LESSON 12

- MODIFIERS AND MODIFIED EXPRESSIONS
  - Common Modifiers
  - Binomial Coefficient
  - Modified Expressions and Superscripts / Subscripts
  - Modified Signs of Comparison
  - Expressions with More Than One Modifier

Format

- Formal Proof

Postscript: Vector Notation Notes

Answers to Practice Material

LESSON PREVIEW

Another type of mathematical notation with vertical components is studied. This lesson also applies the "mathematical statement" format to formal proofs.

Some of the print examples are enlarged in order to more clearly show the modifiers on the printed page. You may also wish to use a magnifier to remove any uncertainty.

MODIFIERS AND MODIFIED EXPRESSIONS

[NC Rule 15]

Some mathematical notation has a vertical aspect that challenges linear braille notation. By using modified expression indicators, the transcriber can relay the material in a compact manner, and the reader can construct the symbols into a meaningful expression.

12.1 Definition

A <u>modifier</u> is a symbol or a combination of symbols occurring *directly over* or *directly under* its related symbol or expression. Here are some typical examples.

$\overset{?}{=}$ a question mark over an equals sign

$3.1\underline{5}$ an underlined digit

$1.\overline{37}$ a line over two numerals (signifying a repeating decimal)

$\overleftrightarrow{AB}$ an arrow over two letters (signifying a line)

$\hat{k}$ a caret over a letter

12.2 Construction of Simple Modified Expressions – The Five-Step Rule

An expression modified using the five-step method is initiated and terminated with special indicators.

⠸	Multipurpose Indicator
⠸⠸	Termination Indicator

The position of the modifier (above or below the expression) is also shown with the use of an indicator.

⠸⠁	Directly-Over Indicator
⠸⠂	Directly-Under Indicator

The process of constructing a modified expression is known as "The Five-Step Rule".

(1) The *multipurpose indicator* is placed immediately before the expression to be modified.

(2) The *expression* to be modified is written second.

(3) The *directly-over indicator* or the *directly-under indicator* is written third to show the position of the modifier.

(4) The *modifying symbol* is written fourth.

(5) The *termination indicator* is written last to show the end of the modification.

Notice that the multipurpose indicator, dot 5, signals the beginning of the modified expression and the termination indicator signals the completion of the modified expression.

Prototype for an expression with a modifier printed directly above it: ⠨⠤⠤⠤⠨⠤⠤⠤⠨

Prototype for an expression with a modifier printed directly below it: ⠨⠤⠤⠤⠨⠤⠤⠤⠨

All components of an expression modified according to the five-step rule should not be divided between lines. If that is not possible, strategies will be presented in Lesson 15.

Common Modifiers

The most commonly used modifiers are presented in this lesson. You have seen many of these symbols in other contexts in previous lessons. Symbols of the code not shown here may also be used as modifiers.

12.3 Arrows as Modifiers

Any of the arrows of the Nemeth Code may be a modifier. Those used in this section are shown below.

⠨⠨	Arrow barbed at right, contracted form	→
⠨⠨⠐⠐⠐⠨	Arrow barbed at both ends	↔
⠨⠐⠐⠐⠨⠨	Arrow shaft with hollow dot at right	⊸
⠨⠐⠐⠐⠨⠐	Upper barb only, right pointing	⇀
⠨⠐ ⠐⠨	Arrow barbed at right, dashed shaft	⇢

12.3.1 **Right-Pointing Arrow.** A right-pointing arrow in regular type with a full barb and single shaft of ordinary length is transcribed in its contracted form when used as a modifier above or below a math expression. The shaft length in print is determined by the width of the expression it modifies. If the modified expression is wide, the arrow shaft will be long in print, but this is not considered to be a "longer than ordinary" arrow shaft in this context.

⫸ $\overrightarrow{YZ}$

YZ is the expression being modified. It is modified directly over with a right-pointing arrow.

⫸ $\underset{\rightarrow}{T}$

T is the expression being modified. It is modified directly under with a right-pointing arrow.

⫸ $\overrightarrow{M'N'}$

The contracted form of the right-pointing arrow is used. The arrow shaft covers a wide expression and so is longer in print.

Show exactly what portion of an expression is being modified. In the next example, only the expressions between the grouping symbols are modified. The grouping symbols are placed outside of the multipurpose and termination indicators.

⫸ $\left|\overrightarrow{O_1P}\right| + \left|\overrightarrow{P_1P_2}\right|$

O_1P and P_1P_2 are the expressions being modified. Each is modified directly over with a right-pointing arrow.

12.3.2 **Other Arrows.** All other types of arrows require the appropriate arrow symbol.

⫸ $\overleftrightarrow{AB}$

AB is the expression being modified. It is modified directly over with an arrow barbed at both ends.

⫸ $\overrightarrow{AB}^{\circ}$

AB is the expression being modified. It is modified directly over with an arrow with a hollow dot at its right end, no barb.

⫸ $\overrightarrow{\mathbf{OT}}$

OT (boldface) is the expression being modified. It is modified directly over with an arrow with an upper right-pointing barb.

⋙ $\overset{-->}{\text{OB}}$

OB is the expression being modified. It is modified directly over with a right-pointing arrow with a dashed shaft.

Example 12-1

$\overleftrightarrow{\text{PQ}}$ is a line through points P and Q.

Example 12-2

We now conclude that $\overrightarrow{\text{OP}} = \overrightarrow{\text{OT}} + \overrightarrow{\text{TP}}$

Reminder from Lesson 1: If a math expression will fit on one line but there is not room for one or both of the switch indicators, one or both switch indicators may stand alone on a line. Keeping the mathematical expression intact on one line is the priority.

12.3.3 **Arrows Used in Vector Notation.** When identical arrows are used above vectors in boldface type throughout the text, they are omitted from the braille transcription. The boldface font attribute of the vectors is considered to be sufficient identification. A transcriber's note is required to explain the presence of the arrows in print. Sample transcriber's note: "The right-pointing arrows printed over boldface vectors are omitted."

Example 12-3

Vector Addition. By adding vectors $\overrightarrow{\mathbf{CD}}$ and $\overrightarrow{\mathbf{EF}}$, the resultant vector, **r**, is found.

$$\overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{EF}} = \mathbf{r}$$

The arrows printed above CD and EF are not transcribed. The bold typeform is retained. (Assume the required transcriber's note appears on the Transcriber's Notes page.)

See the Postscript at the end of this lesson for notes regarding other types of vector notation.

12.3.4 **When the Arrow Itself is Being Modified.** A right-pointing arrow in regular type with a full barb and single shaft of ordinary length is transcribed in its uncontracted form when the arrow itself is being modified.

⠸⠸⠸⠸ Arrow barbed at right, uncontracted form →

$x \xrightarrow{f \circ g} y$ ⠸⠸ ⠸⠸⠸⠸⠸⠸⠸⠸⠸⠸⠸⠸ ⠸⠸

A right-pointing arrow is the expression being modified. It is modified directly over with the operation $f \circ g$ Reminder: In function notation, $\circ$ is a hollow dot, not the letter "o".

12.3.5 **When Other Rules Apply.** Some arrangements that may appear to be modified arrows are not transcribed using the Five-Step Rule.

a. **Signs of Comparison Compounded Vertically.** Horizontal arrows printed one below the other are a sign of comparison compounded vertically. Review this topic in Lesson 9.

b. **Chemistry.** Words or symbols printed above reaction arrows are not treated as modifiers. Rules for these constructions are found in *Chemical Notation Using the Nemeth Braille Code.*

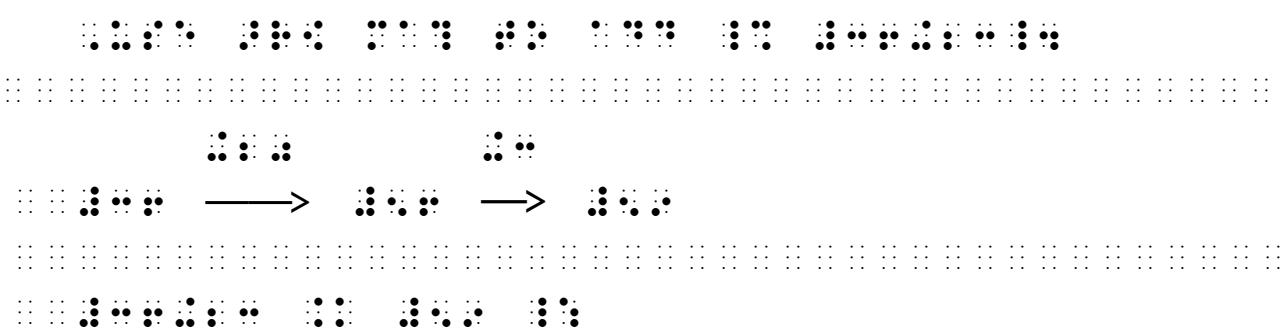
$$\mathrm{Na_2SO_4} \xrightarrow{\mathrm{H_2O}} \mathrm{Na^+} + \mathrm{SO_4^{2-}}$$

c. **Elementary Mathematics—"ARROW MATH" in Lower Grades.** These arrows are not "modified arrows". They are not being used as a sign of comparison, but rather as a "process". Draw these arrows as a tactile graphic. Refer to BANA's *Guidelines and Standards for Tactile Graphics* for techniques.

Example 12-4

Use arrow math to add 36 + 23.

$$36 \xrightarrow{+20} 56 \xrightarrow{+3} 59 \qquad 36 + 23 = 59$$



Instructions: Review arrow construction in Lesson 9. *Reminder:* In braille, identifiers for nonspatial itemized material must all begin on a new line in cell 1.

In the sentence at the end of this practice, assume that all vectors in the document are shown using that particular arrow notation. Show the proper way to omit the vector arrows in the transcription. Include the required transcriber's note after the topic heading.

PRACTICE 12A

Here are two modified arrows: $x \overset{g}{\rightarrow} y \overset{f}{\rightarrow} z$

Arrows as Modifiers

1. $\overleftarrow{\mathrm{F}}$ 2. $\overleftarrow{\mathrm{AB}}$ 3. $\overset{\leftarrowtail}{\mathrm{CD}}$ 4. $\overset{\mapsto}{\mathrm{OB}} \cup \overset{\mapsto}{\mathrm{OC}}$ 5. $\overset{\circ\!-\!\bullet}{\mathrm{EF}}$

6. $\underset{\leftarrow}{\mathrm{T}}$ 7. $\overleftrightarrow{\mathrm{XZ}} \parallel \overleftrightarrow{\mathrm{RS}}$ 8. $\overleftrightarrow{\mathrm{AB}} + \overleftrightarrow{\mathrm{CD}}$

Vector Addition

$\mathbb{R}$ equals $\overrightarrow{\mathbf{OP}}$ equals $\overrightarrow{\mathbf{OM}}$ plus $\overrightarrow{\mathbf{MC}}$ plus $\overrightarrow{\mathbf{CP}}$.

12.4 Carets as Modifiers

⠨⠁	Caret	∧
⠨⠬	Inverted Caret	∨

Note that the symbol for the inverted caret is the same construction as the opening Nemeth Code indicator. Context will make its meaning clear.

⫸ $\hat{\boldsymbol{k}}$ ⠨⠁⠘⠅

Bold letter k is modified directly over with a caret.

⫸ $\underset{\vee}{x}$ ⠨⠬⠭

Letter x is modified directly under with an inverted caret.

Example 12-5

All values of y are located on the regression line $\hat{y} = \alpha + \beta x$.

⠠⠽ ⠠⠗⠑⠛⠗⠑⠎⠎⠊⠕⠝ ⠠⠍⠊⠝⠑ ⠊⠎ ⠠⠁⠍⠕⠝⠛ ⠠⠁⠍⠕⠝⠛ ⠠⠁⠍⠕⠝⠛ ⠠⠁
⠠⠍⠕⠙⠑⠗⠝ ⠠⠁⠍⠕⠝⠛ ⠠⠁⠍⠕⠝⠛ ⠠⠁⠍⠕⠝⠛ ⠠⠁⠍⠕⠝⠛ ⠠⠁⠍⠕⠝⠛ ⠠⠁⠍⠕⠝⠛

12.5 Horizontal Bar as a Modifier

Recall that underlining is not considered to be a typeform in the Nemeth Code. When a number or letter with mathematical meaning is underlined, a switch to Nemeth is required and the methods discussed in this section are applied.

12.5.1 **Bar Over or Under More than One Digit or Letter.** When more than one digit or letter is modified by a single horizontal bar, the five-step rule is applied. Only one bar symbol is transcribed.

⠨	Horizontal Bar	—

⫸ $\overline{99}$ ⠨⠔⠘⠔

The 2-digit number 99 is modified directly over with a horizontal bar. Note that a numeric indicator is not needed because the number is not preceded by a space.

⫸ $\underline{419}$ ⠨⠔⠘⠁⠊⠓

The 3-digit number 419 is modified directly under with a horizontal bar.

⋙ $\$11.\underline{25}$

The 2-digit number 25 is modified directly under with a horizontal bar. Note that the decimal point is not part of this modified expression. The bar is under the digits only.

⋙ $1.\overline{37}$

The 2-digit number 37 is modified directly over with a horizontal bar.

Example 12-6

Prove that $\overline{OP} + \overline{QR} = \overline{OR}$.

Example 12-7

The fraction $\frac{3}{7}$ expressed as a decimal is .428571428571428571 … . Written as a repeating decimal, $\frac{3}{7} = .\overline{428571}$.

The decimal point is not part of the modified expression. The multipurpose indicator is transcribed in the cell immediately preceding the numeral 4.

12.5.2 **Bar Over or Bar Under Only One Digit or Letter.** When a single digit or a single letter in any typeform or any alphabet is modified only by a single horizontal bar either directly over or directly under it, the five-step rule is not applied. Instead, a contracted form is used.

a. **Bar Above a Single Digit or Letter.** When a single digit or a single letter (in any typeform or any alphabet) is modified only by a single horizontal bar directly over it, the symbol for the bar is placed immediately after the digit or letter modified.

⋙ $\overline{x}$

⋙ $\overline{3}$

⫸ $\overline{\mathbf{C}}'$ ⠼⠉⠄⠄

⫸ $\overline{\lambda}_t$ ⠼⠇⠄⠞

Example 12-8

The $\overline{7}$'s are repeated.

⠼⠛ ⠼⠛ ⠼⠛⠄⠄⠄ ⠼⠛ ⠼⠛ ⠼⠛⠄⠄⠄⠄⠄

The apostrophe-s relates to the numeral, therefore it is transcribed inside the switches. The punctuation indicator prevents the apostrophe from being misread as a prime sign.

Example 12-9

Arabic	Roman
5,000	$\overline{\text{V}}$
10,000	$\overline{\text{X}}$
50,000	$\overline{\text{L}}$
100,000	$\overline{\text{C}}$

⠼⠑⠚⠚⠚ ⠼⠩⠩⠩⠩⠩
⠼⠑⠚⠚⠚⠚⠚⠚⠚ ⠼⠩⠩⠩⠩⠩
⠼⠛

⠼⠁⠚⠚⠚⠚⠚ ⠼⠩⠩⠩
⠼⠃⠚⠚⠚⠚⠚ ⠼⠩⠩⠩
⠼⠉⠚⠚⠚⠚⠚ ⠼⠩⠩⠩
⠼⠙⠚⠚⠚⠚⠚ ⠼⠩⠩⠩
⠼⠋

The Arabic numbers in the first column can be transcribed in either UEB or Nemeth. The Roman numerals in column two require a code switch because they are modified. Smoother reading is accomplished by transcribing both columns in Nemeth. Review simple table format in Lesson 5.

b. **Bar Below a Single Digit or Letter.** When a single digit or a single letter (in any typeform or any alphabet) is modified only by a single horizontal bar directly under it, the directly-under indicator followed by the symbol for the bar is placed immediately after the digit or letter modified.

⠨⠨	Directly-Under Indicator followed by Horizontal Bar

⫸ $\underline{x}$ $\underline{\mathrm{R}}^{+}$ ⠨⠨⠭ ⠸⠗⠬⠖⠐⠔

⫸ $\underline{3}$ $5\underline{7}9$ ⠨⠨⠼ ⠼⠑⠛⠔

Study the examples below. Notice when the contracted form of the bar can be used. The simulated braille is in Nemeth. Code switch indicators are omitted.

Example 12-10

(1) $(\overline{x}, \overline{y})$ ⠨⠨⠷⠭⠄ ⠨⠽⠾

(2) $\overline{x}$'s and $\overline{y}$'s ⠨⠨⠭⠄⠎ ⠯⠝⠙ ⠨⠽⠄⠎

(3) $\overline{x+y}$ ⠨⠨⠭⠬⠽⠨⠨

(4) $\underline{x+y}$ ⠨⠨⠭⠬⠽⠨⠨

(5) $A(\underline{s}n)$ ⠨⠁⠷⠎⠝⠨⠾

(6) $\dfrac{\overline{\mathrm{PR}}}{\overline{\mathrm{OR}}}$ ⠨⠨⠏⠗⠄⠄⠸⠌⠨⠨⠕⠗⠄⠄

(7) $P(\overline{x}) = \overline{P(x)}$ ⠨⠨⠏⠷⠭⠄⠾ ⠶ ⠨⠨⠏⠷⠭⠨⠨

Instructions: Treat the three examples of unit vectors in problem #2 as displayed mathematical material.

PRACTICE 12B

Carets and Bars as Modifiers

1. Unit vectors can be denoted with normal vector notation, $\boldsymbol{u}$ or $\vec{u}$, or with standard unit vector notation, $\hat{\boldsymbol{u}}$, spoken "u-hat".
2. Unit vectors in various coordinate systems use Greek and English letters.

   Cartesian coordinate system: $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$

   Cylindrical coordinate system: $\hat{\boldsymbol{\rho}}, \hat{\boldsymbol{\varphi}}, \hat{\mathbf{z}}$

   Spherical coordinate system: $\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\varphi}}$

3. $1.142857\overline{142857}$
4. $\frac{7}{15} = .4\overline{6}$
5. $2 \cdot 3 = \overline{2} \cdot 3 = \overline{2 \cdot 3}$
6. $\overline{\mathrm{PQ}}$, $\overline{x'}$, $\overline{\mathrm{R''S''}}$
7. $\overline{\mathbf{s}}, \overline{\alpha}, \overline{m}\,'$
8. $\mathrm{m}\overline{\mathrm{BC}} = a$
9. $\overline{\mathrm{C}} = 100 \times 1000$
10. $F = 2\pi\overline{r}l$
11. $P\left(\overline{a + bi}\right) = \overline{0} = 0$
12. $\hat{x}_i = 0.5\left(\overline{x}_i + \underline{x}_i\right)$
13. $3.141\underline{59}$

12.6 Other Symbols Used as Modifiers

Apply the five-step rule for the construction of simple modified expressions when the following symbols are used as modifiers.

12.6.1 **Arc**

	Arc Concave Upward	$\frown$

Example 12-11

Consider $\angle ABD = \frac{1}{2}\widehat{DC}$.

12.6.2 **Dot**

	Dot	$\cdot$

In print, the recurrence of one digit in a decimal numeral may be indicated by one dot over the recurring digit.

⪢ $.\dot{6}$

"6" is the expression being modified. It is modified directly over with a single dot.

⪢ $\frac{1}{6} = 0.66\dot{6}\ldots$

The last "6" is the expression being modified. It is modified directly over with a single dot.

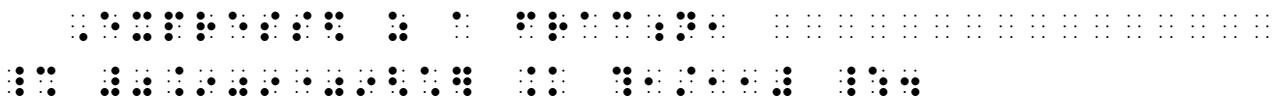
The recurrence of one or more digits in a decimal numeral may be indicated by one dot over each recurring digit. In braille, only one dot is used as a modifier. The dot is placed after the last modified digit in the recurring sequence.

⪢ $1.\dot{3}\dot{7}\dot{5}$

A single dot is printed over each of the three digits 3, 7, and 5. In braille, only one dot is used as a modifier. The multipurpose indicator and termination indicator clearly show what is included within the modified expression.

Example 12-12

Expressed as a fraction, $0.909\dot{0}\dot{9} = \frac{1}{11}$.

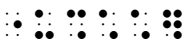


In print, a single dot is shown over each of the last two digits (0 and 9).

When one or more dots occur over or under a single letter or numeral, the symbol for the dot is used as many times as necessary to conform with the print text.

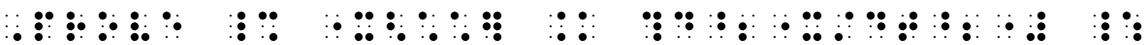
⩾ $\dddot{x}$

⩾ $\underset{\cdot\cdot}{u}$



Example 12-13

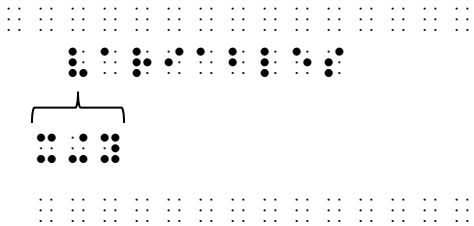
Prove $\ddot{x} = \frac{d^2x}{dt^2}$



12.6.3 **Horizontal Grouping Sign.** When a horizontal grouping sign occurs over or under a mathematical expression, it is either a part of the expression or is pointing to a label.

a. **As a Pointer.** When the grouping sign points to a label or to explanatory text, it must be drawn as a tactile graphic. Refer to *Guidelines and Standards for Tactile Graphics* for drawing techniques.

⩾ $\overbrace{x + y}^{\textit{variables}}$

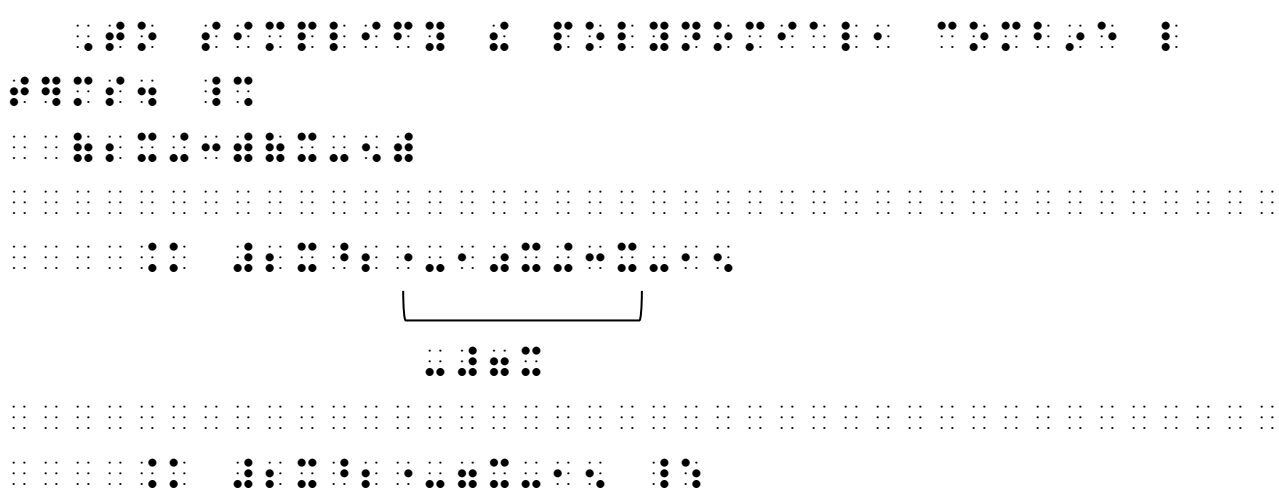


Example 12-14

To simplify the polynomial, combine like terms.

$$(2x+3)(x-5) = 2x^2 \underbrace{-10x + 3x} - 15$$

$$= 2x^2 \quad -7x \quad -15$$



b. **As a Modifier.** When a horizontal grouping sign is part of the math expression, it is regarded as a modifier. Although it is preferable to draw the symbol, the transcriber can represent the construction as shown below, following the five-step rule of modified expressions. The left grouping symbol is transcribed when the modifier is directly over the expression, and the right grouping symbol is transcribed when the modifier is directly under the expression.

A brace is printed above $x + y$. *A brace is printed below $x + y$.*

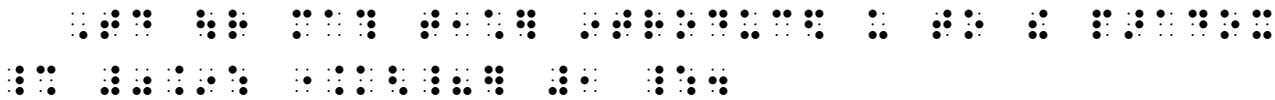
12.6.4 **Question Mark.** When the question mark is not functioning as a sign of omission, the punctuation mark is transcribed. In mathematical context, a punctuation indicator precedes the question mark to prevent it from being misread as the numeral 8.

Question Mark	?	(not an omission)

An equals sign is the expression being modified. It is modified directly over with a question mark.

Example 12-15

Today our math teacher introduced us to the paradox $0.\overline{9} \stackrel{?}{=} 1$.

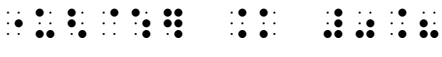


12.6.5 Tilde

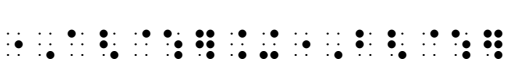
	Single Tilde	$\sim$
	Extended Tilde	$\widetilde{}$

The extended tilde has more than one peak.

⫸ $\tilde{u} = 0.8$



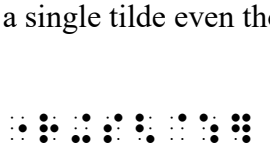
⫸ $\widetilde{\mathrm{A}} \cup \widetilde{\mathrm{B}}$



⫸ $\widetilde{r + s + t}$

The next example is a single tilde even though it is covering several items. (The single tilde has only one peak.)

⫸ $\widetilde{r + s}$



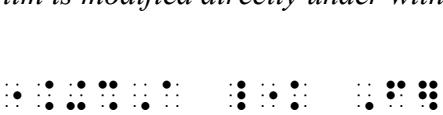
12.7 Expressions as Modifiers

When a modifier is, itself, a mathematical expression, follow Nemeth rules regarding spacing and use of indicators. Each example below contains a comparison sign in the modifier. Nemeth rules applied include the following. A space is required before and after a sign of comparison; an English-letter indicator is not needed when a letter immediately precedes or follows a sign of comparison or when a letter touches an indicator; a numeric indicator is required when a numeral follows a space. The termination indicator marks the end of the modifier.

⫸ $\lim\limits_{x \searrow a}$

lim is modified directly under with the expression $x \searrow a$.

⫸ $\bigcup\limits_{A \subset F}$



The union ("cup") symbol is modified under with the expression $A \subset F$.

Greek capital letters *Sigma* and *Pi* are often encountered in this notation. Sigma is used in summation notation and Pi is used in product notation.

⠰⠎⠛	Sigma	Σ	(capital)
⠰⠏⠊	Pi	Π	(capital)

⟫ $\sum_{i<j}$ ⠰⠎⠛⠊⠍⠍⠁ ⠰⠊ ⠰⠚⠑⠙

Sigma is modified directly under with the inequality $i < j$.

⟫ $\prod_{k=1}$ ⠰⠏⠊⠍⠍⠁ ⠰⠅ ⠰⠁⠑⠙

Pi is modified directly under with the equality $k = 1$.

12.7.1 **Binomial Coefficient.** A binomial coefficient is written as two expressions, one atop the other, enclosed in parentheses. For example, *n choose q* is written like this:

$$\binom{n}{q}$$

The binomial coefficient does not follow the five-step rule for modified expressions. The opening parenthesis is followed by the upper expression; the directly-under indicator is placed next, followed by the lower expression. The closing parenthesis ends the expression.

⠰	Directly-under indicator

⟫ $\binom{n}{q}$ ⠷⠝⠰⠷⠟⠾

⟫ $\binom{n-1}{k-1}$ ⠷⠝⠤⠁⠰⠷⠅⠤⠁⠾

Example 12-16

The **recursive formula** states that $\binom{n}{0} = \binom{n}{n} = 1$ for all integers $n, k : 1 \leq k \leq n - 1$.

⠰⠎ ⠷⠝⠰⠷⠅⠾⠷⠝⠰⠷⠝⠾⠷⠝⠰⠷⠅⠾ ⠷⠝⠰⠷⠅⠾⠷⠝⠰⠷⠝⠾⠷⠝⠰⠷⠅⠾ ⠰ ⠀⠀⠀⠀⠀⠀⠀
⠷⠝ ⠷⠝⠰⠷⠅⠾ ⠷⠝ ⠷⠝⠰⠷⠝⠾ ⠷⠝ ⠷⠝ ⠷⠅ ⠷⠝⠷⠅ ⠷⠝⠰⠷⠝⠤⠁⠰⠷⠅⠤⠁⠾
⠷⠝ ⠷⠝⠷ ⠷⠝⠷⠅⠰ ⠷⠝ ⠷⠝⠷⠅ ⠷⠝ ⠷⠝⠷⠅⠰ ⠷⠝⠷⠅⠰⠷

Reminder from Lesson 5: A colon that means "such that" is transcribed unspaced from the letter it follows, and is preceded by a punctuation indicator. This spacing is applied regardless of the spacing shown in print. Spacing after the colon follows print.

Example 12-17

Use this formula to calculate the binomial coefficient.

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

⠠⠁⠇⠇ ⠠⠃ ⠨⠍⠕⠝⠕⠍⠊⠁⠇ ⠨⠷ ⠠⠊⠝⠞⠑⠛⠑⠗⠎ ⠯ ⠠⠁⠇⠇⠕⠺⠑⠙
⠠⠊⠝⠞⠑⠛⠑⠗⠎ ⠷⠷ ⠨⠷⠷⠊⠝⠞⠑⠛⠑⠗⠎ ⠷⠷

PRACTICE 12C

1. In Figure 7.3, if $\overset{\frown}{AB} = \overset{\frown}{CD}$ in circle O, then $\angle AOB = \angle BOC$.
2. $x \overset{?}{=} y$ means "does x equal y?"
3. $\sum_{d|n}$ (where $d|n$ means "d divides n").
4. $\binom{t}{p} = R_t^p$
5. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ for all integers $n \geq 0$.
6. Does $(\tilde{x},\ \tilde{y})$ mean $\tilde{x}$ and $\tilde{y}$?
7. More modified expressions:

   a. $.249\dot{9}$ b. $2.431\dot{3}\dot{1}$ c. $\dot{x}\ddot{y} - \dot{y}\ddot{x}$

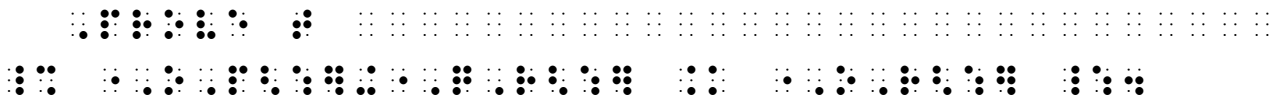
   d. $\underset{\cdot}{a} + \underset{\cdots}{a} = ?$ e. $f \to \tilde{f}$ f. $\widetilde{x+y}$

12.8 Spacing with Modified Expressions

The spacing before and after an entire modified expression is subject to the spacing rules for the symbols preceding or following it.

Example 12-18

Prove that $\overline{\mathrm{OP}} + \overline{\mathrm{QR}} = \overline{\mathrm{OR}}$.



There is no space before or after the operation (plus) sign. There is a space before and after the comparison (equals) sign.

When the modifier is wider than the modified symbol, the print copy will insert extra space to clarify what exactly is being modified. In the next example, only the Sigma is modified, not the letter *a*. The space between the Sigma and the a clarifies the extent of the modifier, i < j. In braille, however, <u>indicators</u> define the extent of a modifier. There is no need for the space in braille.

⫸ $\sum_{i<j} a_{ij}$

Sigma is modified directly under with the inequality $i < j$.
The termination indicator signals the completion of the modifier.

PRACTICE 12D

Spacing with Modified Expressions

A. The probability of the event A, written P(A), is defined as

$$\mathrm{P(A)} = \sum_{\wedge} f(x)$$

where $\sum_{\wedge} f(x)$ means sum $f(x)$ over those values x_i that are in A.

B. $\underset{\alpha \in \mathrm{A}}{\times} \mathrm{A}_{\alpha}$

C. $\prod_{i>j} (x_i - x_j)$

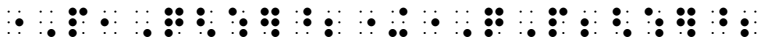
D. $\overline{7} + 2 \stackrel{?}{=} \overline{7 + 2}$

12.9 Modified Expression on the Baseline of Writing

A modifier and its related expression must be placed on the same level of writing.

12.9.1 **Superscript/Subscript After the Modification.** When a modifier affects only the expression and not the superscript or subscript, the modified expression terminates before the superscript (or subscript) is transcribed.

⋙ $\overline{P_1Q}^{\,2} + \overline{QP_2}^{\,2}$



A termination indicator precedes each superscript.

⋙ $\overline{x}^{\,2}$

The contracted form of "bar over" does not have an explicit termination indicator. The superscript follows the "bar" symbol, unspaced.

Note that, when a numeric subscript follows a modified letter, a subscript indicator is required. This rule applies whether the letter is modified by the five-step rule or by the contracted form.

⋙ $\hat{\beta}_3$

⋙ $\overline{\alpha}_2$

⋙ $\underline{\mathrm{N}}_1$

12.9.2 **Superscript/Subscript Within the Modification.** When a modifier affects both an expression on the baseline of writing <u>and</u> its superscript or subscript, the baseline indicator is placed before the directly-over (or the directly-under) indicator. This assures that the expression as a whole appears on the same level of writing.

⋙ $\overline{x^2}$

The five-step method is used because the modified expression (x^2) is not a single letter. The first dot 5 begins the modified expression. The second dot 5 is a baseline indicator following the superscript "2".

⫸ $\overline{\mathrm{OD}^2} + \overline{\mathrm{OP}^2}$

⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆

The first dot 5 begins the first modified expression. The second dot 5 is a baseline indicator following the first superscript "2". Similarly, the third dot 5 begins the second modified expression and the fourth dot 5 is a baseline indicator following the second superscript.

⫸ $\overline{x_1}$ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆

You may wish to review Section 6.11 regarding nonuse of the subscript indicator.

a. **Binomial Coefficient.** Although a binomial coefficient is not technically a modified expression, notice how this rule applies.

⫸ $\begin{pmatrix} a_x \\ b_y \end{pmatrix}$ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆ ⠈⠂⠈⠆

The baseline indicator precedes the "directly under" indicator. This keeps that indicator on the same level of writing as the letters "a" and "b".

PRACTICE 12E

Superscripts and Subscripts

A) $\overline{\mathrm{AB}}^2 + \overline{\mathrm{BC}}^2$

B) $\overline{\mathrm{A}} = [\,\overline{a}_i]$

C) $\sqrt{\ddot{x}^2 + \ddot{y}^2}$

D) $\underline{\mathrm{Z}}^\circ$

E) If $\overline{a}_1 = 72$, find $\overline{a}_7$.

F) Draw $\overrightarrow{\mathrm{P_1P_2}}$ if $\mathrm{P_1}$ is the point $(1, 3)$ and $\mathrm{P_2}$ is the point $(2, -1)$.

G) $\left(\overline{3^{-1}}\right) \in \mathrm{P}$

H) $\overline{x_1} + \overline{y_1}$

12.9.3 **Modified Expression on the Baseline That Follows a Superscript or a Subscript.** When a modified expression written on the baseline of writing immediately follows a superscript or a subscript, several details must be considered in order to determine the necessity of level indicators. Because braille dot 5 has several meanings besides that of baseline indicator (hence the name "multipurpose indicator") mindful use of that symbol is required in order to give the reader the correct information. Several examples will illustrate. Each dot-5 is highlighted in the examples.

a. If a level indicator is needed for the superscript or subscript, the baseline indicator is transcribed before starting the modified expression.

⋙ $x^2\overline{y}$

This dot 5 is a baseline indicator, following the superscript 2.

⋙ $\overline{x}_n\overline{z}_m$

This dot 5 is a baseline indicator, following the subscript n.

b. If a subscript indicator is not needed, a baseline indicator is not needed before starting the modified expression.

⋙ $x_1\overline{z}_1$

Each 1 is printed at the subscript level.There are no dot 5s in this example because a baseline indicator is not needed when a subscript indicator is not used, and the contracted form of the "bar over" does not use a dot 5.

⋙ $x_1\overline{z_1}$

Each 1 is printed at the subscript level. A baseline indicator is not needed when a subscript indicator is not used. This dot 5 is the start of the modified expression.

c. Two indicators may be needed. First, a baseline indicator (dot 5) is used to terminate the effect of the superscript or subscript level indicator. Next, a multipurpose indicator (dot 5) is required to begin the five-step modification.

⋙ $3x^2\overline{\Delta x} + 3x\overline{\Delta x}^{\,2}$

Because x^2 requires a superscript indicator and the five-step rule is used for the modified expression, two dot 5s are needed. The baseline indicator is immediately followed by the dot 5 which begins the modified expression.

⫸ $a_k\overline{b_1 \ldots b_p}$

Because a_k requires a subscript indicator and the five-step rule is used for the modified expression, two dot 5s are needed. The baseline indicator is immediately followed by a dot 5 beginning the modified expression. Finally, the directly-over indicator is preceded by a baseline indicator to assure that the entire modified expression is transcribed on the same level.

⫸ $a_i \prod_{j \neq i} (A - r_j I)$

Because a_i requires a subscript indicator and the five-step rule is used for the modified expression, two dot 5s are needed. The baseline indicator is immediately followed by a dot 5 beginning the modified expression. The last dot 5 is a baseline indicator following the subscript "j".

12.10 Modified Expression Within a Superscript or Subscript

Recall that a modifier and its related expression must be placed on the same level of writing. When a modified expression occurs as a superscript or subscript, or as the first part of a superscript or subscript, the level indicator is transcribed first, followed by the multipurpose indicator which begins the modified expression. This assures that the expression as a whole appears on the same level of writing.

⫸ $S^{\tilde{x}}$

This dot 5 begins the modified expression $\tilde{x}$ (which is in the superscript position). It will not be misread as a baseline indicator because nothing comes between it and the superscript indicator.

⫸ $S_{\tilde{x}}$

This dot 5 begins the modified expression $\tilde{x}$ (which is in the subscript position). It will not be misread as a baseline indicator because nothing comes between it and the subscript indicator.

⫸ ${}_{\tilde{x}}A_1$

The first dot 5 begins the modified expression $\tilde{x}$ (which is a left subscript to the letter A). The second dot 5 is a baseline indicator.

If the modified expression occurs in the middle or at the end of the superscript or subscript, the appropriate level indicator must be repeated before the multipurpose indicator to show continuation of the same level of writing. This assures that the multipurpose indicator will not be misread as a baseline indicator.

⫸ $S_{\tilde{x}+\tilde{y}}$ ⠎⠩⠻⠔⠻⠢⠔⠻⠔⠻⠢⠔⠻⠔⠻

The first dot 5 begins the modified expression $\tilde{x}$. The second dot 5 begins the modified expression $\tilde{y}$. To assure that the second dot 5 is not read as a baseline indicator, the subscript level is restated before the dot 5.

Since the multipurpose indicator is absent in the contracted form of "bar over" or "bar under", the level continues with certainty. It is not necessary to repeat the level indicator when the contracted form is used.

⫸ $e^{a\overline{x}}$ ⠑⠻⠁⠔⠻⠭⠔⠻

PRACTICE 12F

More About Superscripts and Subscripts

(1) $a_0\overline{x}^{\,n} + a_1\overline{x}^{\,n-1}$

(2) $W = \frac{2}{3}\pi r^3\underline{w}\left(h + \frac{3}{8}r\right)$

(3) $S^{\tilde{x}}$ and $S^{\tilde{x}+\tilde{y}}$

(4) $\mathrm{D}_{\overline{\mathrm{x}}}$ or $\mathrm{D}_{\overline{\mathrm{x}}+\overline{y}}$

(5) $3_{\overline{x}} - 2_{\overline{x}}$

(6) ${}_{\overline{n}}A_{\,1}$

Modified Signs of Comparison

12.11 Definition

A modified sign of comparison consists of a simple sign of comparison, such as the equals sign or the tilde, modified by a caret, dot, triangle, question mark, vertical bar, or any symbol <u>except</u> another sign of comparison.

> *When a simple sign of comparison occurs above or below another simple sign of comparison the combination is transcribed as a sign of comparison compounded vertically. See Section 5.8 for a review of that construction. Note that many of those signs are printed with a single horizontal line "bar over" or "bar under". For example, "bar over greater than, inclusion with bar under, bar over single tilde, logical sum with bar under," etc. These signs are not to be misinterpreted as a horizontal bar modifying a sign of comparison.*

12.12 Transcription

A modified sign of comparison as defined above is transcribed in accordance with the five-step rule for modified expressions.

In addition to the caret and inverted caret seen earlier in this lesson, you may also encounter a left- or right-pointing caret in a modified sign of comparison. Do not confuse these two symbols with the "less than" and "greater than" comparison signs. Ask an expert if context does not clarify the identity of this symbol.

⠈⠣	Left-Pointing Caret	<
⠈⠜	Right-Pointing Caret	>

The following list contains the modified equals signs most commonly used.

Modified Equals Sign		
⠐⠣⠐⠣⠐⠣⠐⠣	Caret Over Equals Sign	≙
⠐⠣⠐⠜⠐⠣⠐⠣	Caret Under Equals Sign ("is projective to")	≚
⠐⠣⠐⠣⠐⠣⠐⠜	Inverted Caret Over Equals Sign	≛
⠐⠣⠐⠣⠐⠣⠐⠣	Left-Pointing Caret Over Equals Sign	≦
⠐⠣⠐⠣⠐⠣⠐⠜	Right-Pointing Caret Over Equals Sign	≧

⠐⠶	Dot Over Equals Sign ("is approximately equal to")	$\doteq$
⠐⠶⠐	Dot Over and Dot Under Equals Sign	$\Doteq$
⠐⠶⠐⠐	Two Dots Over and Two Dots Under Equals Sign	$\ddot{=}$
⠐⠶⠴	Hollow Dot Over Equals Sign ("is equal in degrees to")	$\overset{\circ}{=}$
⠐⠶⠠⠶	Equilateral Triangle Over Equals Sign	$\overset{\triangle}{=}$
⠐⠶⠦	Question Mark Over Equals Sign	$\overset{?}{=}$
⠐⠶⠳	Vertical Bar Over Equals Sign	$\overset{\vert}{=}$

⫸ $\sqrt{5} \doteq 2.236$

⠩⠢⠼ ⠐⠶⠐ ⠼⠂⠆⠉⠋

⫸ $5(3) + 2(-2) \overset{?}{=} 11$

⠼⠢⠼⠶⠼⠆⠷⠼⠆⠶⠼⠂⠷ ⠐⠶⠦ ⠼⠂⠂

If a symbol is encountered which does not appear on this list, construct a symbol in accordance with the same principles. A transcriber-created symbol should be listed on the Special Symbols page.

⫸ $A \overset{m}{=} B$ ⠁⠐ ⠐⠶⠍⠐ ⠁⠐

The remainder of the list provided in the code contains other modified comparison signs most commonly used.

```
Modified Tilde
    ⠈⠄⠈⠒⠈⠶    Dot Under Tilde          ⩰
    ⠈⠄⠈⠖⠈⠶    Dot Over Tilde           ≈̇
Modified Horizontal Bar
    ⠈⠢⠈⠁⠈⠶    Caret Over Bar           ⩡
    ⠈⠢⠈⠈⠈⠶    Caret Under Bar          ⩠
                ("is perspective to")
    ⠈⠢⠈⠈⠈⠶    Dot under Bar            ⩢
```

If the horizontal bar is modified by a dot *over* it, the combination is a modified sign of operation ("minus with dot over" signifying "proper difference"). The five-step rule is not used for this symbol. See Section 5.4.7.

Instructions: Use the five-step rule to show the horizontal grouping sign in the last item.

PRACTICE 12G

Modified Signs of Comparison and More

1. $A \doteq 3.14r^2$
2. $\angle b \overset{\circ}{=} \frac{1}{2}\widehat{\mathrm{EB}}$
3. The symbol $\triangleq$ is used to make a definition.
4. $x \dot{\sim} \mathcal{N}(0,1)$
5. $x^n = \underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}$

Expressions with More Than One Modifier

12.13 Modifiers of Higher Order

When two or more modifiers occur one above the other and apply to <u>exactly</u> the same expression, the second, third, etc. modifiers are "modifiers of higher order." In such cases, the directly-over indicator or the directly-under indicator is doubled, tripled, etc., before each modifier to indicate its position. The termination indicator is used only after the last modifier shown.

```
Directly-over indicators
     ⠨⠨      Second order
     ⠨⠨⠨     Third order
Directly-under indicators
     ⠰⠰      Second order
     ⠰⠰⠰     Third order
⠸  Termination indicator
```

⫸ $\overline{\overset{a=3}{x+y}}$

⠨⠨⠁⠨⠨⠁⠨⠨⠁⠨⠨⠁⠨⠨⠁⠨⠨⠁ ⠨⠨ ⠨⠨⠁⠨⠨⠁

Analysis: The expression x + y is modified directly over with a horizontal bar which, in turn, is modified directly over with the expression a = 3.

⫸ $\underset{\substack{a=3\\b=2}}{\underline{x+y}}$

⠨⠨⠁⠨⠨⠁⠨⠨⠁⠨⠨⠁⠨⠨⠁⠨⠨⠁ ⠨⠨ ⠨⠨⠁⠨⠨⠁⠨⠨⠁⠨⠨⠁ ⠨⠨ ⠨⠨⠁⠨⠨⠁

Analysis: The expression x + y is modified directly under with a horizontal bar which, in turn, is modified directly under with the expression a = 3 which is modified directly under with the expression b = 2.

⟫ $\sum_{\substack{i,j,k \\ i<j<k}}$

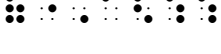
Analysis: The Sigma is modified directly under with letters i, j, k . That, in turn, is modified directly under with the expression i < j < k. Review the rules regarding use of the English-letter indicator in Lesson 3. An English-letter indicator is needed for the first letter "j" because it is a "single letter" according to the definition in the Nemeth Code. The other letters are either touching indicators or are preceded or followed by a sign of comparison, so an English-letter indicator is not needed.

12.13.1 **Parallel Horizontal Bars.** When two or more parallel horizontal bars are the same length and apply to <u>exactly</u> the same expression, they are treated as a single modifier. In such cases, the directly-over or the directly-under indicator is used only once, and the symbol for the bar is used as many times as necessary to correspond to the print text.

⟫ $\bar{\bar{9}}$

⟫ $(\bar{\bar{A}})$

⟫ $\underline{\underline{\underline{x}}}$

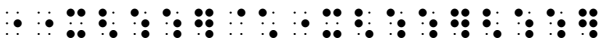


Attention: Two parallel horizontal bars must not be misinterpreted as the equals sign, and three parallel horizontal bars must not be misinterpreted as the identity sign. The symbol's meaning can be determined by reading the surrounding text for context. See Section 5.8 ("Signs of Comparison Compounded Vertically") to review equals sign printed above or below a sign of comparison.

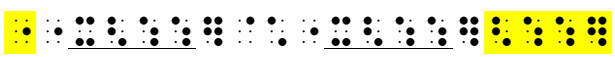
12.14 Individual Modifiers

When two or more modifiers do not apply to exactly the same symbols but cover different portions of the same expression, the longer modification encloses the entire modified expression. Within the long expression, each inner expression is modified individually.

⟫ $\overline{\overline{\bar{\bar{x}} \times \bar{\bar{x}}}}$



Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expressions are underlined.



The five-step rule and the contracted form of transcription may be used together.

⩾ $\overline{(\bar{a}A + \bar{c}C)}$

Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expressions are underlined.

⩾ $\overline{P_{\left(\overline{x^2}\right)}}$

Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expression is underlined.

Reminder: The subscript level indicator must be repeated before the modified expression indicator so the dot 5 is not misread as a baseline indicator.

12.15 Simultaneous Modifiers

When an expression is simultaneously modified both above and below, the modifier *below* is transcribed first and the modifier *above* is transcribed second. The termination indicator is used only at the end of the entire modification.

⩾ $\sum_{3}^{7}$

⩾ $\prod_{k=2}^{6} a_k$

Recall from 12.8 that, when the modifier is wider than the modified symbol (in this case, $k = 2$ is wider than Π), the print copy might insert extra space to clarify what exactly is being modified. In braille, the modified Pi is unspaced from the letter "a".

⩾ $\underline{\bar{x}}$

⩾ $\underline{\overline{x + y}}$

As you study the eight constructions in the next example, note the use or nonuse of the English-letter indicator and the use or nonuse of the numeric indicator. The simulated braille is in Nemeth. Code switch indicators are omitted.

Example 12-19

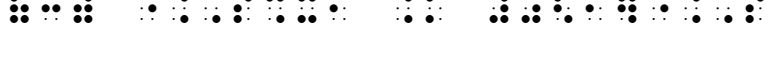
(1) $\sum_{k=1}^{n} (2k) = n(n+1)$

(2) $\sum_{i,j=1}^{n} a_{ij} x_i x_j$

(3) $\sum_{x_1=0}^{1} \sum_{x_2=0}^{1}$

(4) $\bigcup_{r=1}^{n} A_r = A_1 \cup A_2 \ldots$

(5) $\overline{\overline{\prod_{x \in a} c'x}}$



(6) $\dfrac{\frac{b=2}{x+y}}{a=3}$

(7) $\sum_{i=1}^{6} \overline{P_{i-1} P_i}$

(8) $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$

⠷⠦⠾ ⠷⠭⠬⠽⠾⠘⠝
⠀⠀⠨⠅ ⠐⠨⠠⠎⠩⠅ ⠨⠅ ⠼⠴⠣⠝⠻⠷⠝⠩⠅⠾⠭⠘⠝⠤⠅⠐⠽⠘⠅

This is a linked expression where $(x+y)^n$ is the anchor and $= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$ is the link. Recall from Lesson 8 that when a linked expression will not fit on one braille line, start a runover line with the link. The equals sign within the modification is not a suitable division site because the components of an expression modified according to the five-step rule should not be divided between lines. (See 12.2.)

PRACTICE 12H

Expressions with More Than One Modifier

1. $\bar{\bar{9}} \cdot \bar{\bar{3}} = \overline{\overline{9 \cdot 3}}$

2. $\overline{\overline{A^n}}$

3. $\overline{A \cap \overline{\overline{B \cap \bar{\bar{C}}}}}$

4. $\overline{a_{\bar{n}} + b_{\bar{p}}}$

5. $\overline{x_{\bar{a}} + y^{\bar{n}}}$

6. $\underline{\bar{\bar{\mathrm{N}}}}$

7. $\sum_{i=1}^{k} \sum_{j=1}^{k}$

8. $\prod_{\substack{j=1 \\ j \neq k}}^{n}$

12.16 Formal Proof [NC Rule 26.7]

A proof is a valid argument that establishes the truth of a mathematical statement. It is often introduced by a heading such as *Theorem*, *Proposition*, or *Lemma*. A progressive sequence of statements leads to the conclusion. In a formal proof, every step of the argument is shown and each step is supported by a definition or by a previously proven statement.

Lesson 11 explained how to transcribe a mathematical statement. Those guidelines are summarized in steps a-c, below, in the context of a proof.

a. A blank line precedes the beginning of the proof.

b. *Heading:* The heading can be formatted as a paragraph heading or as a cell-5 or cell-7 heading, at the transcriber's discretion. Review Section 11.38.b for details.

c. *Statement:* Continue with the text, using normal (3-1) paragraph style. When the statement is printed in a variant typeform and the proof follows, in regular type, it is recommended that the typeform be preserved for the statement in order to retain distinction.

d. *Auxiliary Captions:* Paragraph headings such as Given, Hypothesis, Prove, or Conclusion begin in cell 3, without a blank line before the paragraph. Associated material follows the caption. Runovers go in cell 1.

e. *Two-Column Proof:* See 12.16.1, below.

f. *End of Proof Icon:* See 12.16.2, below.

g. When the proof is complete, insert a blank line before continuing with the text.

12.16.1 **Two-Column Proof.** When a formal proof is presented by numbered steps printed in two columns, the layout is changed as follows.

a. If there is a caption such as "Proof", follow the same pattern established in step d, above ("Auxiliary Captions").

b. The column format is changed to a list in braille. A transcriber's note must call attention to the change in format. See step c, below, for a sample transcriber's note.

   A blank line is inserted before the list. If there are column headings, such as "Statement" and "Reason", see step c, below. Each step begins in cell 1, starting with the first item from the left column. Runovers are in cell 3. The related item from the right column begins in cell 1 on the next line, with runovers in cell 3.

   Each item must be labeled with an identifier as described below.

c. *Step Identifiers:* Typically, the print copy includes the column headings "Statements" and "Reasons". In braille, the column headings are replaced with a letter – "S" or "R", respectively – as part of each step number. For example, "1S" is Statement 1, followed on the next line by "1R" which is Reason 1. If other column headings appear in print, the transcriber chooses an appropriate letter to represent each heading. If no column headings appear in print, use "S" for the left-column items and "R" for the right-column items.

If the steps are not numbered in print, identifiers are added in the manner stated above. A transcriber's note must specify the meaning of the letter labels. Any transcriber-added numbers or letters must also be explained.

Sample transcriber's note: "Proofs printed with steps in columns headed *Statements* and *Reasons* are transcribed with an S or R immediately following the step number to show the column in which the step appears. Each step from the *Statements* column is immediately followed by the corresponding step from the *Reasons* column."

Identifiers are transcribed in either UEB or Nemeth according to code-switching guidelines.

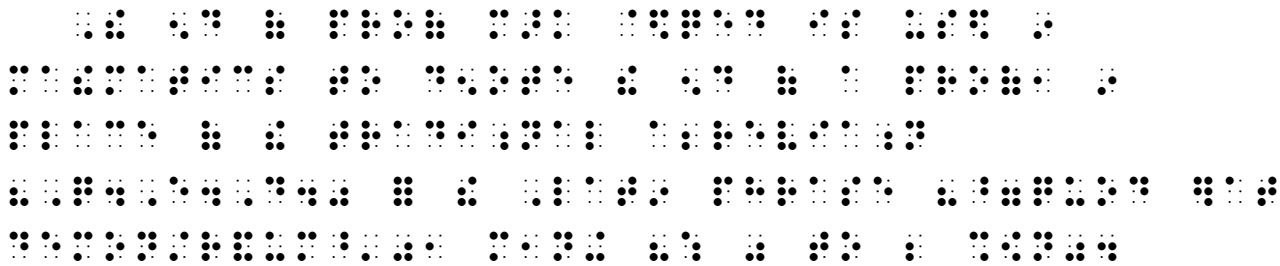
12.16.2 **End of Proof Icon.** Sometimes a shape (often a bold rectangle or square) or the Latin abbreviation "Q.E.D." is used to denote the end of a proof. The shape is often printed at the right margin and may go unnoticed. Watch for it. Regardless of the shape or abbreviation used in print, the icon shown below is used in braille, and is transcribed following the last word or character in the proof, preceded by one blank cell. The icon is created using the UEB transcriber-defined shape indicator and may be used in either Nemeth or UEB without switching. The icon must be listed on the Special Symbols page. (Details regarding a Special Symbols page in a UEB with Nemeth transcription will be covered in the Final Lesson.)

⠸⠶⠶⠶⠶⠶	End of Proof Icon	*for example,* ▌ *or* ■

If the shape indicates the end of a long example or exercise, the symbol for the filled-in or shaded shape is used. See Section 11.6 in Lesson 11.

Example 12-20

The end of proof mark ▌ is used in mathematics to denote the end of a proof, in place of the traditional abbreviation "Q.E.D." for the Latin phrase "**quod erat demonstrandum**", meaning "which was to be shown".

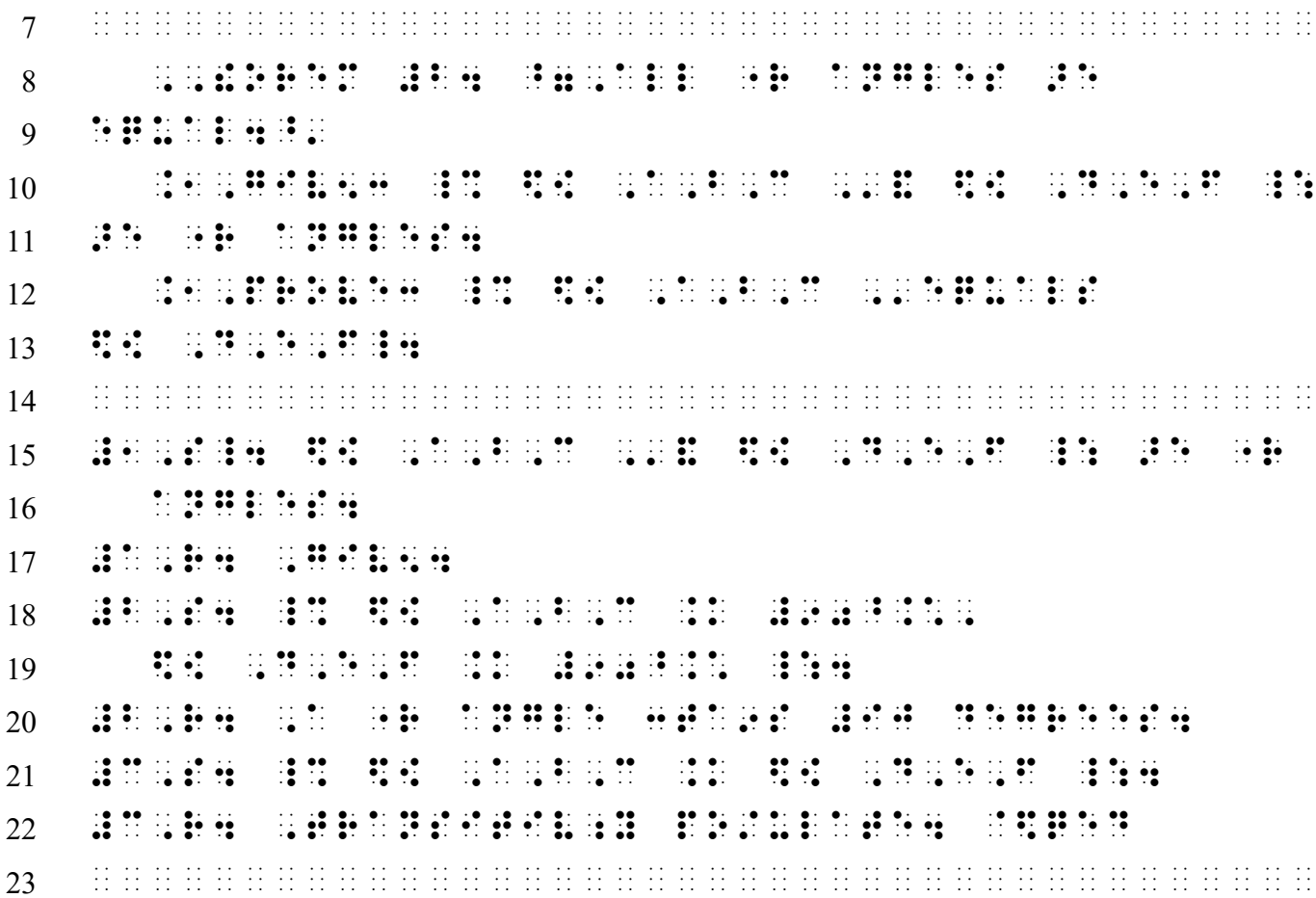


Example 12-21

(Assume the required transcriber's note regarding the two-column proof appears on the Transcriber's Notes page.)

THEOREM 2. All right angles are equal.
Given: ∠ ABC and ∠ DEF are right angles.
Prove: ∠ABC equals ∠DEF.

Statements	*Reasons*
1. ∠ ABC and ∠ DEF are right angles.	1. Given.
2. ∠ ABC = 90°, ∠ DEF = 90°.	2. A right angle contains 90 degrees.
3. ∠ ABC = ∠ DEF.	3. Transitivity postulate. ■



Line 7: A blank line precedes the proof. Box lines are not required.
Line 8: The heading is printed in capital letters and also in a nonregular typeface (boldface). When a paragraph heading is printed in capital letters, typeform is disregarded.
Lines 8-9: The paragraph style is 3-1. Distinctive typeform (boldface) is retained in the statement.
Lines 10-11 and 12-13: Each auxiliary caption follows print regarding typeform (italics, in this example), and uses a 3-1 paragraph style.
Line 14: A blank line precedes the list.
Lines 15-22: Each item in the 2-column proof begins in cell 1, with runovers in cell 3. Identifying letters S and R are combined with each step number.
Line 22: A dark square is printed in the right margin to mark the end of the proof. The "qed" icon is transcribed.
Line 23: A blank line follows the proof.

PRACTICE 12I

Instructions: Create a Transcriber's Notes page that might appear in a volume which contains the proof shown in Practice 12J. A statement citing the code book is required in every volume that uses Nemeth. In the first paragraph, state the title and edition of the Nemeth Code as well as any BANA updates. Something like this:

> Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 2022.*

In the second paragraph, explain the step-number format as described in 12.16.1.c. Refer to *Braille Formats* for further guidelines regarding the structure of a Transcriber's Notes page.

PRACTICE 12J

Given: $3x = 7 - \frac{1}{2}x$

To Prove: $x = 2$

STEP	REASON
1. $3x = 7 - \frac{1}{2}x$	1. GIVEN
2. $6x = 14 - x$	2. Multiplication Property
3. $7x = 14$	3. Addition Property
4. $x = 2$	4. Division Property ▌

POSTSCRIPT

Vector Notation Notes

There is no standard typographic convention for representing a vector letter. It may be printed as an upright or slanted (italic) boldfaced letter, or an upright, italic, bold, or bold-italic letter with a superscribed right-pointing arrow. In braille, the bold is retained but the italic typeform is disregarded. The arrow is transcribed as a modification only if the letter is not bold.

— A bold upright letter: retain the bold typeform. See 7.6.2.a and Example 7-16.
— A bold italic letter: retain the bold typeform; disregard the italics.
— A bold upright or bold italic letter with a right-pointing arrow printed above the letter or letters: retain the bold typeform; disregard italics; disregard the arrow. See 12.3.3.
— A normal upright letter with a superscribed right-pointing arrow: retain the arrow.
— An italic letter with a superscribed right-pointing arrow: disregard the typeform; retain the arrow.

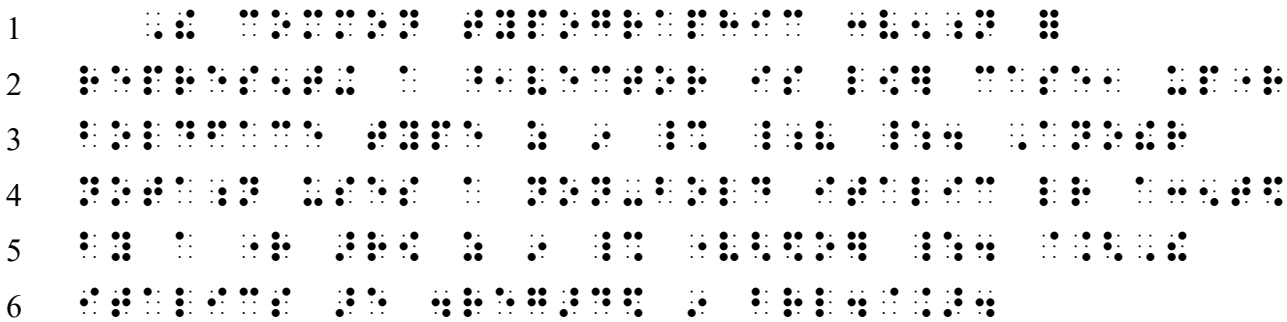
Note:

— The superscribed caret ("hat") must never be omitted from the transcription of unit vectors. See 12.4.
— Boldfaced symbols may also be encountered in vector notation. See Sections 7.9.1 and 7.9.2 in Lesson 7.

Transcriber's Note Required: When the arrow is not transcribed, a transcriber's note must inform the reader of its presence in the print copy. See 12.3.3. When typeform is specifically referred to in the text but is disregarded in braille, a transcriber's note is needed, as in the example below.

Example 12-22

The common typographic convention for representing a **vector** is lower case, upright boldface type as in **v**. Another another notation uses a non-bold italic letter accented by a right arrow as in $\vec{v}$.



Line 5: The contracted form of the right-pointing arrow is used. See 12.3.1.
Lines 5-6: An embedded transcriber's note says "The italics are disregarded in braille."

For further practice, see Appendix A—Reading Practice.

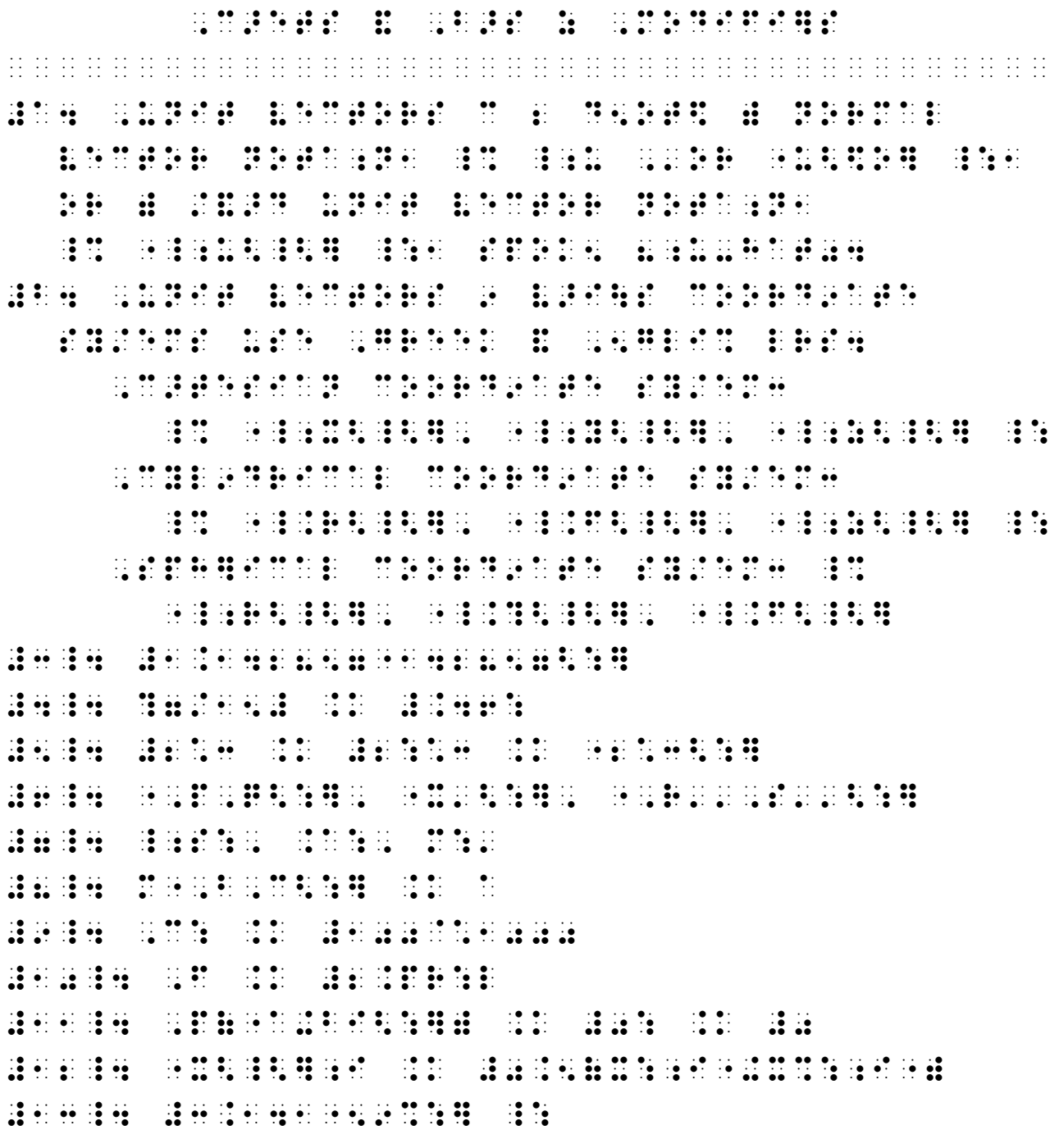
EXERCISE 12

Prepare Exercise 12 for your grader.

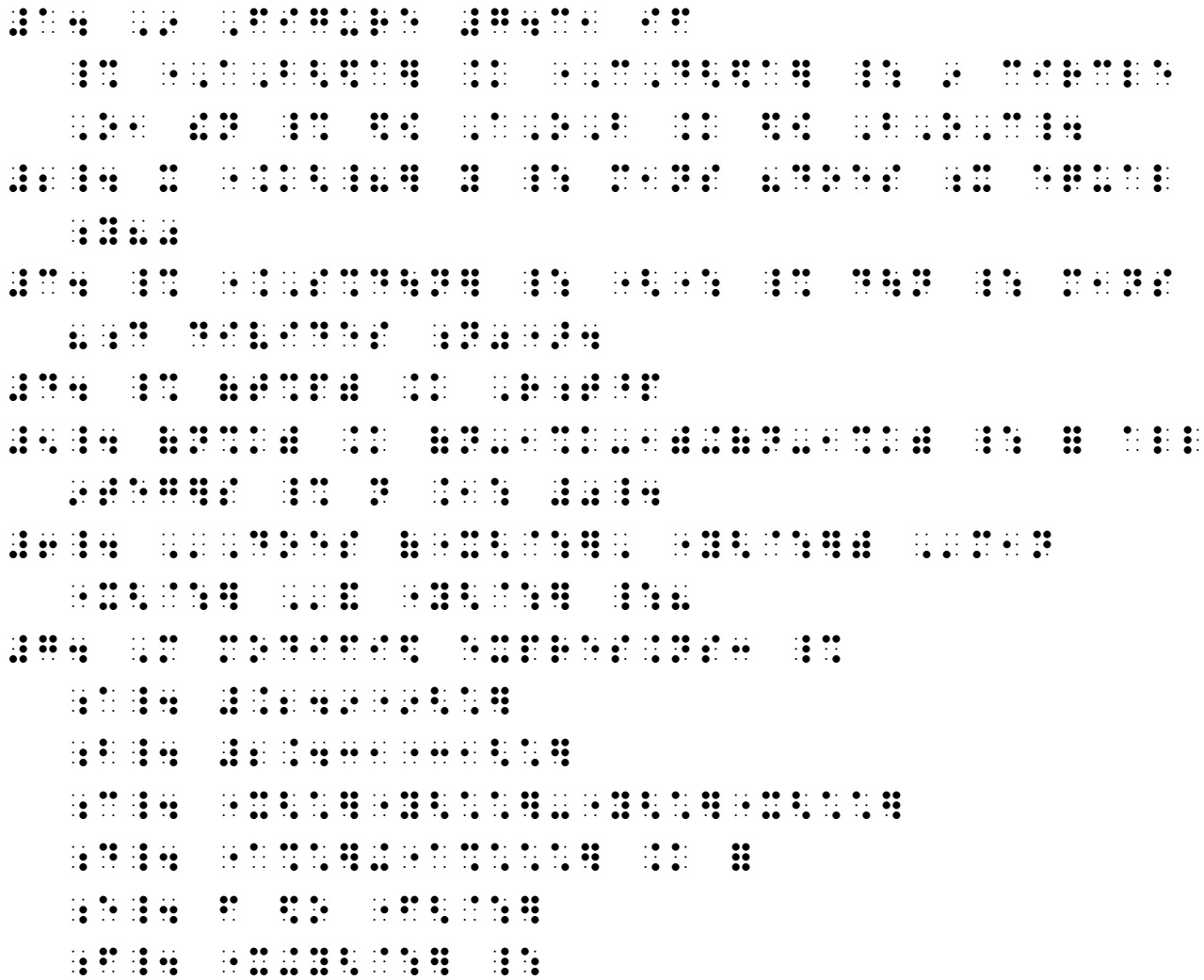
ANSWERS TO PRACTICE MATERIAL

PRACTICE 12A

PRACTICE 12B



PRACTICE 12C



Line 6: Recall from 3.6.7 (Lesson 3) that a single-word switch indicator cannot be placed immediately before an opening parenthesis and that left and right grouping symbols must be transcribed in the same code (Lesson 4). Therefore, Nemeth must be terminated before the left UEB parenthesis.

Lines 6-8 may also be transcribed as follows:

PRACTICE 12D

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PRACTICE 12E

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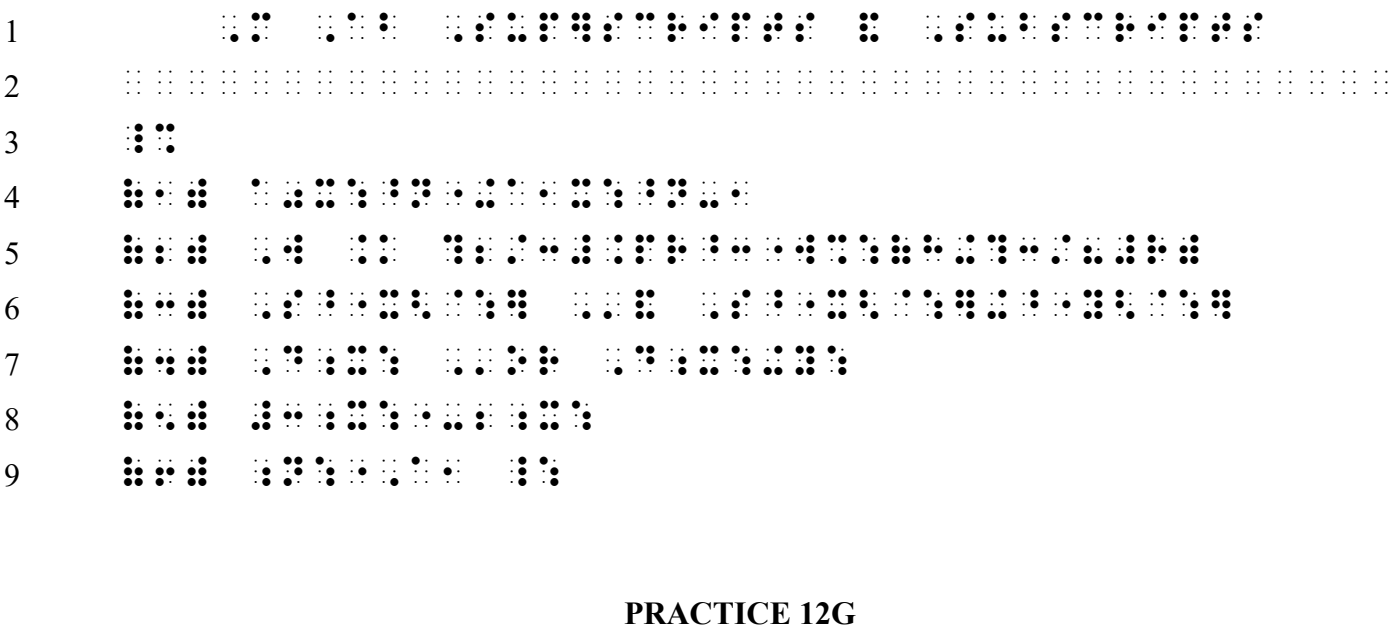
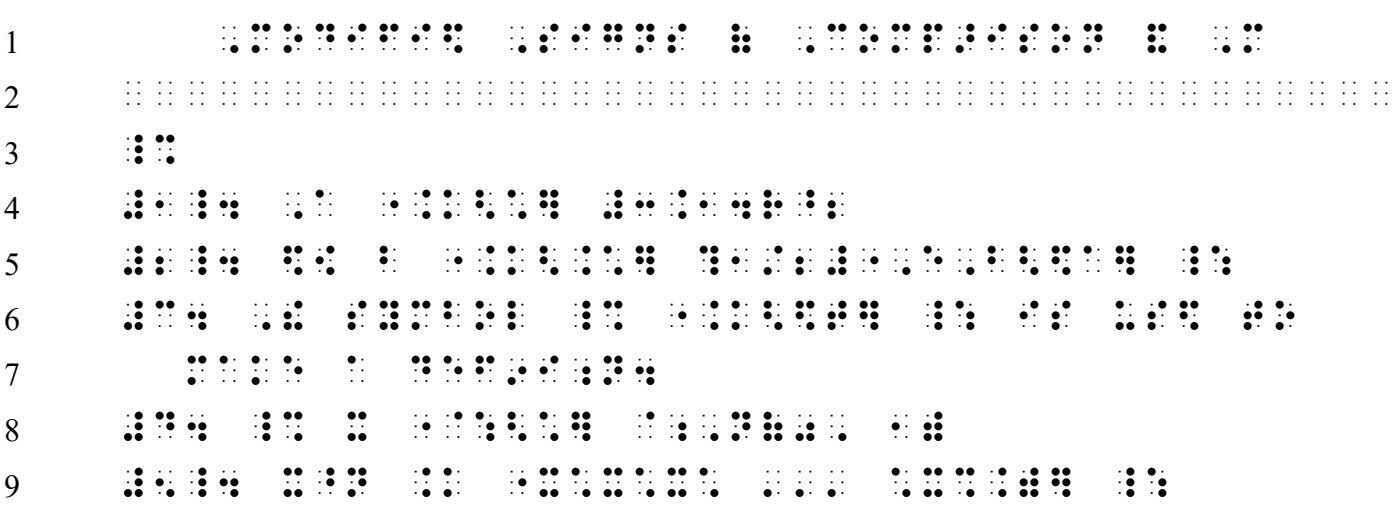
PRACTICE 12F



PRACTICE 12G



Line 8: The opening Nemeth Code indicator is placed after the identifier, according to the suggested guidelines given in Lesson 2 regarding UEB-to-Nemeth within an itemized list. See 2.18.2.c.

PRACTICE 12H

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PRACTICE 12I

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PRACTICE 12J

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