

## **LESSON 10**

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### **LESSON PREVIEW**

The study of spatial arrangements continues with a look at multiplication and division problems. The lesson ends with a review of the rules for use/nonuse of the numeric indicator and a summary of rules regarding signs of grouping.

*NOTE: Code switch indicators are not shown in the isolated Nemeth examples in this lesson.*

## ***SPATIAL ARRANGEMENTS, continued***

Lesson 9 looked at spatially arranged addition and subtraction problems. The list below summarizes format rules presented in that lesson.

### **Review of Format for Spatial Arrangements**

1. A blank line is required above and below a spatial arrangement. It may begin on line 1 and/or end on line 25.
2. Code switch indicators are placed outside of the arrangement.
3. The numeric indicator is not used. (There are some exceptions.)
4. An identifier associated with a spatial arrangement is positioned according to rules applied to the specific type of problem.
5. Side-by-side arrangement is allowed according to certain spacing rules.
6. If a spatial arrangement lies on lines 1-2, any symbol of the arrangement cannot appear within three blank spaces of the first symbol of the print page number on line 1. If a spatial arrangement lies on lines 24-25, any symbol of the arrangement cannot appear within three blank spaces of the first symbol of the braille page number on line 25.
7. Only the general omission symbol is used to show an omission within a spatial arrangement.

## ***SPATIAL ARRANGEMENT WITH MULTIPLICATION***

[NC 25.4]

The parts of a spatial multiplication problem are labeled in this illustration.

$$\begin{array}{r} 2 \\ \times 3 \\ \hline 6 \end{array}$$

*multiplicand*  
*multiplier*  
*separation line*  
*product*

### **10.1 Alignment**

In a spatial arrangement for multiplication, the multiplier and multiplicand must be aligned the same way as in the print copy. Any associated symbols such as dollar signs, commas, and decimal points correspond to the print placement.

### **10.2 Placement of Multiplication Symbol**

In braille, the multiplication symbol must immediately precede the multiplier, regardless of print placement.

The operation sign is not always present. If there is no multiplication sign, examine the surrounding text to determine that this is indeed a multiplication problem. Then apply alignment rules for multiplication.

### 10.3 Separation Line

In braille, the separation line extends one cell to the left and to the right of the longest entry appearing above or below it. If there is more than one separation line in a given arrangement, each must be the same length regardless of the way it is printed.

⠠⠠⠠⠠⠠⠠⠠ Separation Line (varying in length)

#### Example 10-1

$$\begin{array}{r}
 2704 \\
 \times 12 \\
 \hline
 \end{array}$$

*Alignment:* Digits in the multiplier and multiplicand (lines 1 and 2) are vertically aligned the same way as in the print copy. *Placement of Multiplication Symbol:* The multiplication cross is unspaced from the multiplier (line 2) even though this symbol is printed further to the left. *Separation Line:* The separation line extends one cell to the left and to the right of the longest entry appearing above or below it.

#### Example 10-2

$$\begin{array}{r}
 132 \\
 \times 300 \\
 \hline
 39600
 \end{array}$$

*Alignment:* Digits in the multiplier and multiplicand (lines 1 and 2) and in the product (line 4) are vertically aligned the same way as in the print copy. *Placement of Multiplication Symbol:* The multiplication cross is unspaced from the multiplier (line 2) even though this symbol is printed further to the left. *Separation Line:* The separation line extends one cell to the left and to the right of the longest entry appearing above or below it.

#### Example 10-3

$$\begin{array}{r}
 \$421 \\
 \times 6 \\
 \hline
 \$2526
 \end{array}$$

*Spacing and Alignment:* The dollar signs correspond to the print placement.

Example 10-4

$$\begin{array}{r}
 1,623 \\
 \times 5.27 \\
 \hline
 \end{array}$$

*Spacing and Alignment: The commas and decimal points correspond to the print placement.*

**Alignment of Partial Products**

**10.4 Partial Products**

When partial products are shown in a sample problem, note that the final answer (the product) is obtained by adding the partial products. Hence, partial products and the final product (the answer) must be aligned for addition. All of the separation lines in one multiplication problem are the same width and in the same cell column, even though they may not appear this way in print.

2704	<i>multiplicand</i>	
× 12	<i>multiplier</i>	
5408	<i>partial product</i>	}
2704	<i>partial product</i>	
32448	<i>product</i>	
		<i>an addition problem</i>

Example 10-5

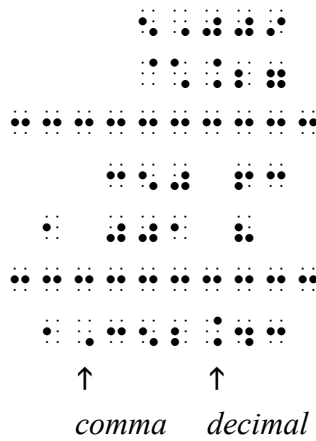
$$\begin{array}{r}
 2704 \\
 \times 12 \\
 \hline
 5408 \\
 2704 \\
 \hline
 32448
 \end{array}$$

- |   |         |  |
|---|---------|--|
| 1 | ⠠⠠⠠⠠    | <i>The multiplicand and the multiplier (lines 1 and 2)</i> |
| 2 | ⠠⠠⠠⠠    | <i>are aligned for multiplication, as printed.</i>         |
| 3 | ⠠⠠⠠⠠⠠⠠⠠ |  |
| 4 | ⠠⠠⠠⠠    | <i>The partial products (lines 4 and 5)</i>                |
| 5 | ⠠⠠⠠⠠    | <i>and the product (line 7) are aligned</i>                |
| 6 | ⠠⠠⠠⠠⠠⠠⠠ | <i>according to the rules for addition,</i>                |
| 7 | ⠠⠠⠠⠠⠠⠠⠠ | <i>that is, by place value.</i>                            |

10.4.1 **Spacing.** If the product contains a comma or a decimal point, the corresponding cells in the partial products above it are left blank. Blank cells are not inserted in the separation lines.

*Example 10-6*

$$\begin{array}{r}
 5,009 \\
 \times .27 \\
 \hline
 35063 \\
 10018 \\
 \hline
 1,352.43
 \end{array}$$



*aligned for  
multiplication*

*The partial products  
and the product  
are aligned  
for addition.*

*Instructions:* Use side-by-side layout, leaving one blank space between the end of one separation line and the beginning of the next. Include switch indicators in your transcription. Review side-by-side format rules in Lesson 9.

**PRACTICE 10A**

$$\begin{array}{r}
 \$98 \\
 \times 100 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \$33 \\
 \times 200 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 12.12 \\
 \times 15.3 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 7,165 \\
 \times 85 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \$98 \\
 \times 100 \\
 \hline
 \$9,800
 \end{array}$$


$$\begin{array}{r}
 \$33 \\
 \times 200 \\
 \hline
 \$6,600
 \end{array}$$

$$\begin{array}{r}
 12.12 \\
 \times 15.3 \\
 \hline
 3636 \\
 6060 \\
 1212 \\
 \hline
 185.436
 \end{array}$$

$$\begin{array}{r}
 7,165 \\
 \times 85 \\
 \hline
 35825 \\
 57320 \\
 \hline
 609,025
 \end{array}$$

## 10.5 Omissions in Spatial Multiplication Problems

As noted in Lesson 9, only the general omission symbol is used to show an omission within a spatial arrangement, regardless of the print sign used. The one-cell symbol allows the transcription to maintain alignment.

 <span style="margin-left: 20px;">General Omission Symbol</span>
---

### Example 10-7

148	⠠⠠⠠
× 15	⠠⠠⠠⠠
-----	⠠⠠⠠⠠⠠⠠
7□0	⠠⠠⠠⠠
□48	⠠⠠⠠
-----	⠠⠠⠠
□2□0	⠠⠠⠠⠠⠠⠠
	⠠⠠⠠⠠

## 10.6 Fractions and Mixed Numbers

In a multiplication problem which contains fractions and mixed numbers, the various parts of the fractions are aligned vertically, both above and below the separation line, as in an addition or subtraction problem. You may wish to review Section 9.22 in Lesson 9.

### Example 10-8

11	⠠⠠⠠⠠
-----	⠠⠠⠠⠠
12	⠠⠠⠠⠠
× 3	⠠⠠⠠⠠
-----	⠠⠠⠠⠠
4	⠠⠠⠠⠠⠠⠠

*The fraction indicators and the fraction lines are vertically aligned. Numerators and denominators touch the fraction line.*

### Example 10-9

1 1/2	⠠⠠⠠⠠⠠⠠
-----	⠠⠠⠠⠠
× 3/8	⠠⠠⠠⠠
-----	⠠⠠⠠⠠
	⠠⠠⠠⠠⠠⠠

*The fraction indicators align, including the fractional part of the mixed number. The multiplication cross touches the multiplier according to rules regarding spatial multiplication.*

**Example 10-10**

54	⠠⠠⠠⠠⠠
× 2	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
$\frac{3}{4}$	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
40	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
$\frac{1}{2}$	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
108	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
148	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
$\frac{1}{2}$	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

*Place values and indicators are aligned throughout.*

**10.7 Polynomials**

In a multiplication problem which contains polynomials, the multiplicand and multiplier (above the first separation line) are aligned as in the print copy. The partial products and final product are aligned following rules for addition.

**Example 10-11**

8r + 9s	⠠⠠⠠⠠⠠⠠⠠
5r - 6s	⠠⠠⠠⠠⠠⠠⠠
40r <sup>2</sup> + 45rs	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
- 48rs - 54s <sup>2</sup>	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
40r <sup>2</sup> - 3rs - 54s <sup>2</sup>	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

1	⠠⠠⠠⠠⠠⠠⠠
2	⠠⠠⠠⠠⠠⠠⠠
3	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
4	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
5	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
6	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
7	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

*Lines 1-2: The multiplicand and multiplier are aligned as printed.*  
*Lines 4-7: The partial products and the product are aligned for addition.*  
*Lines 4-7: Terms, operation signs, superscript indicators, and baseline indicators are aligned.*

## 10.8 Subscripts Denoting Nondecimal Bases

In arrangements which show multiplication of nondecimal bases, the subscript indicators are vertically aligned. The rightmost partial product sets the location of this alignment. This may differ from the spacing shown in the print copy.

### Example 10-12

$$\begin{array}{r}
 54_{\text{eight}} \\
 \times 23_{\text{eight}} \\
 \hline
 204_{\text{eight}} \\
 130_{\text{eight}} \\
 \hline
 1504_{\text{eight}}
 \end{array}$$

*To ensure that the subscript indicators align, a space is inserted in the second partial product.*

## 10.9 Regrouping Numbers with Multiplication

⠠⠠⠠⠠⠠⠠	Regrouping Indicator for Numbers Above the Arrangement (varying in length)
⠠⠠⠠⠠⠠⠠	Regrouping Indicator for Numbers Below the Arrangement (varying in length)

If regrouping numbers are shown, follow the rules for regrouping numbers with addition as outlined in Lesson 9. This indicator is transcribed whether or not the line appears in the print copy. It is inserted between the regrouping numbers and the multiplicand and is one cell wider than the separation line. The regrouping number is placed in the same columnar position as in print.

### Example 10-13

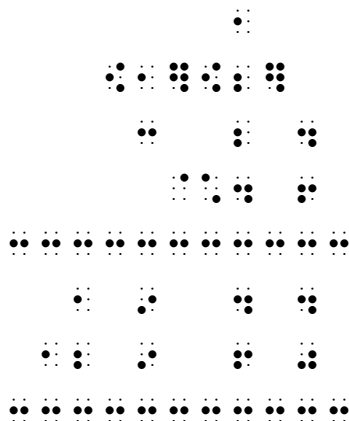
$$\begin{array}{r}
 \overset{1}{5},\overset{15}{319} \\
 \times 6 \\
 \hline
 31,914
 \end{array}$$



- a. When the multiplier consists of more than one digit, the regrouping numbers might be canceled and replaced. Recall that regrouping indicators are not needed when numbers are canceled. Review Section 9.25 in Lesson 9.

*Example 10-14*

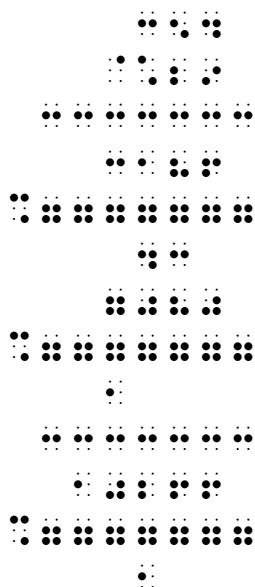
$$\begin{array}{r}
 \overset{1}{\cancel{1}} \\
 324 \\
 \times 46 \\
 \hline
 1944 \\
 12960 \\
 \hline
 \end{array}$$



- b. When the multiplier consists of more than one digit, the regrouping numbers might be written within the partial products and product area. Determine whether the regrouping numbers are written above or below each digit to select the appropriate regrouping indicator – "above" or "below."

*Example 10-15*

$$\begin{array}{r}
 354 \\
 \times 29 \\
 \hline
 3186 \\
 \overset{4}{\underset{3}{}} \\
 7080 \\
 \overset{1}{\underset{1}{}} \\
 \hline
 10266 \\
 \underset{1}{\phantom{0}}
 \end{array}$$



*The appropriate regrouping indicator is inserted between the regrouping numbers and the line in the arrangement to which they apply.*

## 10.10 Placement of Identifiers with Spatial Multiplication

An identifier, if present, is placed on the first line of the multiplication problem (the multiplicand) regardless of its placement in print. If there are regrouping numbers, the identifier is still placed on the line with the multiplicand.

### Example 10-16

$$\begin{array}{r}
 1. \quad 19 \\
 \times 6 \\
 \hline
 54
 \end{array}
 \qquad
 \begin{array}{r}
 2. \quad \begin{array}{c} 15 \\ 319 \end{array} \\
 \times 6 \\
 \hline
 1914
 \end{array}$$

*Reminders: One blank space comes between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines. No symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.*

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*Instructions:* Use side-by-side layout. Include code switch indicators in your transcription.

**PRACTICE 10B**

$$\begin{array}{r} 1) \quad \frac{3}{4} \\ \times \frac{5}{8} \\ \hline 15 \\ \hline 32 \end{array}$$

$$\begin{array}{r} 2) \quad 9\frac{3}{4} \\ \times 4\frac{7}{12} \\ \hline \end{array}$$

$$\begin{array}{r} 3) \quad \overset{3}{9}\overset{3}{9}\overset{3}{9} \\ \times 4 \\ \hline 3,996 \end{array}$$

$$\begin{array}{r} 4) \quad \quad \quad 3p + 6q \\ \quad \quad \quad 11p - 2q \\ \hline 33p^2 + 66pq \\ \quad - 6pq - 12q^2 \\ \hline 33p^2 + 60pq - 12q^2 \end{array}$$

## ***DIVISION PROBLEMS***

[NC 25.5]

### **10.11 Notation Devices**

It will be helpful to refer to the parts by using the following standard terminology. In this sample ("forty divided by ten equals 4"), 40 is the *dividend*, 10 is the *divisor*, and 4 is the *quotient*.

$$\begin{array}{r} \textit{quotient} \\ \textit{divisor} \overline{) \textit{dividend}} \end{array} \qquad 10 \overline{) 40}^4$$

Other print styles which can be found in various publications are illustrated below using the same numbers for dividend, divisor, and quotient. Each style includes a division sign (either straight, slanted, or curved) and a separation line. The quotient can be printed above, below, or next to the dividend. Any one publication will likely use only one style throughout a document.

$$10 \overline{) 40}^4 \qquad 10 \overline{) 40} (4 \qquad 10 \overline{) 40}^4 \qquad 10 \overline{| 40} \qquad \underline{10} \overline{| 40}^4$$

If there is a *remainder*, it may be shown after the quotient, preceded by the letter R or r. In this sample ("forty-seven divided by ten"), 7 is the remainder.

$$\begin{array}{r} \textit{quotient} \textit{ remainder} \\ \textit{divisor} \overline{) \textit{dividend}} \end{array} \qquad 10 \overline{) 47}^4 \text{ R}7$$

### **10.12 Linear (Nonspatial) Representation of Division Problems**

The Preliminary Lesson introduced linear division problems using the symbol  $\div$  for "divided by", as in  $15 \div 3$  ("fifteen divided by three"). A division problem may also be printed using a curved, straight, or slanted line for the division sign, and a horizontal line printed above or below the divisor.

$$3 \overline{) 15}$$

When only a divisor and dividend are printed, the problem is formatted as a linear construction as long as the divisor and dividend are composed entirely of numerals. (A "numeral" may include decimal points and/or commas.) The print style is replicated by using one of the following symbols between the divisor and dividend. Note that the straight and slanted division signs are represented with the same braille symbol.



Example 10-18

1.  $3\overline{)15}$       2.  $0.3\overline{)30}$       3.  $3\overline{)4.5}$       4.  $30\overline{)9,060}$

1    ⠠⠠

2    ⠠⠠⠠⠠    ⠠⠠⠠⠠⠠⠠

3    ⠠⠠⠠⠠    ⠠⠠⠠⠠⠠⠠⠠⠠

4    ⠠⠠⠠⠠    ⠠⠠⠠⠠⠠⠠

5    ⠠⠠⠠⠠    ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

*Reminder: In nonspatial braille, each identifier begins on a new line in cell 1 even though the problems are arranged side by side in print.*

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*Instructions: Use appropriate margins for embedded, displayed, and itemized problems.*

**PRACTICE 10C**

**Converting Fractions to Decimals**

To write the fraction  $1/2$  as a decimal, you can divide "one point zero divided by two", or  $2\overline{)1.0}$ . Put away your calculator! Using mental math, it's easy! The answer is .5 .

Can you divide *these* fractions in your head?

$1/5: 5\overline{)1.0}$

$4/5: 5\overline{)4.0}$

$9/10: 10\overline{)9.0}$

**Practice**

1. Solve these division problems.

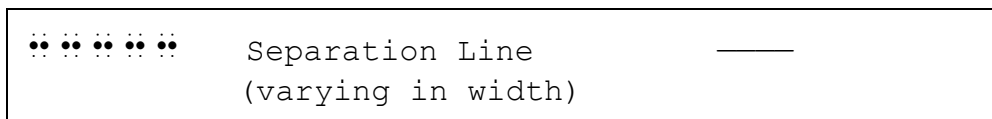
a.  $8\overline{)128}$       b.  $6\overline{)5,304}$

c.  $4\overline{)204}$       d.  $9\overline{)837}$

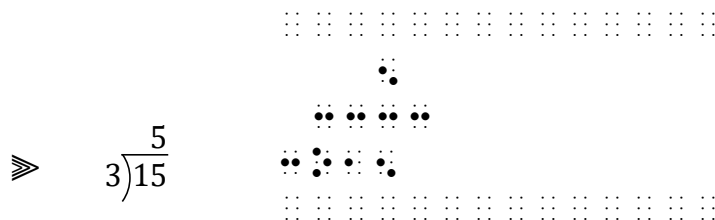
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### 10.13 Spatial Representation of Division Problems

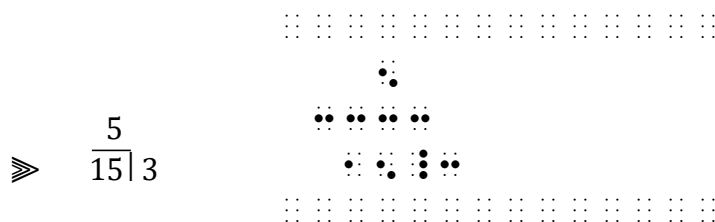
A division problem is transcribed as a spatial (vertical) arrangement if it includes a quotient, partial products and differences, or if there are spaces within the dividend. The quotient is aligned with the dividend as shown in the print copy.



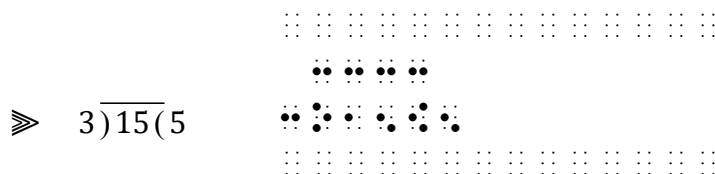
10.13.1 **Width of the Separation Line.** The separation line begins in the column containing a division symbol and ends in the column containing the other division symbol if the latter appears in print. When only one division symbol is printed, the separation line ends in the cell which is one column beyond the overall arrangement. *Reminder:* The division symbol is unspaced from the divisor and the dividend. Several styles of division problems are illustrated below.



*This separation line begins in the cell directly over the (curved) division sign and ends one cell beyond the overall arrangement.*



*This separation line begins one cell beyond the overall arrangement and ends in the cell directly over the (straight) division sign.*



*This separation line begins directly over the "curving right" division sign and ends in the cell directly over the "curving left" division sign. Because a quotient is present, it is transcribed as a spatial arrangement, even though the quotient is printed on the same line as the divisor and the dividend.*

$$\ggg 10 \overline{) 50,000} \begin{array}{r} 5,000 \end{array}$$

*The quotient is aligned with the dividend as shown in the print copy.*

$$\ggg 40 \overline{) \$49.99}$$

*Even though there is no quotient shown, the presence of the dollar sign in the dividend means a spatial arrangement must be constructed.*

$$\ggg \frac{110(10}{11}$$

*This arrangement shows the separation line printed below the dividend, with the quotient printed below that. The separation line begins one cell beyond the overall arrangement and ends in the cell directly under the division sign.*

$$\ggg \underline{10} \overline{) 50,000} \begin{array}{r} 5,000 \end{array}$$

*This arrangement shows a horizontal line printed below the divisor. That line is disregarded in braille.*



10.13.2 **A Quotient with a Remainder.** When a remainder is printed next to the quotient, the capitalized letter R, or uncapitalized letter r, (indicating "remainder") is preceded by a space. When the quotient is arranged above the separation line, the separation line extends one cell to the right of the overall arrangement, including the remainder, even if the line is not printed in this manner.

$$\gg 25 \overline{)452} \begin{matrix} 18 \\ R2 \end{matrix}$$

*Recall from Lesson 6 that a number printed to the right of a letter needs a multipurpose indicator (dot 5) to show that the number is not a subscript.*

The arrangement in braille reflects the print layout. In the next sample, the quotient and remainder are printed to the right of the dividend.

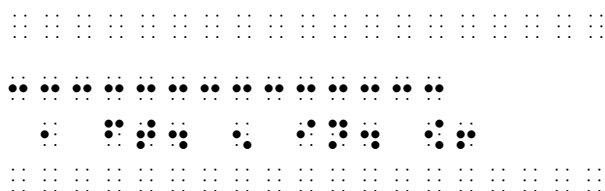
$$\gg 37 \overline{)476 \text{ sq. in.}} (12 \text{ r}32$$

10.13.3 **Abbreviations.** When words or abbreviations appear in the problem, a spatial arrangement is required. When an abbreviation is next to the division symbol, spacing rules for abbreviations next to operation signs are applied. See Lesson 4.

$$\gg 6 \overline{)1 \text{ ft. } 5 \text{ in.}}$$

*The numeric indicator is not used in a spatially arranged problem.*

➤  $\overline{1 \text{ ft. } 5 \text{ in.}} (6$

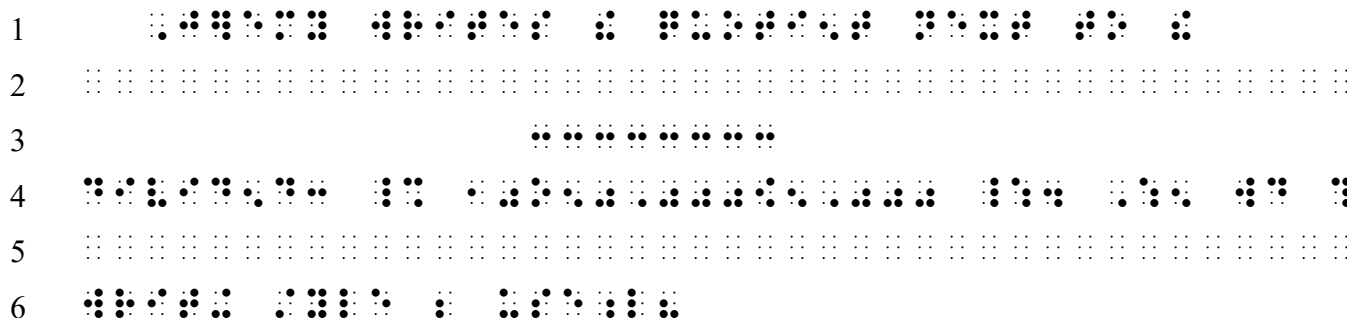


*A space is required between an abbreviation and a sign of operation, in this case "in." and the (curved) division sign.*

10.13.4 **Embedded Division Problem.** A blank line must precede and follow a spatially arranged problem, even when the problem is embedded within narrative.

Example 10-19

Jeremy writes the quotient next to the dividend:  $10 \overline{)50,000} (5,000$ . When would this writing style be useful?



*Lines 2 and 5: Blank lines are required before and after an embedded spatial math expression.*

*Line 4: Alignment: The divisor and dividend are placed on the same line as the text.*

*Line 4: Code switch indicators follow the rules for an embedded math expression.*

*In this example, the two switches fit on the same line as the math.*

10.13.5 **Review of Format for Spatial Arrangements**

- A blank line is required above and below a spatial arrangement.
- The numeric indicator is not used. (There are some exceptions.)
- Side-by-side arrangement is allowed.
- An identifier associated with a spatial problem is positioned according to rules applying to the type of arrangement, not necessarily at the top line of the problem.

10.13.6 **Long Division.** In a long division problem, multiples are written below the dividend. A series of subtraction problems are performed, giving partial remainders and, finally, a remainder. The long division portion of the problem is aligned for subtraction.

$\begin{array}{r} 123 \\ 17 \overline{)2091} \\ 17 \\ \hline 39 \\ 34 \\ \hline 51 \\ 51 \\ \hline 0 \end{array}$	<div style="display: flex; flex-direction: column; gap: 5px;"> <span><i>dividend</i></span> <span><i>multiple (17×1)</i></span> <span><i>partial remainder</i></span> <span><i>multiple (17×2)</i></span> <span><i>partial remainder</i></span> <span><i>multiple (17×3)</i></span> <span><i>remainder</i></span> </div>	}	<i>subtraction problems</i>
---	--	---	-----------------------------

- a. **Alignment.** The components of the problem are aligned in the same manner as they are aligned in the print copy. All separation lines must be the same length, regardless of their relative lengths in the print copy.

Example 10-20

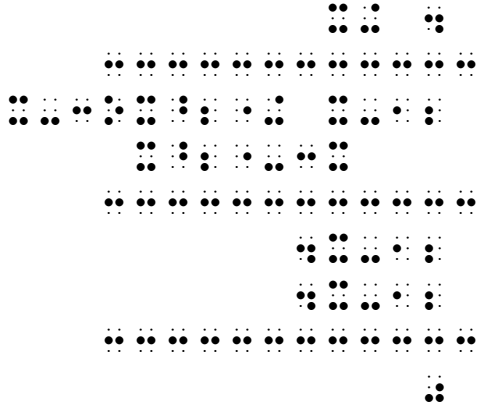
$\begin{array}{r} 123 \\ 17 \overline{)2091} \\ 17 \\ \hline 39 \\ 34 \\ \hline 51 \\ 51 \\ \hline 0 \end{array}$		}	<i>aligned for subtraction</i>
---	--	---	--------------------------------

Example 10-21

$\begin{array}{r} 18r2 \\ 25 \overline{)452} \\ 25 \\ \hline 202 \\ 200 \\ \hline 2 \end{array}$	
--	--

Example 10-22

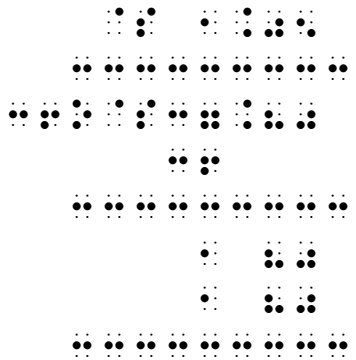
$$\begin{array}{r}
 x + 4 \\
 x - 3 \overline{) x^2 + x - 12} \\
 \underline{x^2 - 3x} \phantom{- 12} \\
 4x - 12 \\
 \underline{4x - 12} \\
 0
 \end{array}$$



- b. **Blank Cells.** When commas or decimal points occur in a dividend, corresponding blank cells should be left throughout the body of the division problem, except in the separation lines.

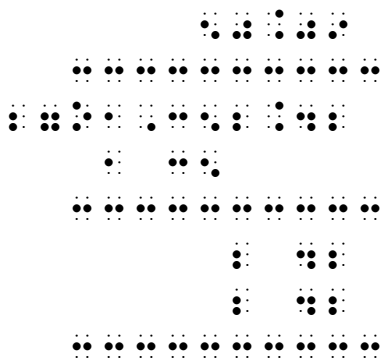
Example 10-23

$$\begin{array}{r}
 \$1.05 \\
 36 \overline{) \$37.80} \\
 \underline{36} \phantom{00} \\
 180 \\
 \underline{180} \\
 0
 \end{array}$$



Example 10-24

$$\begin{array}{r}
 50.09 \\
 27 \overline{) 1,352.42} \\
 \underline{135} \phantom{00} \\
 242 \\
 \underline{242} \\
 0
 \end{array}$$



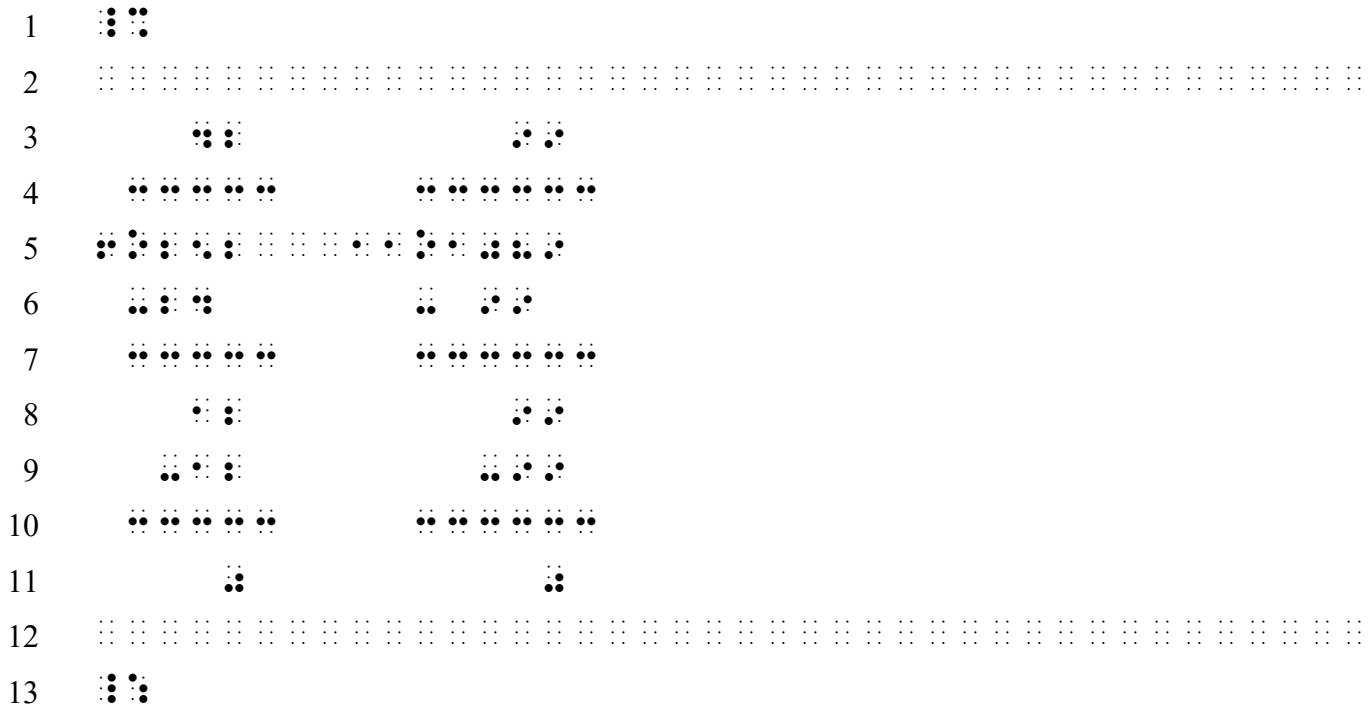
- c. **Caret.** If a caret occurs in a dividend, corresponding blank cells are left throughout the body of the division problem, except in the separation lines. Two cells are required for the caret. The decimal point replacing the caret in the quotient is aligned in the right-hand cell.



- d. **Minus Sign.** A minus sign appearing in the long division portion of the problem is placed according to the rules of spatially arranged subtraction problems—one cell to the left of the leftmost numeric symbol that appears above its separation line. If a minus sign appears on the first line below the dividend, the symbol will be placed in the same column as the division symbol. In that case, the following separation line will begin in the same column.

Example 10-26

$$\begin{array}{r} 42 \\ 6 \overline{)252} \\ -24 \\ \hline 12 \\ -12 \\ \hline 0 \end{array} \qquad \begin{array}{r} 99 \\ 11 \overline{)1089} \\ -99 \\ \hline 99 \\ -99 \\ \hline 0 \end{array}$$



*Line 5: According to the rules regarding side-by-side spatial arrangements, three blank cells come between any symbol on any line of the first problem and any symbol on any line of the second problem, not counting separation lines.*

- e. **Intentional Misalignment.** If the quotient has been intentionally misaligned as an exercise for the student, the same misalignment is shown in the transcription.

Example 10-27

Find the mistake.

$$\begin{array}{r}
 109 \\
 15 \overline{)1635} \\
 \underline{15} \\
 135 \\
 \underline{135}
 \end{array}$$

*The leftmost character in the spatial arrangement is placed in the appropriate cell for displayed material (cell 3).*

---

*Instructions:* Treat the "seven other ways" problems as side-by-side displayed problems. Place each dividend in the same row across the braille line.

**PRACTICE 10D**

Here are seven other ways to write  $11 \overline{)2233}^{203}$  or "2233 divided by 11 equals 203".

$$\frac{203}{2233} \overline{)11}$$

$$11 \overline{) \frac{203}{2233}}$$

$$11 \overline{)2233}^{203}$$

$$\frac{2233}{203} \overline{)11}$$

$$\frac{2233}{203} \overline{)11}$$

$$11 \overline{)2233}^{203}$$

$$11 \overline{)2233}^{203}$$

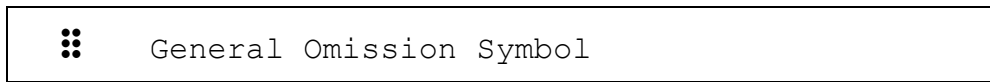
What is  $11 \overline{)2234}$ ? Use long division.

$$\begin{array}{r} 203 \text{ R}1 \\ 11 \overline{)2234} \\ \underline{22} \\ 34 \\ \underline{33} \\ 1 \end{array}$$



## 10.14 Omissions in Spatial Division Problems

As noted in Lesson 9, only the general omission symbol is used to show an omission within a spatial arrangement, regardless of the print sign used. The one-cell symbol allows the transcription to maintain alignment.



### Example 10-28

$\begin{array}{r} \square \\ 7 \overline{)49} \end{array}$	
--	---

### Example 10-29

$\begin{array}{r} ?? \\ 6 \overline{)108} \end{array}$	
--	---

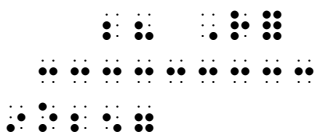
### Example 10-30

$\begin{array}{r} \square \\ 6 \overline{)108} \end{array}$	
---	--

*Recall from Section 9.19.2 in Lesson 9 that when the print omission sign spans more than one column in the arrangement, the general omission symbol aligns beneath the rightmost character of the omission.*

- a. When a remainder is to be filled in, a multipurpose indicator is not needed following the letter "R" because the general omission symbol is nonnumeric, even though it may represent a numeral.

### Example 10-31

$\begin{array}{r} 28 \text{ R} \square \\ 9 \overline{)257} \end{array}$	
--	---

## 10.15 Regrouping in Division

The regrouping indicator is transcribed above or below the dividend, depending on the position of the separation line. The first cell of each regrouping indicator tells whether to read the numbers above or below the line. The regrouping indicator is one cell longer on the left than the separation line. A blank space is left in the dividend and in the quotient where necessary to accommodate a regrouping number. No blank spaces are left in the regrouping line.

- a. When the separation line is printed above the dividend, the regrouping indicator for numbers *below* the arrangement is transcribed below the dividend.

$$\ggg \begin{array}{r} 24 \\ 3 \overline{) 712} \end{array}$$

- b. When the separation line is printed below the dividend, the regrouping indicator for numbers *above* the arrangement is transcribed above the dividend.

$$\ggg \begin{array}{r} 4) 9^1 5^3 0.^2 0 \\ \underline{237.5} \end{array}$$

Example 10-32

Find the error in this division problem. Can you correct it? Check your answer.

$$\begin{array}{r} 767R4 \\ 7 \overline{) 53_4 8_5 3} \end{array}$$

1

2

3

4

5

6

7

8

9

10

## 10.16 Cancellation in Long Division

Cancellation was introduced in Lesson 9.

⠠	Opening Cancellation Indicator
⠡	Closing Cancellation Indicator

If cancellation is shown in long division, the canceled material is enclosed in braille cancellation indicators. Blank cells are left where necessary for proper alignment. The canceled material must exactly represent what is canceled in print.

*Example 10-33*

$  \begin{array}{r}  \phantom{65} \overset{5}{78} \cancel{4} \\  65 \overline{) 51000} \\  \underline{455} \\  550 \\  \underline{520} \\  300 \\  \underline{260} \\  \cancel{40}  \end{array}  $	$  \begin{array}{r}  \phantom{65} \overset{5}{\text{⠠⠠⠠⠠⠠}} \text{⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠} \\  \phantom{65} \text{⠠⠠⠠⠠} \\  \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠}  \end{array}  $
--	---

Example 10-34

$$\begin{array}{r}
 \phantom{x+2} \phantom{)} x - 5 \\
 x + 2 \overline{) x^2 - 3x - 10} \\
 \underline{-x^2 - 2x} \phantom{-10} \\
 \phantom{-x^2 -} -5x - 10 \\
 \underline{+5x + 10} \\
 \phantom{-x^2 - 5x -} 0
 \end{array}$$

1		⠠⠠⠠	⠠⠠	⠠⠠
2	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠			
3	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠	⠠⠠	⠠⠠⠠
4	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠	⠠⠠	
5	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠			
6		⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠	⠠⠠⠠
7		⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠	⠠⠠⠠
8	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠			
9				⠠⠠

- Line 1: "x-5" is aligned as printed.
- Line 3: The division sign is unspaced from the divisor.
- Line 4: The minus sign is transcribed in the same column (cell) as the division sign.
- Line 6: Space inserted above cancellation indicators.
- Line 7: Aligned for subtraction.
- Line 9: "0" is aligned by place value.

a. When aligning parts of a polynomial problem with cancellation, do not separate a coefficient from its variable, as illustrated in this portion extracted from a long division problem.

		⠠⠠⠠⠠⠠⠠
		⠠⠠⠠⠠⠠⠠
⠠⠠⠠	$  \begin{array}{r}  3x^2 \\  -2x^2 \\  \hline  x^2 \\  -x^2 \\  \hline  \end{array}  $	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
		⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
		⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
		⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

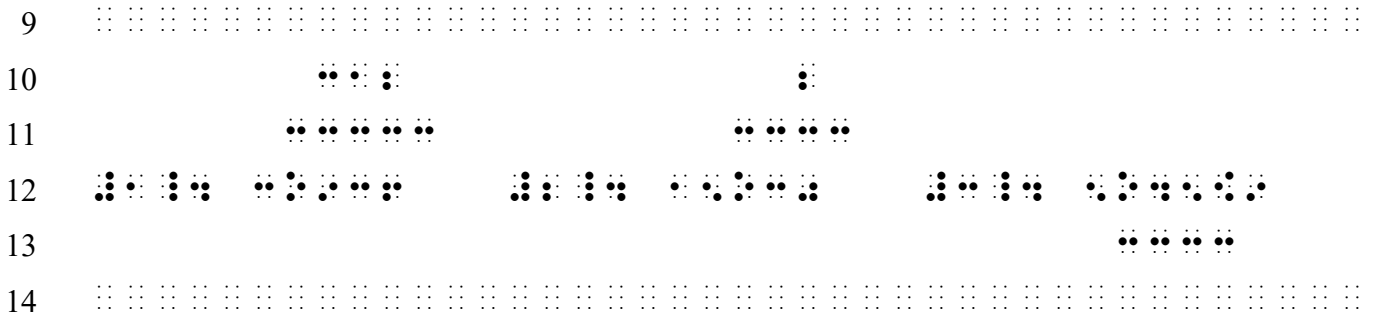
Lines 4-5: A space is inserted where the coefficients lie above the separation line.

**10.17 Placement of Identifiers with Spatial Division**

An identifier, if present, is placed on the line with the dividend. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines. When arranged side by side across the width of the page, no symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.

Example 10-35

1.  $3 \overline{) 312}$       2.  $15 \overline{) 30} \frac{2}{}$       3.  $5 \overline{) 45} (9$



**10.18 Other Layouts**

Other types of division problems will be taught in a later lesson.

*Instructions:* Do not use side-by-side layout in this transcription.

**PRACTICE 10E**

Fill in the boxes with the correct numeral.

(a)  $6 \overline{) 23^5 5^1 813} R \square$       (b)  $25 \overline{) 80} \frac{3 \frac{1}{5}}{\frac{5}{25}}$       (c)  $4 \overline{) 19} R \square$

75  
□



- (e)  $3 * 9 = 27$                      ⠠⠚ ⠠⠨ ⠠⠷ ⠠⠨ ⠠⠢ ⠠⠠⠨ ⠠⠠⠢ ⠠⠠⠠
- (f) **183**                                     ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- (g) **14625**                               ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠

10.19.2 **Nonuse of the Numeric Indicator.** [NC Rule 3.4] A numeric indicator is not needed in the following circumstances, as long as the numeral is in regular type.

- h. when a numeral is unspaced from and follows a mathematical character or a slash.
- i. when a numeral is unspaced from and follows a Nemeth indicator.
- j. when a numeral is unspaced from and follows a left grouping sign.
- k. when a numeral follows a hyphen which follows a numeral or other mathematical symbol.
- l. when a numeral in regular type is part of an "enclosed list".
- m. when a numeral is partitioned into segments with spaces.
- n. when a numeral is part of a spatially arranged addition, subtraction, multiplication, or division problem.

The following samples illustrate points h-n.

- (h)  $-7 + 12 - 3 = +2$                    ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- \$1.5K-\$2.5B                     ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠
- 2/3                                   ⠠⠠⠠ ⠠⠠⠠
- (i)  $x^2 - 2 \text{ R}5$                          ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- $\frac{3}{4} \sqrt[3]{8}$                            ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- (j)  $(-4 \text{ and } -5)$                      ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- $(0 = x)$                            ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- $(7\%)$                                ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- (k)  $3.4-3.8 \text{ cc}$                      ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- 80%-90%                         ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- (l)  $\{.5, .7, .9, 1.1\}$                  ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- (m)  $987 \text{ } 656 \text{ } 000$                  ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- (n)         53                               ⠠⠠⠠
- +51                               ⠠⠠⠠ ⠠⠠⠠
- 
- 104                               ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠
- ⠠⠠⠠

## 10.20 Review of Rules for Signs of Grouping

A review of the cited sections is recommended.

- a. Inside the switches, grouping symbols (parentheses, brackets, braces) are not considered to be punctuation; they are signs of grouping. [2.9]
- b. The numeric indicator is not used before a numeral which immediately follows a grouping symbol. [2.10]
- c. Nemeth grouping symbols are punctuated mathematically. [2.12.1]
- d. Inside the switches, grouping symbols are transcribed using the Nemeth symbols even when they have no mathematical meaning. [2.12.a]
- e. When an isolated math symbol is enclosed within parentheses, brackets, or braces, the paired grouping symbols are transcribed in Nemeth. [2.12]
- f. Paired grouping symbols must be transcribed in the same code. [2.13]
- g. The English-letter indicator is not used when a single English letter or a Roman numeral is entirely enclosed within signs of grouping. [3.12.2, 4.6] This rule applies to mathematical "single letters" only, not to single-letter abbreviations. [4.21]
- h. The appropriate alphabetic indicator must be used with any letter from the German, Greek, Hebrew, or Russian alphabets. [4.10.1] This rule applies even when non-English letters are enclosed within, or in contact with, signs of grouping.
- i. A sequence of mathematical items enclosed in signs of grouping (an "enclosed list") must use the Nemeth grouping symbols. [4.17]
- j. When a single English letter or a Roman numeral is in direct contact with only one sign of grouping, the English-letter indicator is or is not used as though the grouping sign was not present. If the grouping sign carries a prime or other modifying symbol, the English-letter indicator is not used. [4.18]
- k. When a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed before the sign of grouping. If no subscript indicator is used, a return to the baseline is implied. [6.21]
- l. When brackets or vertical bars are printed in mathematically significant boldface, dots 456 are used before the grouping symbol. [7.9.3]

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*For further practice, see Addendum 1—Reading Practice.*

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Submit Exercise 10 to your instructor.
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PRACTICE 10B

1  
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## PRACTICE 10C

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The Braille characters represent arithmetic problems and their solutions. The problems are arranged in a grid, with some starting on new lines in cell 3. The characters are arranged in a regular pattern of groups, representing numbers and operations in a specific Braille notation.

*Lines 14-17: Each subdivision begins on a new line in cell 3 even though the problems are arranged side by side in print.*

## PRACTICE 10D

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*Lines 2 and 6 are blank because a spatial arrangement must be preceded and followed by a blank line, even when embedded within a narrative paragraph.*

*Line 5: The code switch indicators are placed on the main line of the embedded math problem.*

*Lines 10 and 15: The leftmost symbol of the first arrangement in each row is placed in cell 3 because it is displayed to a 3-1 paragraph.*

*Lines 9, 11, and 14: Three blank cells between symbols in neighboring arrangements is the minimum number allowed (separation lines excluded).*

*Line 18: The Nemeth Code terminator is placed in cell 1 following the blank line after the completion of the spatially arranged material.*

*Line 19: The next paragraph begins on a new line.*



