LESSON 16

- SPATIAL ARRANGEMENTS OF FRACTIONS
- HYPERCOMPLEX FRACTIONS
- CONTINUED FRACTIONS
- INSTRUCTIONAL COMMENTARY
- STEM-AND-LEAF PLOTS

Answers to Practice Material

LESSON PREVIEW

The remaining lessons take a look at a variety of formatting topics. In this lesson, spatially arranged fractions are studied, a braille format for instructional commentary is given, and layouts for stem-and-leaf plots are introduced.
Fractions are usually transcribed linearly, as discussed in Lesson 8. However, certain situations suggest or require a spatial arrangement.

Recall the terminology used for the parts of a fraction.

\[ \frac{3}{4} \]

16.1 Spatial Fraction Line

The spatial fraction line is comprised of a series of dots 25. An opening fraction indicator (1456) marks the beginning of the fraction line; a closing fraction indicator (3456) signals the end.

In a spatially arranged fraction the numerator is placed above the spatial fraction line and the denominator is placed below it. The fraction line is comprised of the same number of cells as the widest item above (numerator) or below (denominator), with the fraction indicators placed one cell beyond the width on either end, as illustrated in the next section.

Note: Code switch indicators are absent in many of the isolated examples in this lesson.

16.2 Numerator and Denominator

The numerator is centered above the fraction line; the denominator is centered below the fraction line. If exact centering is not possible, the item is moved to the left one cell. Since fractions do not contain material aligned for computation, the numeric indicator is used in the numerator and/or denominator according to the rules of the Nemeth Code. For the same reason, the English-letter indicator is used where necessary in the numerator and/or denominator.

**Example 16-1**

\[
\begin{align*}
\text{(a)} & \quad \text{(b)} & \quad \text{(c)} \\
\frac{3}{4} & \quad \frac{x}{y} & \quad \frac{3}{100}
\end{align*}
\]
16.3 Placement of Identifiers with Spatially Arranged Fractions

An identifier, if present, is placed on the same braille line as the fraction line. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement.

Example 16-2

5. \( \frac{(9x) + 3}{(3x + 15) + 3} \)  
6. \( \frac{3xy + 9x^2}{3x} \)

Recall from Lesson 9 that three blank cells come between the rightmost symbol of an arrangement and the next identifier.
Situations Requiring Spatial Presentation of Simple Fractions

16.4 Fractions Arranged Spatially for Illustration

When the parts of a simple fraction are labeled, use a spatial arrangement. The labels are uncontracted because they are within the code switches.

Example 16-3

Four children share a dozen cookies equally. Expressed as a fraction, how many cookies does each child receive?

$$\frac{12}{4} \quad \text{number of cookies}$$

$$\frac{3}{4} \quad \text{number of children}$$
PRACTICE 16A

1. Shandra invited seven friends to a pizza party. Two pizzas were ordered. Each pizza had eight slices. Which fraction shows how many slices of pizza each child can have if they share equally?

   a. \( \frac{2}{8} \) number of pizzas

   b. \( \frac{8}{8} \) number of slices

   c. \( \frac{16}{8} \) number of children

16.5 Cancellation with Replacement Values [NC 12.1]

Recall that a spatial arrangement is required when numbers, letters, or abbreviations are canceled in print and replacement values are shown. Reminder: The print copy may show any type of stroke to indicate cancellation.

![Opening Cancellation Indicator](image1)

![Closing Cancellation Indicator](image2)

When cancellation occurs within a fraction, replacement items are placed above canceled items in the numerator and below canceled items in the denominator. Each replacement item is centered with respect to the canceled term. If exact centering is not possible, the term is moved to the left one cell.

Example 16-5

\[ \frac{25}{50} \]

1. \( \frac{1}{2} \)

2. \( \frac{5}{10} \)

3. \( \frac{3}{6} \)

4. \( \frac{2}{4} \)

5. \( \frac{1}{2} \)
Line 1: Replacement "1" is centered above the canceled number. A numeric indicator is required.
Line 2: A numeric indicator is not required because the numeral is not preceded by a space.
Line 3: The fraction line is as long as the widest item in the fraction. In this case, both numerator and denominator have a 4-cell item.
Line 4: A numeric indicator is not required because the numeral is not preceded by a space.
Line 5: Replacement "2" is centered below the canceled number. A numeric indicator is required.

Example 16-6

\[
\frac{25}{50} = \frac{1}{2}
\]

Note that the second fraction ("one-half") is transcribed as a linear fraction because it doesn't contain cancellation. The equals sign and the linear fraction are placed on the same line as the spatial fraction line.

16.5.1 Alignment Considerations with Cancellation. Vertical alignment of place value is not a consideration in spatially arranged fractions when the material is not aligned for computation. Replacement items are centered above or below their canceled item. Placement of the replacement item is relative to the width of the canceled item, without respect to place value. Note the location of the replacement items in the next example.

Example 16-7

\[
\frac{1449}{207} = 7
\]
a. **Alignment of Indicators.** When the canceled number and its replacement contain the same number of digits, the numeric indicator and the opening cancellation indicator will align.

### Example 16-8

\[
\frac{\frac{1}{6}}{\frac{36}{6}} = \frac{1}{6}
\]

<table>
<thead>
<tr>
<th>Line 1</th>
<th>Line 2</th>
<th>Line 3</th>
<th>Line 4</th>
<th>Line 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replacement &quot;1&quot; is centered above the canceled number on line 2. In this case, the numeric indicator lies directly above the opening cancellation indicator below it.</td>
<td>Canceled &quot;6&quot; is centered above the fraction line. Exact centering is not possible so it is moved left one cell.</td>
<td>The fraction line is as long as the widest item in the fraction (the denominator).</td>
<td>Canceled &quot;36&quot; is centered below the fraction line.</td>
<td>Replacement &quot;6&quot; is centered below the canceled number above it.</td>
</tr>
</tbody>
</table>

16.5.2 **Stacked Cancellation.** More than one level of cancellation may be encountered.

### Example 16-9

\[
\frac{15 \times 5280}{60 \times 60} = \frac{22}{4} = 5
\]
PRACTICE 16B

Simplify using cross canceling. Then multiply the numerators and multiply the denominators.

1. \( \frac{\frac{1}{3}}{\frac{5}{15}} \times \frac{\frac{1}{5}}{\frac{6}{2}} = \frac{1}{6} \)

2. \( \frac{\frac{1}{16}}{\frac{8}{1}} \times \frac{\frac{8}{1}}{\frac{8}{2}} = \frac{1 \times 8}{1} = 8 \)

16.5.3 Cancellation and Level Indicators. Care must be taken to place the opening cancellation indicator and its paired closing cancellation indicator on the same level.

Both the base and its superscript are canceled with one stroke. A baseline indicator places the cancellation terminator on the same level as the paired opening cancellation indicator.

Only the superscript is canceled. The opening and closing cancellation indicators are at the superscript level.

Example 16-10

The denominator is centered under the fraction line.
Example 16-11

\[ \frac{4x^2q}{p^{\frac{3}{2}}} \]

The replacement numeral "2" is placed below the canceled superscript in the denominator, even though it is printed beside the canceled superscript.

PRACTICE 16C

1. \[ \frac{2b + a}{b} \div \frac{4b^2 - a^2}{b^2} = \frac{1}{b} \cdot \frac{b}{\frac{b}{1}} = \frac{b}{2b - a} \]

16.6 Cancellation Without Replacement Values [NC 12.1]

Material containing cancellation with no replacement may be transcribed linearly.

16.6.1 Linear Presentation of Simple Fractions with Canceled Items. When replacement items are not given for canceled material in the numerator and denominator, a linear fraction may be transcribed. The cancellation indicators must exactly represent what is canceled in print.

Example 16-12

Each letter "r" "s" and "t" is printed with a slash through it.

Example 16-13

Each letter combination "rst" as well as the parentheses are printed with a line through them.
Example 16-14

Use factorization and cross cancellation to simplify \( \frac{x^2 - 5x + 6}{-x + 2} \).

\[
\frac{x^2 - 5x + 6}{-x + 2} = \frac{(x-2)(x-3)}{-(x-2)}
\]

\[
= \frac{(x-2)(x-3)}{-(x-2)}
\]

\[
= -x + 3
\]

### 16.6.2 Canceled Abbreviations

When canceled abbreviations show no replacements, linear transcription may be applied. The abbreviation is unspaced from the cancellation indicators. As a whole, however, the abbreviation must follow the spacing rules for abbreviations (Lesson 3). If a space is required, the space comes before and/or after the related indicator or indicators.

- \( \text{in.} \)
- \( \text{in.} \)
- \( 7888 \text{ in}^3 + n \)
- \( \frac{1 \text{ ft}^3}{1728 \text{ in}^3} \)
Example 16-15

By canceling the cubic inches, the answer can be expressed in cubic feet.

\[
\frac{7888 \text{ in}^3}{1728 \text{ in}^3} \times \frac{1 \text{ ft}^3}{1728 \text{ in}^3} = \frac{7888 \text{ ft}^3}{1728}
\]

Lines 3-4: The fractions are transcribed linearly because the canceled portions do not show replacement values.

Line 3: A space is required between each abbreviation and its related numeral, even when the abbreviation is canceled. A space is required between the abbreviation and the operation symbol (multiplication dot) even when the abbreviation is canceled.

Line 4: A space is required between the numeral and its associated abbreviation.

PRACTICE 16D

How many meters are in 100 yards? By multiplying 100 yd by several expressions of "1" and canceling units of measure, a solution is found. Answer: There are 91.44 m in 100 yd.

\[
? \text{ m} = 100 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}}
\]

\[
= \frac{100 \times 3 \times 12 \times 2.54 \times 1 \text{ m}}{100}
\]

\[
= 91.44 \text{ m}
\]
HYPERCOMPLEX FRACTIONS

[NC 13.7 and 13.8]

16.7 Definition and Recognition

The term "hypercomplex" fraction is used only in the context of the Nemeth Code—it is unlikely that you will encounter this term in a math book. First, we will review the definition of a complex fraction.

Recall from Lesson 8 that a complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers. In other words, a complex fraction is a fraction within a fraction.

\[ \frac{\frac{1}{2}}{5} \]

This is a complex fraction.

If a simple fraction contains a complex fraction at the superscript or subscript level, it is still a simple fraction. In this example, the numerator is "1" and the denominator is "2 raised to the (one-half over three-fourths) power."

\[ \frac{1}{\frac{\frac{1}{2}}{2^{\frac{3}{4}}}} } \]

This is a simple fraction.

The complex fraction is at the superscript level.

A hypercomplex fraction is one whose numerator or denominator, or both, contain at least one complex fraction. Here is a print example of a hypercomplex fraction. The denominator is "10" and the numerator contains the complex fraction shown above, "one-half over five."

\[ \frac{\frac{1}{2}}{\frac{5}{10}} \]

This is a hypercomplex fraction.

The numerator contains a complex fraction.
16.8 Transcription of Hypercomplex Fractions

Although a linear transcription is possible, using a linear arrangement for the numerator and a linear arrangement for the denominator, within a spatial arrangement overall, gives a clearer representation of the hypercomplex fraction.

16.8.1 Hypercomplex Fraction Indicators. Opening and closing hypercomplex fraction indicators mark the beginning and end of a spatial hypercomplex fraction line.

Here is the transcription of the hypercomplex fraction shown on the previous page. The complex fraction in the numerator is transcribed as a linear arrangement within the overall spatial arrangement.

Example 16-16

\[
\begin{array}{c}
\frac{1}{2} \\
\frac{5}{10}
\end{array}
\]

Terms of the fraction are centered above and below their fraction lines.

If the numerator and denominator are too long to fit on one line, follow the rules regarding division of mathematical expressions between braille lines. (Lesson 15) The next example shows an expression which will fit in 40 cells as first transcribed, but if it is displayed, division of the denominator is necessary.
Example 16-17

1. Show that \( \frac{3 \frac{1}{3}}{7 \frac{1}{5}} \times \frac{1}{\frac{2.2}{7}} \) of \( \frac{11 \frac{1}{5}}{17} \times \frac{4 \frac{4}{11}}{4} = \frac{1}{2} \).

16.8.2 Other Layouts. The combined transcription method illustrated above is the preferred way of transcribing a hypercomplex fraction. However, it is permissible to use an entirely spatial arrangement or an entirely linear arrangement. In particular, linear hypercomplex fractions are difficult to assemble mentally. Example 16-16 is illustrated below using these two alternate methods. Symbols and indicators used in the linear method are as follows.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Opening Hypercomplex Fraction Indicator</td>
</tr>
<tr>
<td></td>
<td>Horizontal Hypercomplex Fraction Line</td>
</tr>
<tr>
<td></td>
<td>Diagonal Hypercomplex Fraction Line</td>
</tr>
<tr>
<td></td>
<td>Closing Hypercomplex Fraction Indicator</td>
</tr>
</tbody>
</table>
Example 16-18

\[
\begin{array}{c}
\frac{1/2}{5} \\
\frac{10}{3}
\end{array}
\]

(entirely spatial)

\[
\begin{array}{c}
\frac{1/2}{5} \\
\frac{10}{3}
\end{array}
\]

(entirely linear)

\[
\begin{array}{c}
\frac{1/2}{5} \\
\frac{10}{3}
\end{array}
\]

16.9 Higher Orders of Complexity

Hypercomplex fractions of higher order may be transcribed in the manner described above, with dot 6 added the appropriate number of times before the fraction indicators and the matching fraction lines. No examples are shown.

Instructions: Transcribe this hypercomplex fraction using the combined method—that is, transcribe each complex fraction as a linear arrangement within the overall spatial arrangement. Numerators and denominators can be determined by noting the length of each fraction line in print.

PRACTICE 16E

\[
\begin{array}{c}
\frac{r^2 - 4s^2}{s^2} \\
\frac{r + 2s}{s} \\
\frac{4r - 2s^2}{3s} \\
\frac{2s^2 - 3r}{4r}
\end{array}
\]

1. Solve for \( r \) in terms of \( s \).
CONTINUED FRACTIONS

[NC 13.9]

16.10 Definition and Recognition

In essence, a continued fraction is a fraction within a fraction. Each denominator is the sum of a whole number and a fraction. The digit in the denominator of one fraction becomes the digit in the numerator of the next fraction. You may find them in the study of irrational numbers such as pi and certain square roots.

The continued fraction shown below can be described as follows. The fraction begins with \( 1 - \frac{7}{3} \). The first denominator is \( 3 + \frac{3}{2} \). The next denominator is \( 2 + \frac{2}{2} \). The final denominator is \( 2 - \frac{2}{3} \).

\[
n = 1 - \frac{7}{3 + \frac{3}{2 + \frac{2}{2 - \frac{2}{3}}}}
\]

16.10.1 Transcription Rules. A continued fraction is transcribed as a spatial arrangement. The layout essentially mimics the print arrangement. Each fraction line begins in the cell after the plus or minus sign (unspaced). Opening and closing fraction indicators are not used. All fraction lines end in the same cell, aligned with the rightmost symbol in the arrangement.

Each new denominator's whole number is aligned directly below the first cell of the separation line above it. Its numerator is centered with respect to the fraction line below it. If that number cannot be exactly centered, it is placed one cell to the left of center.

The rules will make sense when you study the example below, which is the transcription of the continued fraction shown above.
Example 16-19

\[ n = 1 - \frac{7}{3 + \frac{3}{2 + \frac{2}{2 - \frac{2}{3}}} \right] \]

Line 1: The opening Nemeth Code indicator is transcribed on the line preceding the required blank line.
Line 3: The numerator is centered over the fraction line below it.
Line 4: The length of this fraction line extends to the rightmost character in the arrangement, which is on lines 9 and 11.
Line 5: This numerator "3" is centered over the fraction line below it. Likewise for the numerators on lines 7 and 9.
Line 6: The numeric indicator is directly aligned with the first cell of the fraction line two lines above. Likewise for lines 8 and 10.
Line 13: The Nemeth Code terminator is in cell 1 on the line following the required blank line.
The square root of 3 can be expressed as a continued fraction.

\[ \sqrt{3} = 1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cdots}}} \]
16.11 Format for Instructional Commentary

When instructional commentary appears alternated with math problems, place the comment on the line following the related expression, blocked 4 cells to the right of the runover position for the expression. Explain this format in a transcriber's note.

Sample transcriber's note: "Comments printed beside related math problems are placed on the line following the expression, blocked four cells to the right of the runover position of the expression."

When switching into or out of Nemeth before a change of margins, place the switch indicators after the last item of the line rather than at the beginning of the next line. This maintains clarity in the indented margin pattern. The single-word switch indicator is used, as needed.

Comments are often printed in a different color or typeface. (In the examples in this section, instructional commentary is printed in italics.) The variant typeform is disregarded in the braille transcription.

Notice in the examples how the comments are clearly set off by indentation.

Example 16-20

If $y$ varies inversely as $x$, and $y = 3$ when $x = 4$, find $y$ when $x = 18$.

\[
\frac{x_1}{y_2} = \frac{x_2}{y_1} \quad \text{Given.}
\]

\[
4 = \frac{18}{y_2} \quad \text{Substitute the known values.}
\]

\[
18y_2 = 12 \quad \text{Now cross multiply.}
\]

\[
y_2 = \frac{12}{18} = \frac{2}{3} \quad \text{Divide each side by 18 and simplify.}
\]

The value of $y$ when $x = 18$ is $\frac{2}{3}$. 

Lines 1-3: The narrative begins in cell 3, with runovers in cell 1.
Lines 2-3: This phrase can also be transcribed as follows:

Lines 4, 6, 8, 10: Each displayed math expression begins in cell 3.
Line 5: The comment begins in cell 9, which is four cells to the right of the runover cell of the displayed material even though there is no runover present. A single-word switch is used for this comment. The italic typeform is disregarded.
Lines 6, 8, 10: A Nemeth Code terminator ends each line, preparing the reader for the following comment.
Lines 7, 9, 11: Each comment begins in cell 9.
Line 12: The runover is blocked (cell 9).
Lines 7 and 9: The opening Nemeth Code indicator is placed at the end of the line of text preceding the Nemeth material to assure that all of the displayed math expressions begin in the same cell.
Lines 13-14: The narrative begins in cell 3, with its runover in cell 1. No blank line is needed.

16.11.1 Alignment of Equals Signs. If the print copy aligns equals signs of separate displayed equations, the layout is disregarded in the braille transcription. This assures that each step begins in the same cell, as the first example below illustrates. On the other hand, if the print layout meets the definition of a nested list expression, braille format for nested list expressions is applied. (Lesson 8) The second example below illustrates this layout.
Example 16-21

We write the equation in slope-intercept form by solving for \( y \).

\[-5x - 2y = 6\]

\[-2y = 5x + 6 \quad \text{Add 5x to both sides.}\]

\[-2y = \frac{5x+6}{-2} \quad \text{Divide both sides by } -2.\]

\[y = \frac{5x}{-2} + \frac{6}{-2} \quad \text{Divide each term by } -2 \text{ and simplify.}\]

\[y = -\frac{5}{2}x - 3 \quad \text{Slope-intercept form, } y = mx + b.\]

Overall: Although the equals signs are aligned in print, these are five separate equations, not a nested linked expression. Alignment of the equals signs is disregarded. Each expression will start in the same cell (cell 3, in this case—displayed to a narrative paragraph). Study the placement of the code switch indicators to see how they do not interfere with the starting cell of each displayed expression.

Lines 1-2: Narrative paragraph in (3-1).
Lines 3, 4, 6, 8, 11: Each displayed equation is placed in cell 3. There are no runovers, but if there were, the runover would start in cell 5.
Lines 5, 7, 9, 12: Each comment begins in cell 9, which is four cells to the right of the runover position of the previous line, even though no runovers are present.
Lines 10 and 13: Runovers to comments are blocked (cell 9, in this case).
Example 16-22

1. Reduce \( \frac{330}{4950} \) to lowest terms using the "prime factors" method.

\[
\frac{330}{4950} = \frac{2 \times 3 \times 5 \times 11}{2 \times 3 \times 3 \times 5 \times 5 \times 11}
\]

Express numerator and denominator as prime factors.

\[
= \frac{\cancel{2} \times \cancel{3} \times \cancel{5} \times \cancel{11}}{\cancel{2} \times \cancel{3} \times \cancel{3} \times \cancel{5} \times \cancel{5} \times \cancel{11}}
\]

Cancel common factors 2, 3, 5, and 11.

\[
= \frac{1}{3 \times 5}
\]

\[
= \frac{1}{15}
\]

Overall: The displayed math is a nested linked expression. The pattern for this nested linked expression is 5-9, 7-9 (displayed to 1-3 itemized text). The comments are blocked in cell 13. Study the placement of the code switch indicators to see how they do not interfere with the starting cell of each pattern (anchor in cell 5, links in cell 7, comments in cell 13).
PRACTICE 16G

What percent of 48 is 54?

\[ n \cdot 48 = 54 \quad \text{Write an equation.} \]

\[ \frac{48n}{48} = \frac{54}{48} \quad \text{Divide each side by 48.} \]

\[ n = 1.125 \quad \text{Simplify.} \]

\[ = 112.5\% \quad \text{Change the decimal to a percent.} \]

Answer: 54 is 112.5% of 48.
STEM-AND-LEAF PLOTS

16.12 Recognition

A stem-and-leaf plot is a method of showing data distribution in columns and rows. A vertical line partitions the information into data on the left, called the stem, and data on the right, called the leaf. Stem and leaf data may consist of numbers, letters, and/or blank entries and may be spaced or unspaced in the print copy. Stem data are right justified to the vertical line; leaf data are left justified to the vertical line. The columns may include a heading. Here is an example.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 4</td>
</tr>
<tr>
<td>9</td>
<td>3 4 5 7 8 9</td>
</tr>
<tr>
<td>8</td>
<td>0 2 2 9</td>
</tr>
<tr>
<td>7</td>
<td>5 8 9</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

A distinctive feature of a stem-and-leaf plot is the vertical line separating the stems from the leaves.

16.13 The Table

A stem-and-leaf plot is transcribed in Nemeth notation using the guidelines for tables outlined in *Braille Formats*. The entire table is transcribed in Nemeth, including the column headings (if present), even if the numbers or letters are unmodified. Column headings begin in cell 1 and no contractions are used. Column separation lines follow on the next line below the headings, covering the same number of cells as the widest data entry in each column. Column headings are not transcribed if they do not appear in print.

The vertical line between the stem column and the leaf column is transcribed as dots 456, preceded and followed by one blank cell. This symbol is transcribed in every row, including between column headings and between column separation lines.

<table>
<thead>
<tr>
<th>Vertical Line (preceded and followed by a space)</th>
</tr>
</thead>
</table>

The following Nemeth rules apply to this example.

- The numeric indicator is omitted in the body of the plot.
- Spaces are not inserted between single-digit entries.
- The data are justified to the vertical line as shown in print.
### Example 16-23

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 4</td>
</tr>
<tr>
<td>9</td>
<td>3 4 5 7 8 9</td>
</tr>
<tr>
<td>8</td>
<td>0 2 2 9</td>
</tr>
<tr>
<td>7</td>
<td>5 8 9</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

_The table is preceded and followed by a blank line. Code switch indicators do not replace required blank lines._

#### 16.14 The Key

A key is often provided in the print copy and may be located above, below, next to, or within the table. The key is transcribed first, regardless of its placement in the print copy. The key should be placed on the same braille page as the table whenever possible.

Numeric indicators are not used for the portion of the key that replicates a numeric entry in the plot. (In this illustration, 10|0 is the portion of the key replicating the stem "10" and the leaf "0".) The value assigned to the key is transcribed using the numeric indicator as required by the Nemeth Code. (In this illustration, a numeric indicator is used for the value "100".)

\[
\Rightarrow 10 \mid 0 = 100
\]

If the key is printed in a variant typeform, the typeform is disregarded in braille. A key is not constructed by the transcriber if one does not appear in print.
The following Nemeth rules apply to this example.

- The key is transcribed first, beginning in cell 1.
- The numeric indicator is omitted in the body of the plot.
- Spaces are not inserted between single-digit entries.

Example 16-24

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0 4</td>
</tr>
<tr>
<td>9</td>
<td>3 4 5 7 8 9</td>
</tr>
<tr>
<td>8</td>
<td>0 2 2 9</td>
</tr>
<tr>
<td>7</td>
<td>5 8 9</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

10 | 0 = 100

16.14.1 Runovers in the Key. If the key requires a runover line, the runover begins in cell 3. When there are words in the key, they may be transcribed in UEB, switching to Nemeth for the portion that reflects the stem and leaf.

The key in the next example includes a descriptive label ("Key:"). The label is transcribed only if it appears in print. If the key is set off by shading or in a box, box lines are not transcribed.
16.15 Data Consisting of More Than One Character; Punctuation Between Entries

The next example shows leaf entries with a decimal point and more than one digit. It also shows commas between leaf entries. The key is printed within the body of the table. The following rules are applied:

- The key is transcribed first.
- The numeric indicator is omitted in the body of the plot.
- Data consisting of groups of two or more digits require one blank cell between entries.
- Punctuation printed between units of data is omitted in the transcription.

Example 16-26

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8.3</td>
</tr>
<tr>
<td>6</td>
<td>4.3, 5.1, 5.5, 6.7, 7.0, 8.7, 9.3</td>
</tr>
<tr>
<td>7</td>
<td>0.0, 2.8, 3.2, 7.4, 7.4</td>
</tr>
</tbody>
</table>

Key: 2 | 2 represents 2,150,000 to 2,249,999 copies sold.
16.16 **Alphabetic Data**

For alphabetic data, the following rules apply.

- The English-letter indicator is omitted in the body of the plot.
- Spaces are not inserted between single-letter data entries.
- Data consisting of groups of two or more letters require one blank cell between entries.

The leaves in the next example consist of pairs of lowercase letters. There is no key.

**Example 16-27**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>aa ab ac ba bd cc cd ef en er</td>
</tr>
<tr>
<td>42</td>
<td>bd bd dc</td>
</tr>
</tbody>
</table>

Reminder: *The numeric indicator is omitted in the body of the plot (Stem column).*

16.16.1 **Alphabetic Key.** In an alphabetic key, the English-letter indicator is not used for the portion of the key that replicates an entry in the plot. The value assigned to the key is transcribed using the English-letter indicator as required by the Nemeth Code.

\[
\begin{align*}
\rightarrow & \quad c \mid d = cd \\
\end{align*}
\]

16.17 **Blank Entries**

A blank entry in a stem-and-leaf plot is shown as blank space in braille. Do not transcribe a general omission symbol. Do not fill the width of the column with dot 5s.

The following rules also apply to the next example.

- The numeric indicator and the English-letter indicator are omitted in the body of the plot.
- Spaces are not inserted between single-letter data entries.
- Capital letters are capitalized individually.
This plot has no key and no column headings—none are added in braille. The blank space in the second row indicates a blank entry in the leaf column. Single letters are unspaced and are capitalized individually.
16.18 Runovers Within the Table

16.18.1 Print Runovers. A runover of leaves shown in the print copy should be ignored if all leaves will fit on one row in braille. Use the full width of the available cells in the row.

Example 16-29

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 2 2 2 3 4 4 5 5 5 6 6 7</td>
</tr>
<tr>
<td></td>
<td>7 7 7 8 8 9 9 9 9</td>
</tr>
<tr>
<td>2</td>
<td>0 1 1 1 2 6 6 8</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

The first row of leaf data requires two lines in print, but not in braille.
16.18.2 **Braille Runovers.** If leaf data will not fit on one line in the braille transcription, indent the runover line two cells to the right after using as much of the braille line as possible. Do not repeat the stem row heading. The next example illustrates, expanding the two-letter data example shown previously.

**Example 16-30**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>aa ab ab ac ba ba bd cc cd cd ef en er er</td>
</tr>
<tr>
<td>42</td>
<td>bd bd dc</td>
</tr>
</tbody>
</table>

The vertical line is transcribed on every line in the table.

16.19 **Back-To-Back Plot**

A back-to-back stem-and-leaf plot is used when two sets of data are being compared. There are three columns. The stem is the middle column. Data is read from the stem outward, which means that data in the left leaf is read from right to left. There will be two keys. Here are the rules as they apply to the next example.

- **The Keys:** The left column's key is transcribed first, followed on the next line by the right column's key.
- **The Columns:** Column separation lines are transcribed below only the lines that contain column headings.
- **The Runovers:** Right leaf data are read from left to right; runovers to the right leaf data column are shifted two cells to the right. Left leaf data are read from right to left; runovers to the left leaf data column are shifted two cells to the left.
Example 16-31

Ms. Abel's Test Scores

<table>
<thead>
<tr>
<th>Second Grade Classes</th>
<th>Fifth Grade Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4220</td>
<td>6 2469</td>
</tr>
<tr>
<td>987776655521</td>
<td>7 111223334556667899900</td>
</tr>
<tr>
<td>999998888776655444332110</td>
<td>8 1222444555789</td>
</tr>
<tr>
<td>98877753320</td>
<td>9 223577780</td>
</tr>
<tr>
<td>10</td>
<td>00</td>
</tr>
</tbody>
</table>

0 | 6 | denotes a score of 60     | 6 | 2 denotes a score of 62

Line 3: Key to left column. Italic typeform is disregarded.
Line 4: Key to right column. Italic typeform is disregarded.
Line 5: A blank line precedes the plot.
Lines 6-7: Column headings are uncontracted in a stem-and-leaf plot. These column headings require two lines. Runovers to column headings are blocked.
Line 8: The stem column (middle column) has no heading, so no column separation line is transcribed.
Line 11: Reading left to right, runovers to the right leaf data column are shifted two cells to the right.
PRACTICE 16H

A pet store owner constructed the following stem-and-leaf plot showing the number of guinea pigs at each of her seventeen stores.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7 8</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 6 8 8 8</td>
</tr>
<tr>
<td>3</td>
<td>0 2 6 6 7 8</td>
</tr>
<tr>
<td>4</td>
<td>1 2 6 6</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Key: 2 | 0 represents 20 guinea pigs

How many stores have fewer than 36 guinea pigs?

For further practice, see Appendix A—Reading Practice.

EXERCISE 16

Prepare Exercise 16 for your grader.
ANSWERS TO PRACTICE MATERIAL

PRACTICE 16A

1. Immy invites six friends to a pizza party.
2. Two pizzas are ordered: one pizza is 12 slices.
3. Slices are divided among 6 children.
4. Each slice is divided equally.

10. Number of pizzas
11. Number of children
12. Number of slices
13. Number of children
14. Number of slices
15. Number of children

Lines 18-19: Recall from Lesson 9 that, following a spatial arrangement, the required blank line comes first, followed by the Nemeth Code terminator on the next line in cell 1.
Lines 1-3: Nemeth "Instructions" format is applied. Margins are 5-3 and the bold typeform is disregarded.

Line 4: A blank line must precede the spatial material which follows.
PRACTICE 16C

1. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

2. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

3. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

4. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

5. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

6. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

7. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

8. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

9. \[ \frac{\sqrt{B}}{\sqrt{2B}-A} \]

PRACTICE 16D

1. The lm meters are 3281.09 yds. This

2. multiply yd by scale expressions

3. was y cuem units a measure, a solution is

4. find the answer, is the lm yd...

5. in ...

6. ...

7. ...

8. ...

9. ...

10. ...

11. ...

12. ...

Line 5: Excessive code switching is avoided by staying in Nemeth Code to transcribe "100 yd."

Lines 6-12: This is a nested list format because the equals signs are aligned in the print copy.

Runovers to the first link are in cell 7 (lines 8-10).
PRACTICE 16E

1. \( \sqrt{3} \cdot \cos \theta + \sin \theta + \tan \theta \)
2. \( \sqrt{4} \cdot \sin \theta + \cos \theta + \tan \theta \)
3. \( \sqrt{5} \cdot \cos \theta + \sin \theta + \tan \theta \)
4. \( \sqrt{6} \cdot \sin \theta + \cos \theta + \tan \theta \)
5. \( \sqrt{7} \cdot \cos \theta + \sin \theta + \tan \theta \)
6. \( \sqrt{8} \cdot \sin \theta + \cos \theta + \tan \theta \)

PRACTICE 16F

1. \( \sqrt{3} \cdot \sqrt{2} \cdot \sqrt{4} \cdot \sqrt{5} \cdot \sqrt{6} \cdot \sqrt{7} \cdot \sqrt{8} \)
2. \( \sqrt{9} \cdot \sqrt{10} \cdot \sqrt{11} \cdot \sqrt{12} \cdot \sqrt{13} \cdot \sqrt{14} \cdot \sqrt{15} \)
3. \( \sqrt{16} \cdot \sqrt{17} \cdot \sqrt{18} \cdot \sqrt{19} \cdot \sqrt{20} \cdot \sqrt{21} \cdot \sqrt{22} \)
4. \( \sqrt{23} \cdot \sqrt{24} \cdot \sqrt{25} \cdot \sqrt{26} \cdot \sqrt{27} \cdot \sqrt{28} \cdot \sqrt{29} \)
5. \( \sqrt{30} \cdot \sqrt{31} \cdot \sqrt{32} \cdot \sqrt{33} \cdot \sqrt{34} \cdot \sqrt{35} \cdot \sqrt{36} \)
6. \( \sqrt{37} \cdot \sqrt{38} \cdot \sqrt{39} \cdot \sqrt{40} \cdot \sqrt{41} \cdot \sqrt{42} \cdot \sqrt{43} \)
7. \( \sqrt{44} \cdot \sqrt{45} \cdot \sqrt{46} \cdot \sqrt{47} \cdot \sqrt{48} \cdot \sqrt{49} \cdot \sqrt{50} \)
8. \( \sqrt{51} \cdot \sqrt{52} \cdot \sqrt{53} \cdot \sqrt{54} \cdot \sqrt{55} \cdot \sqrt{56} \cdot \sqrt{57} \)
9. \( \sqrt{58} \cdot \sqrt{59} \cdot \sqrt{60} \cdot \sqrt{61} \cdot \sqrt{62} \cdot \sqrt{63} \cdot \sqrt{64} \)
10. \( \sqrt{65} \cdot \sqrt{66} \cdot \sqrt{67} \cdot \sqrt{68} \cdot \sqrt{69} \cdot \sqrt{70} \cdot \sqrt{71} \)
11. \( \sqrt{72} \cdot \sqrt{73} \cdot \sqrt{74} \cdot \sqrt{75} \cdot \sqrt{76} \cdot \sqrt{77} \cdot \sqrt{78} \)
12. \( \sqrt{79} \cdot \sqrt{80} \cdot \sqrt{81} \cdot \sqrt{82} \cdot \sqrt{83} \cdot \sqrt{84} \cdot \sqrt{85} \)
PRACTICE 16G

1. Write an equation.
2. Divide each side by \( D H \).
4. Change a decimal to a fraction.
5. Answer: \( \frac{1}{14} \) is \( \frac{1}{1} \) at \( D H \).

Line 1: The narrative paragraph begins in cell 3. There is no runover.
Line 2, 4, 6: Each displayed expression begins in cell 3.
Lines 2 and 4: Alignment of the equals signs in print is disregarded.
Lines 3, and 5: Each comment is blocked in cell 9, which is four cells to the right of the runover cell (cell 5). The italic typeform is disregarded.
Lines 6-9: This is a nested linked expression. Each link begins in the runover cell (cell 5) according to the rules for nested linked expressions.
Lines 8 and 10: Each comment is blocked in cell 11, which is four cells to the right of the runover cell in this nested list format (cell 7).
Line 8: The single-word switch indicator is used even though there are no contractions in the word. Nemeth continues on the next line.
Line 11: The narrative paragraph continues from line 1, in the runover cell (cell 1).
PRACTICE 16H

1. A pet store owner is tracking the sale of guinea pigs. She collects
2. stem-and-leaf plot data to summarize the number of guinea pigs
3. sold at each location. The data is as follows:

4. Key: LM S L = LE represents 27 guinea pigs
5. pigs.

6. **STEM** | **LEAF**
7. | ---- | ------
8. | 6 | 7
9. | 7 | 8
10. | 8 | 9
11. | 9 | 0
12. | 0 | 1
13. | 1 | 2
14. | 2 | 3
15. | 3 | 4
16. | 4 | 5
17. | 5 | 6
18. | 6 | 7
19. | 7 | 8

20. This stem-and-leaf plot summarizes the sales data for the guinea pigs.