over Pointing Left	↔
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Pointing Left Over Pointing Right	↔
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Long Pointing Right Over Short Pointing Left	↔
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Short Pointing Right Over Long Pointing Left	↔
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Pointing Right, Upper Barb Only Over Pointing Left, Lower Barb Only	↔
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Pointing Right Over Boldface Pointing Left	↔
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Boldface Pointing Left Over Pointing Right	↔
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Boldface Pointing Right Over Boldface Pointing Left	↔

9.12 Arrows Used as Signs of Comparison Compounded Horizontally [NC 21.11]

Lesson 5 also introduced the topic of signs of comparison compounded horizontally. When arrows are arranged side by side, the same principle applies. The combination of symbols becomes a single comparison sign "compounded horizontally". A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that the symbols are printed side by side. Arrows not shown in this list should be transcribed in accordance with this principle.

⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Pointing Up Followed by Pointing Down	↕
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Pointing Down Followed by Pointing Up	↕
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Pointing Up Followed by Boldface Pointing Down	↕
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Boldface Pointing Down Followed by Boldface Pointing Up	↕

Arrows compounded horizontally are most often encountered in the subject of chemistry. When the rules of *Chemical Notation Using the Nemeth Braille Code* are being followed, use the arrow constructions from that code, which differ from the Nemeth symbols.

9.13 Nonmathematical Arrows

Nonmathematical arrows should not be used for nonmathematical purposes. Rules and guidelines in BANA publications examine flowchart arrows, arrows in graphic organizer diagrams, lead lines in tactile graphics, etc. Refer to *Guidelines and Standards for Tactile Graphics* for techniques regarding arrows in specific applications such as box-and-whisker plots, clock faces, graphs, line plots, measurement tools, number lines, and spinners.

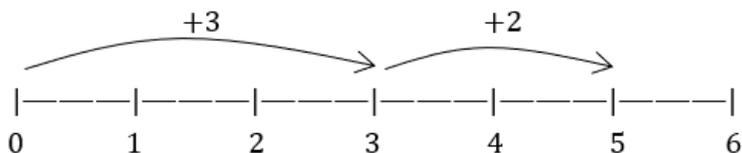
Here are some samples. (Transcriptions are not shown.)

Example 9-6

$$(2x + 3)(x - 5) = 2x^2 - 10x + 3x - 15$$

These "pointing" arrows should be drawn as a raised-line tactile graphic.

Example 9-7



Methods for depicting arrows above number lines are given in Guidelines and Standards for Tactile Graphics.

Example 9-8

$$\frac{3}{4} \leftarrow \text{fraction line}$$

The lead line in this diagram could be omitted without a loss of information.

INTRODUCTION TO SPATIAL ARRANGEMENTS

9.14 Background

Up to this point we have looked at mathematical material that is read linearly, from left to right, whether it appears embedded within the narrative or set apart from the text as displayed material. When material is arranged on more than one line in print and there is essential vertical alignment of place values or characters, a spatial arrangement is required in braille. ("Alignment" refers to vertical location in the same column or cell.)

9.14.1 **Numeric Indicator.** The numeric indicator is generally not used in arrangements which are aligned for computation. Use of the numeric indicator in certain situations abides by rules particular to that specific focus. This distinction will be clarified as each topic is presented.

9.14.2 **Format.** A blank line is required above and below a spatial arrangement. However, transition to a new braille page before beginning or after ending a spatial arrangement takes the place of the required blank line. When no running head is used, a spatial arrangement can begin on line 1 unless a code switch indicator is required at the page turn, or unless the print page number interferes with the alignment. Similarly, a spatial arrangement can end on line 25 unless a Nemeth Code terminator is required or unless the braille page number interferes with the alignment. These exceptions are illustrated later in this lesson.

This lesson teaches rules for the transcription of spatially arranged addition and subtraction problems.

Spatial Arrangements with Addition and Subtraction

The Preliminary Lesson introduced linear problems using the symbol + for plus, – for minus, and = for equals.

$$\gg 2 + 3 = 5 \quad \begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{matrix}$$

$$\gg 7 - 6 = 1 \quad \begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{matrix}$$

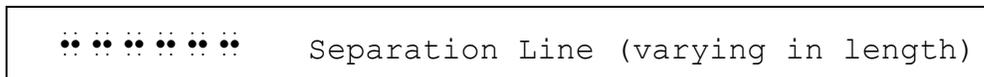
When addition and subtraction problems are printed in a vertical arrangement, Nemeth rules for spatial arrangements apply. We will use standard terminology to refer to the parts.

$$\begin{array}{r} 2 \\ + 3 \\ \hline 5 \end{array} \quad \begin{array}{l} \textit{addend} \\ \textit{addend} \\ \textit{separation line} \\ \textit{sum} \end{array}$$

$$\begin{array}{r} 7 \\ - 6 \\ \hline 1 \end{array} \quad \begin{array}{l} \textit{minuend} \\ \textit{subtrahend} \\ \textit{separation line} \\ \textit{difference} \end{array}$$

9.15 Separation Line

The line printed between the addends and the sum or between the subtrahend and the difference (signifying "equals") is called the "separation line" and is transcribed as a line of dots 25. In braille, the separation line must be made one cell longer at either end than the overall width of the arrangement.



9.16 Alignment with Addition and Subtraction

In spatial arrangements for addition and subtraction, the corresponding digits, commas, and decimal points are vertically aligned one below the other. In other words, digits under digits, commas under commas, decimal points under decimal points.

The Examples Placement of code switch indicators with spatial arrangements is discussed later in this lesson. Assume Nemeth context in these examples. Switch indicators and the blank line before and after each example are not shown.

Example 9-9

$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$	$\begin{array}{r} \cdot\cdot \\ \cdot\cdot \cdot\cdot \\ \hline \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \end{array}$	$\begin{array}{r} 7 \\ -6 \\ \hline 1 \end{array}$	$\begin{array}{r} \cdot\cdot \\ \cdot\cdot \cdot\cdot \\ \hline \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \end{array}$
--	---	--	---

Numeric indicators are not used. Digits are aligned by place value, as printed. The separation line extends one cell to the left and one cell to the right of the overall arrangement.

Example 9-10

$\begin{array}{r} 2,017 \\ +1,963 \\ \hline 3,980 \end{array}$	$\begin{array}{r} \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \hline \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \end{array}$	$\begin{array}{r} 7.5 \\ -6.7 \\ \hline 0.8 \end{array}$	$\begin{array}{r} \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \\ \hline \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \cdot\cdot \\ \cdot\cdot \cdot\cdot \cdot\cdot \end{array}$
--	--	--	--

Commas and decimal points are aligned as printed. The separation line extends one cell to the left and one cell to the right of the overall arrangement.

9.16.1 **Intentional Misalignment.** If items have been intentionally misaligned as an exercise for the student, the misalignment is preserved in the transcription. In the next example, the student has been instructed to arrange the digits for proper place value.

Example 9-11

$$\begin{array}{r}
 25.92 \\
 10.4 \\
 +3.796 \\
 \hline
 \end{array}$$

The decimal points are intentionally misaligned. The braille transcription matches the print arrangement.

9.16.2 **Operation Sign is Absent.** The plus sign is not always present in addition problems. If there is no operation sign, examine the surrounding text to determine that this is indeed an addition problem. Then apply alignment rules for addition.

Example 9-12

$$\begin{array}{r}
 .36 \\
 7.02 \\
 3.04 \\
 \hline
 \end{array}$$

9.17 Placement of Symbols

9.17.1 **Operation Symbols.** The plus or minus symbol indicating addition or subtraction is placed one column of cells to the left of the leftmost numeric symbol in the part of the arrangement which lies *above* the separation line, regardless of print layout.

Example 9-13

$ \begin{array}{r} 97 \\ +1 \\ \hline \end{array} $	$ \begin{array}{r} 97 \\ +1 \\ \hline 98 \end{array} $
---	---

The plus sign is printed within the column reserved for the tens place. In braille, the plus sign is placed one column to the left of the leftmost numeric symbol that appears above the separation line.

Example 9-14

$$\begin{array}{r}
 877 \\
 -16 \\
 \hline
 861
 \end{array}$$

The minus sign is printed under the "8" in the minuend. In braille, the minus sign is placed one column to the left of the leftmost numeric symbol that appears above the separation line.

- a. **Considerations Below the Separation Line.** Placement of the operation symbol is in relation to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may fall beneath symbols which appear above the separation line.

Example 9-15

$$\begin{array}{r}
 65 \\
 +56 \\
 \hline
 121
 \end{array}$$

The "ones" and "tens" place values in the sum are aligned with those place values in the addends. Because the plus sign is placed one cell to the left of the "tens" column, the "hundreds" place value in the sum (below the separation line) falls in the same column as the plus sign.

9.17.2 **Monetary Symbols.** Monetary symbols which appear above the separation line must also be similarly placed—that is, one column of cells to the left of the leftmost numeric symbol in the part of the arrangement which lies *above* the separation line. A plus or minus symbol (one cell) aligned below a monetary symbol (two cells) is right adjusted in the column.

Example 9-16

$$\begin{array}{r}
 \$.36 \\
 7.02 \\
 +3.04 \\
 \hline
 \end{array}$$

- a. **Considerations Below the Separation Line.** Placement of the monetary symbol applies to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may be shown beneath the symbols which appear above it.

Example 9-17

\$.36	⠠⠠⠠⠠⠠⠠
7.02	⠠⠠⠠⠠⠠⠠
+ 3.04	⠠⠠⠠⠠⠠⠠
-----	⠠⠠⠠⠠⠠⠠
\$10.42	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
	⠠⠠⠠⠠⠠⠠

The decimal point and place values in the sum align with the decimal points and place values in the addends. The dollar signs above and below the separation line are not aligned in print and so need not be aligned in braille.

- b. **Further Considerations.** The alignment rules above the separation line apply regardless of print layout, as illustrated in the following examples. Below the separation line, normal spacing rules for monetary symbols apply.

Example 9-18

\$1.45	⠠⠠⠠⠠⠠⠠
- 1.05	⠠⠠⠠⠠⠠⠠
-----	⠠⠠⠠⠠⠠⠠
	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

The minus sign is printed to the left of the dollar sign. This alignment is not duplicated in braille.

Example 9-19

\$1.45	⠠⠠⠠⠠⠠⠠
- 1.05	⠠⠠⠠⠠⠠⠠
-----	⠠⠠⠠⠠⠠⠠
\$.40	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
	⠠⠠⠠⠠⠠⠠

Below the separation line, the dollar sign in print is spaced away from the decimal in order to align with the dollar sign above the separation line. In braille, no space comes between the dollar sign and the decimal.

9.18 Side-by-Side Layout

When there is just one spatial problem, it is placed following established margin rules for embedded or displayed expressions. When there is more than one spatial problem, it is preferred that they be transcribed side by side across the page with the leftmost symbol in cell 1. In side-by-side layout, care must be taken to leave at least three blank cells between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement. "Any symbol" does not refer to a separation line. This means that one blank space is left between the end of one separation line and the beginning of the next when neighboring arrangements contain separation lines.

Example 9-20

$$\begin{array}{r}
 35 \\
 -24 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 975 \\
 +25 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 12,618 \\
 -1,414 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 17.17 \\
 +.17 \\
 \hline
 \end{array}$$

The first separation line begins in cell 1. Each separation line extends one cell to the left and one cell to the right of its overall arrangement. One clear column of blank space is between the ends of the separation lines.

9.18.1 **Top Alignment.** The first addend in each problem is transcribed on the same line, regardless of the print arrangement. When the separation lines are not on the same braille line, at least one clear column of blank space (one cell's width) must be maintained between the ends of the separation lines. This assures that at least three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring spatial arrangement (separation lines excluded).

Example 9-21

$$\begin{array}{r}
 \$3.00 \\
 .99 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 \$7.39 \\
 .25 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 \$.56 \\
 .37 \\
 .93 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 \$6.26 \\
 5.83 \\
 .27 \\
 \hline
 \end{array}$$

Both polynomials are right justified in print. The student is instructed to align the terms. (This example illustrates the proper placement of switch indicators and margins for displayed spatial material. Both topics are discussed later in this lesson.)

9.21 Abbreviations

When abbreviations occur in a spatially arranged addition or subtraction problem, they are vertically aligned and transcribed according to the rules for abbreviations outlined in Lessons 3 and 4.

Example 9-31

$$\begin{array}{r}
 4 \text{ ft. } 7 \text{ in.} \\
 + 3 \text{ ft. } 5 \text{ in.} \\
 \hline
 7 \text{ ft. } 12 \text{ in.}
 \end{array}$$

9.22 Fractions

In a spatial addition or subtraction arrangement containing fractions, the fraction lines and fraction indicators are vertically aligned throughout the problem. Also, each numerator and denominator is in contact with its fraction line, regardless of the number of digits. (Digits in numerators and denominators do not align by place value.) Indicators and numbers must not appear in the same column.

Spaces may need to be inserted to achieve alignment.

Example 9-32

$$\begin{array}{r}
 1/4 \\
 + 2/4 \\
 \hline
 \end{array}$$

Recall from 8.4.2 that fraction indicators are not transcribed when a fraction is printed in this manner.

Example 9-33

$$\begin{array}{r}
 13/16 \\
 + 5/16 \\
 \hline
 \end{array}$$

The opening and closing fraction indicators are aligned. The fraction lines are aligned. A space is inserted before the 1-digit numerator in order to have it touch the fraction line.

- a. **Alignment with Mixed Numbers.** The whole number part of a mixed number is vertically aligned by place value.

Example 9-34

$$\begin{array}{r}
 2 \frac{15}{16} \\
 + 10 \frac{5}{8} \\
 \hline
 \end{array}$$

Whole numbers 2 and 10 align by place value.

- b. **Alignment of Different Indicators.** When different kinds of fraction indicators occur in the same problem, align the (1456) and (3456) symbols. A space is inserted in order to achieve alignment.

Example 9-35

$$\begin{array}{r}
 \frac{3}{8} \\
 + 9 \frac{1}{8} \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 5 \frac{1}{2} \\
 + \frac{11}{12} \\
 \hline
 \end{array}$$

Notice the blank cell above or below the (456) part of the mixed number indicators to assure that indicators and numbers do not appear in the same column. Reminder: Each numerator and denominator is in contact with its fraction line.

9.22.1 Alignment with the Answer

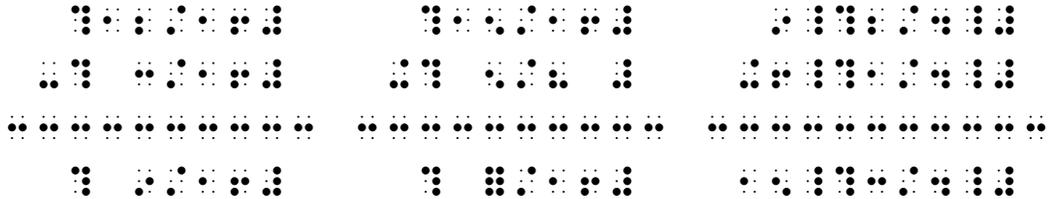
The fraction indicators and fraction lines in the answer below the separation line must also align with those above the separation line. Blank cells are inserted in order to achieve alignment.

Example 9-36

$$\begin{array}{r} 12 \\ \hline 16 \\ - 3 \\ \hline 16 \\ \hline 9 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 15 \\ \hline 16 \\ + 5 \\ \hline 8 \\ \hline ? \\ \hline 16 \end{array}$$

$$\begin{array}{r} 9 \frac{2}{4} \\ + 6 \frac{1}{4} \\ \hline 15 \frac{3}{4} \end{array}$$

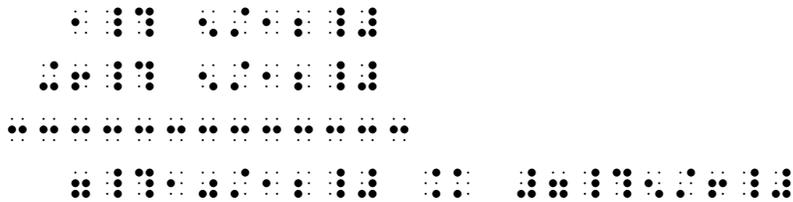


Fraction indicators and fraction lines are aligned. Each numerator and denominator is in contact with its fraction line. Take particular notice of the single-digit numerators and denominators as well as the general omission symbol in the second problem. Recall from [9.17.1.a](#) that the plus sign in the third problem can be in the same column as the tens place of the 2-digit whole number in the answer.

9.22.2 A Linear Portion of a Spatial Arrangement. An equals sign printed within a spatial arrangement follows linear spacing rules for signs of comparison.

Example 9-37

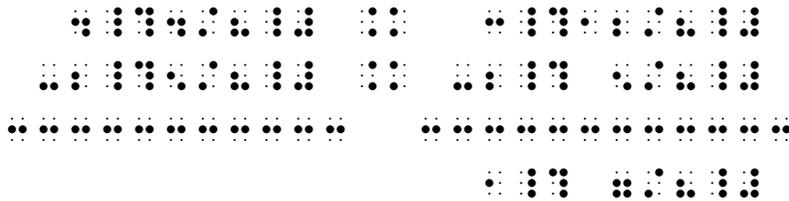
$$\begin{array}{r} 1 \frac{5}{12} \\ + 6 \frac{5}{12} \\ \hline 7 \frac{10}{12} = 7 \frac{5}{16} \end{array}$$



A numeric indicator is required for the final answer because it is not part of the vertical computation.

Example 9-38

$$\begin{array}{r}
 4 \frac{4}{8} = 3 \frac{12}{8} \\
 - 2 \frac{5}{8} = - 2 \frac{5}{8} \\
 \hline
 1 \frac{7}{8}
 \end{array}$$



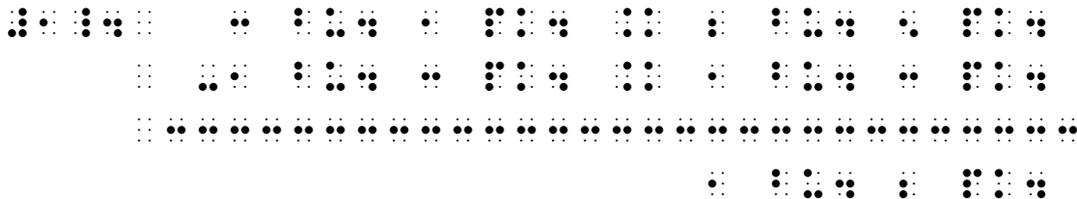
The equals signs show a relationship between the two side-by-side spatial arrangements.

9.23 Placement of Identifiers

The numeric or alphabetic identifier associated with a spatially arranged addition or subtraction problem is positioned at the top line of the problem—that is, with the first addend or with the subtrahend. One clear column of blank space (one cell's width) must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines.

Example 9-39

$$\begin{array}{r}
 1. \quad 3 \text{ bu. } 1 \text{ pk.} = 2 \text{ bu. } 5 \text{ pk.} \\
 \quad - 1 \text{ bu. } 3 \text{ pk.} = 1 \text{ bu. } 3 \text{ pk.} \\
 \hline
 \quad \quad \quad 1 \text{ bu. } 2 \text{ pk.}
 \end{array}$$



9.23.1 **Side-by-Side Layout.** Spatially arranged itemized material may be arranged side by side. The identifiers are located on the same braille line across the width of the page, in sequential order. Problems are spaced so that no less than three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded) including the neighboring identifier.

Note that this is a special rule pertaining to spatially arranged itemized material. As taught in Lesson 1, each main item in *nonspatial* itemized material must start in cell 1, even if the print copy arranges the items side by side across the page.

Example 9-40

$$\begin{array}{cccccc}
 1) & 15 & 2) & 15 & 3) & 15 & 4) & 15 & 5) & 150 & 6) & 150 \\
 & \underline{+3} & & \underline{-3} & & \underline{+13} & & \underline{-13} & & \underline{+130} & & \underline{-130}
 \end{array}$$

10

11

12

13

14

15

16

17

18

Lines 11 and 15: Three blank cells come between the rightmost symbol in each problem (the "ones" column) and the numeric indicator of the next identifier.

Lines 11-13 and 15-17: One blank cell comes between the right parenthesis of each identifier and the leftmost cell of the separation line on line 3.

Line 14: A blank line precedes the second group of spatially arranged problems.

Line 15: No attempt is made to align identifiers with those on line 11.

Instructions: Transcribe the opening Nemeth Code indicator on a line by itself, in cell 1. The next line should be blank. Begin the problems on the line following the blank line. Use side-by-side layout. After completion of the last problem, leave one line blank and transcribe a Nemeth Code terminator in cell 1.

PRACTICE 9E

1)
$$\begin{array}{r} 621 \\ + ??? \\ \hline 1096 \end{array}$$

2)
$$\begin{array}{r} 17x - 8y - z \\ - 2x + 17y + 6z \\ \hline 15x + 9y + _ _ \end{array}$$

3)
$$\begin{array}{r} 3x^2 - 5x + 4 \\ - 5x^2 + 12x - 12 \\ \hline -2x^2 + 7x - 8 \end{array}$$

4)
$$\begin{array}{r} 1 \frac{2}{3} \text{ yr} \\ + 5 \frac{5}{12} \text{ yr} \\ \hline \end{array}$$

$$6 \frac{13}{12} \text{ yr} = 7 \frac{1}{12} \text{ yr}$$

9.24 Regrouping Numbers in Addition Problems

Small numbers printed above or below digits in a problem are referred to as "regrouping numbers". (Older texts may call them "carried numbers".) The transcription places the regrouping numbers in the same location as printed, inserting the appropriate regrouping indicator between the regrouping numbers and their associated digits. A regrouping line usually does not appear in print, since the smaller font is enough to distinguish them.

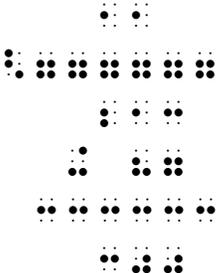
The first cell of the regrouping indicator tells the reader to read the numbers above the line or below the line.

	Regrouping Indicator for Numbers Above the Arrangement (varying in length)
	Regrouping Indicator for Numbers Below the Arrangement (varying in length)

The regrouping indicator will be one cell longer on the left than the separation line. It is important to place the regrouping numbers in the same columnar positions as in print.

In the first example, the regrouping numbers are printed above the first addend. In the second example, the regrouping numbers are printed above the separation line.

Example 9-41

$\begin{array}{r} \overset{1}{2}13 \\ + 87 \\ \hline 300 \end{array}$	
---	---

In the print copy, the regrouping numbers are in a smaller typeface and a regrouping line is not present.

Example 9-42

	7 9	⠠⠠⠠⠠
	2 1 4	⠠⠠⠠⠠⠠
+	1 8	⠠⠠⠠⠠
	<u>3 1 1</u>	⠠⠠⠠⠠⠠⠠⠠⠠
		⠠⠠
		⠠⠠⠠⠠⠠⠠
		⠠⠠⠠

In the print copy, the regrouping numbers are in a smaller typeface and a regrouping line is not present.

9.24.1 **With Identifiers.** If an identifier is present, it is placed on the first line of the addition problem (the first addend) regardless of the presence of regrouping numbers. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines and regrouping indicators.

Example 9-43

	25.9	⠠⠠⠠⠠⠠
1.	49.4	⠠⠠⠠⠠⠠
	87.2	⠠⠠⠠⠠⠠
	<u>162.5</u>	⠠⠠⠠⠠⠠⠠⠠⠠
		⠠⠠⠠⠠⠠

Introduction to Cancellation

[NC Rule 12]

A spatial arrangement is required when numbers, letters, abbreviations, or words in a mathematical expression are canceled in print by any type of stroke through them, and replacement values are shown. The opening and closing cancellation indicators enclose the material being canceled.

⠠	Opening Cancellation Indicator
⠡	Closing Cancellation Indicator

➤	6	⠠⠠⠡
➤	6	⠠⠠⠡
➤	ft	⠠⠠⠠⠠⠡
➤	mol	⠠⠠⠠⠠⠡

9.25 Cancellation in Subtraction Problems

When cancellation is shown in subtraction, a regrouping indicator is not needed. Each regrouping number is aligned with the corresponding canceled digit in the problem.

Digits, decimal points, or other symbols above and below the line with cancellation must be carefully aligned for computation. Spaces are inserted where necessary to achieve alignment. Numerals or other symbols above *and* below the separation line must not appear in the same columns as cancellation indicators.

Example 9-44

$ \begin{array}{r} \overset{4}{1} \overset{10}{\cancel{0}} \\ - 39 \\ \hline 111 \end{array} $	
---	--

Place values align throughout the arrangement, including the regrouping numbers. To assure a clear column above the cancellation indicators, a space is inserted before the two-digit regrouping number on line 1.

Example 9-45

$$\begin{array}{r}
 \overset{11}{7} \cancel{2} \overset{13}{3} \\
 - 254 \\
 \hline
 569
 \end{array}$$

Cancellation indicators align with cancellation indicators. The minus sign on line 4 is placed one column to the left of the opening cancellation indicator on line 3 to assure a clear column below that indicator.

9.25.1 **With Identifiers.** If an identifier is present, it is placed on the first line of the subtraction problem (the minuend) regardless of the presence of canceled items. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 9-46

A.

$$\begin{array}{r}
 \overset{7}{\cancel{2}} \overset{12}{\cancel{3}} \overset{4}{\cancel{5}} \overset{16}{\cancel{6}} \\
 - 2347 \\
 \hline
 5909
 \end{array}$$

PRACTICE 9F

Regrouping Numbers and Cancellation

1.

$$\begin{array}{r}
 \overset{1}{9} \overset{1}{4} 8 \\
 + 75 \\
 \hline
 1023
 \end{array}$$

2.

$$\begin{array}{r}
 \$ \overset{12}{2} \overset{2}{.} 52 \\
 65.89 \\
 91.99 \\
 \hline
 32.03 \\
 \hline
 \$192.43
 \end{array}$$

3.

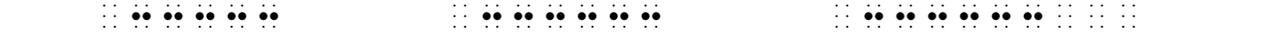
$$\begin{array}{r}
 \overset{4}{\cancel{5}} \overset{16}{\cancel{6}} \overset{1}{\cancel{7}} \overset{13}{\cancel{8}} \\
 - 804 \\
 \hline
 4819
 \end{array}$$

9.26.3 **Pagination and Blank Lines.** Transition to a new braille page before beginning or after ending the transcription of a spatial arrangement takes the place of the required blank line. *The discussion in this section assumes that code switch indicators are not needed, imagining that Nemeth material precedes and follows each example.*

- a. **Starting a Braille Page with a Spatial Arrangement.** When no running head is in use, a spatial arrangement may begin on line 1. There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the print page number. If a running head is present, line 2 must be blank and the spatial arrangement will begin on line 3.

Example 9-48

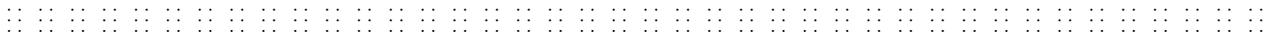
1) $\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$	2) $\begin{array}{r} 155 \\ +3 \\ \hline \end{array}$	3) $\begin{array}{r} 155 \\ +133 \\ \hline \end{array}$
--	---	---

1			
2			
3			
4			

This example has no running head. The problems begin on line 1.

Example 9-49

1) $\begin{array}{r} 1 \\ +1 \\ \hline \end{array}$	2) $\begin{array}{r} 2 \\ +2 \\ \hline \end{array}$	3) $\begin{array}{r} 13 \\ +3 \\ \hline \end{array}$	4) $\begin{array}{r} 14 \\ +4 \\ \hline \end{array}$
---	---	--	--

1				
2				
3				
4				
5				
6				

This example has a running head. Line 2 is blank.

- b. **Ending a Braille Page with a Spatial Arrangement.** A spatial arrangement may be placed on the last line of the braille page. There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the braille page number.

Example 9-50

$$\begin{array}{r}
 1) \quad 15 \\
 \quad + 3 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 2) \quad 451 \\
 \quad -301 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 3) \quad 959 \\
 \quad + 13 \\
 \hline
 \end{array}$$

22

23

24

25

Line 25: There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the braille page number.

9.27 Wide Arrangements [NC 26.6.1]

As noted above, if any portion of a spatial arrangement falls on lines 1, 2, 24, or 25 of the braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement and the first symbol of the page number. This restriction includes the rightmost symbol of a separation line. If this spacing cannot be achieved, the arrangement must begin on line 3 or end on line 23, respectively.

The following example illustrates a wide arrangement that begins in cell 3, assuming it is displayed to narrative.

Example 9-51

$$\begin{array}{r}
 S = 1 + 3 + 5 + \dots + 45 + 47 + 49 \\
 S = 49 + 47 + 45 + \dots + 5 + 3 + 1 \\
 \hline
 2S = 50 + 50 + 50 + \dots + 50 + 50 + 50
 \end{array}$$

Example 9-51-a (Arranged near the top of a braille page, with no running head.)

1

2

3

4

5

6

Lines 1-2: The separation line (line 5) invades the three-cell area before the print page number on line 1, so this problem is not allowed to fall on lines 1 or 2.

Example 9-53

1. a.
$$\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$$
 b.
$$\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$$
 c.
$$\begin{array}{r} 15 \\ +13 \\ \hline \end{array}$$
 d.
$$\begin{array}{r} 15 \\ -13 \\ \hline \end{array}$$

Braille representation of the arithmetic problems above, including the numbers 15, 3, 13, and 18, and the plus and minus signs.

9.28.2 **With Narrative.** If there is narrative associated with the main division number, the subdivisions begin on the next available line (following the required blank line that precedes spatial material) starting in cell 3.

Example 9-54

1. Solve: a.
$$\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$$
 b.
$$\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$$
 c.
$$\begin{array}{r} 15 \\ +13 \\ \hline \end{array}$$
 d.
$$\begin{array}{r} 15 \\ -13 \\ \hline \end{array}$$

Braille representation of the arithmetic problems above, including the numbers 15, 3, 13, and 18, and the plus and minus signs. The Braille includes a blank line before the problems and a blank line after the last problem.

Placement of Code Switch Indicators

Each code switch indicator should fall on the same braille page as the mathematical material to which it applies. (There are times when this is not possible.) A code switch indicator does not take the place of a blank line. To keep the code switch indicators from interfering with the layout of the problem, they are placed outside of the arrangement. Position the required blank lines *between* the Nemeth switches as described below.

9.29 Code Switching with Unitemized Spatial Arrangements

- i. If the problems are not preceded by text, the opening Nemeth Code indicator is placed in cell 1 on a line by itself.

If there is text before the problems, the opening Nemeth Code indicator is placed on the same line with the end of the text. If there is not room on that line for the switch indicator, it is placed in cell 1 on the next line. “Text” includes instructions and transcriber’s notes.

For spatial material following a heading, see Section [9.31](#).

- ii. The required blank line comes next.
- iii. When the spatial problems are transcribed side by side, the full width of the page is utilized; that is, the leftmost symbol of the first problem in each row is placed in cell 1. This layout is applied regardless of the margins applied to the surrounding text.

When there is only one spatial problem, or when they are not transcribed side by side, the leftmost symbol of each arrangement is placed in the appropriate cell for displayed material.

- iv. When Nemeth Code terminates after a spatial problem, the required blank line comes first, followed by the Nemeth Code terminator in cell 1 on a line by itself.
- v. Resume UEB text on the line following the terminator, unless a format change that requires a leading blank line, such as a heading, follows.

Example 9-55

15	115	155
+3	+30	+13
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>

Braille representation of the example above, showing the placement of Nemeth Code indicators and blank lines between the arithmetic problems.

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

21

22

23

- Line 5: A blank line is required following the centered heading on line 4.*
- Line 6: The opening Nemeth Code indicator is placed in cell 1 on a line by itself.*
- Line 7: A blank line precedes the spatial problem set.*
- Line 11: A blank line follows the spatial problems.*
- Line 12: The Nemeth Code terminator is placed in cell 1 on a line by itself.*
- Line 14: A blank line precedes the centered heading on line 15.*
- Lines 16-23: The same rules apply as to lines 5-12.*

For further practice, see Appendix A—Reading Practice.

EXERCISE 9

Prepare Exercise 9 for your grader.

