

# **AN INTRODUCTION TO BRAILLE MATHEMATICS USING UEB WITH NEMETH**

## **A Course for Transcribers**

**Revised by Lindy B. Walton**

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**[www.loc.gov/nls](http://www.loc.gov/nls)**

*National Library Service  
for the Blind and Print Disabled*

*The Library of Congress*

I believe that I could not have reached my potential in mathematics without the Nemeth Code. With it, I am able to read and write mathematics, as well as other sciences, at all levels, limited only by my talent and my ambition.

—Dr. Abraham Nemeth, creator of the braille code for mathematics and science notation

#### DEDICATION

I credit my interest in the continued training of braille transcribers in the Nemeth Code to my friend and mentor, Helen Hay, whose fascination and enthusiasm about this braille code was contagious.

#### THANKS

I offer my gratitude to the original authors of this lesson manual, Helen Roberts, Bernard M. Krebs, and Barbara Taffet, for their insight into the learning process and for their eye for detail. Many of the excellent examples from the original book are preserved in this edition. I also wish to thank my supervisors and colleagues in the Madison Metropolitan School District for realizing the importance of the development of this curriculum. And to my husband, whose patience with my obsession has been tireless—thank you for endless cups of coffee.

—Lindy Walton

## **ABOUT THE PROGRAM**

This course is designed for the UEB certified transcriber who is ready to take on the challenge of transcribing print mathematics and science materials into braille using *The Nemeth Braille Code for Mathematics and Science Notation 2022*. The program operates under a contract with the National Library Service for the Blind and Print Disabled, Library of Congress (NLS). All transcribing and proofreading course lessons and tests are administered under the National Federation of the Blind Braille Certification Training Program (BCTP). The following information is copied from the cover letter that the enrolled student will receive when accepted into the course. Up-to-date instructions will be sent at the time of enrollment.

The course is based on *The Nemeth Braille Code for Mathematics and Science Notation 2022* which is a publication of the Braille Authority of North America (BANA). Course materials do not supersede the authority of the official BANA code book.

### *Eligibility*

- United States citizenship or residency
- High school diploma or equivalent
- Knowledgeable in recommended braille formats for textbooks

### *Prerequisite*

- Library of Congress certification in literary braille transcribing (UEB) for a minimum of six months

### *Equipment*

Any of the following methods may be used in order to submit lesson exercises in braille: a forty-cell slate, a braillewriter, or a computer application that allows for direct input of 6-key braille. Use of back translation is allowed but 6-key entry is a necessity as well. A line length of forty cells is required, regardless of production method.

### *The Lesson Material*

The lessons are available by following the "Mathematics Braille Transcribing" link at [www.nfb.org/transcribers](http://www.nfb.org/transcribers). If you are unable to utilize the material from the website, please contact us at [transcribers@nfb.org](mailto:transcribers@nfb.org) or (410) 659-9314, extension 2510, and we will work with you to ensure you receive the material in a format that is usable for you.

Most lessons conclude with an exercise which is to be submitted to your grader for evaluation. You are encouraged to submit your work on a regular basis (at least monthly). You may submit only one lesson at a time.

Revisions to the online course lessons are occasionally necessary. The student should check the website to ensure that any lesson being completed is the current version. To report errors in this instruction manual, please send your message to [transcribers@nfb.org](mailto:transcribers@nfb.org).

### *Your Grader*

Students whose goal is to acquire certification in UEB with Nemeth must enroll through the NFB. After receiving your transcription of the first exercise, a grader will be assigned. Your grader will evaluate your first submission and will be your point of contact for the rest of the course.

### *How to Enroll*

Whether intending to take the course with a local teacher or through correspondence, prospective students must submit the application form before beginning the course. Applicants can enroll in the course in one of two ways.

- Complete the application online at <https://nfb.org/programs-services/braille-certification/mathematics-braille-transcribing>
- Or mail a print application to the address shown below.

National Federation of the Blind  
Braille Certification Training Program  
200 East Wells Street at Jernigan Place  
Baltimore, MD 21230

### *The Certification Test*

Upon completion of the course, the student may apply for the certification test. When requesting the test, students who have taken the course locally must include a letter from their Library of Congress certified mathematics braille instructor attesting that the student has successfully completed the course. Certification tests are distributed and evaluated by the National Federation of the Blind. Instructions for preparation and submission will accompany the exam. The Library of Congress is the certifying authority. Candidates scoring a passing grade will receive a Library of Congress certificate.

### *Further Information*

For more information about the braille certification training program, you may email, call, or write:

- email: [transcribers@nfb.org](mailto:transcribers@nfb.org)
- call: 410-659-9314, extension 2510
- write to the address shown above.

## **FOREWORD TO THE 2024 EDITION**

The first edition of the *Introduction to Braille Mathematics* was published in 1978 and was written by the late Helen Roberts and Bernard M. Krebs. It was my privilege to complete the text with Mr. Krebs after Helen passed away. Since that time, numerous corrections and updates have been made both to the Nemeth Code itself and to this manual. Now, however, a major change has necessitated a complete rewriting of the lessons. 2016 was the implementation year in the United States for new transcriptions to be produced using the Unified English Braille Code. Because Nemeth Code works *within* UEB, many of the rules of Nemeth Code must be modified.

After the first lesson most examples, practices, and exercises are shown in a text-like context. In this way, the student can see how the Nemeth Code works in a real setting such as found in texts of many grade levels and complexities.

The practices within each lesson are available for self checking by the student. Answers to the practices are given at the end of each lesson. Braille reading practice is offered in Addendum 1. Most lessons conclude with an exercise which will be sent to you and evaluated by your teacher or by your NFB-assigned grader.

The student should understand that the Nemeth Code itself is the authoritative source for all UEB with Nemeth transcriptions. The student should also be thoroughly familiar with the sourcebooks listed in the PREREQUISITES which follow this Foreword.

It has long been my hope that this manual could be brought into the present era. Lindy Walton, an experienced transcriber who works with the NLS Nemeth certification program, led the writing of this Second Edition. Once again, it is my honor to work with an exceptional member of the braille transcriber community.

Both Lindy and I thank the following for their support and help: Mary Denault, Peggy Jackson, Bill Jackson, Kyle DeJute, Julie Sumwalt, Lynnette Taylor, the members of the BANA Nemeth Code Technical Committee, the Grafton Braille Service Center, and the many students who participated in the beta testing of this course. Lastly, we direct our sincerest gratitude to the National Federation of the Blind which has lent support to the development and publication of this comprehensive manual.

—Barbara Taffet

## **PREREQUISITES**

A prerequisite to the study of UEB with Nemeth is certification in Unified English Braille, adequate experience in literary braille transcription, and confidence in your production method. Before beginning this course of study the student should also be thoroughly familiar with current methods for transcribing a textbook. Rules and guidelines are found in the following sourcebooks, all of which are available from the Braille Authority of North America (BANA) at [www.brailleauthority.org](http://www.brailleauthority.org). Dates shown below are the editions used as a resource in this lesson manual.

*The Rules of Unified English Braille, Second Edition 2013*

*Braille Formats: Principles of Print-to-Braille Transcription 2016*

*The Nemeth Braille Code for Mathematics and Science Notation 2022*

*Guidelines and Standards for Tactile Graphics 2022*

*Chemical Notation Using Nemeth Braille Code 2023*

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