*AN INTRODUCTION TO BRAILLE MATHEMATICS USING UEB WITH NEMETH*
*A Course for Transcribers*

# LESSON 14

- FUNCTION NAMES AND THEIR ABBREVIATED FORMS

*Spatial Arrangements, continued*

- SQUARE ROOT DIVISION
- OTHER PRINT LAYOUTS SHOWING DIVISION

Answers to Practice Material

## LESSON PREVIEW

Rules regarding function names and their abbreviated forms are presented. Many examples are shown. The study of spatial arrangements continues with other forms of division problems: square root division, partial quotient layout, synthetic division, and others.

# *FUNCTION NAMES AND THEIR ABBREVIATED FORMS*

**[NC Rule 18]**

## 14.1 List of Common Function Names and Their Abbreviated Forms

The most common function names and their abbreviated forms are listed below. Function names that do not appear in this list are subject to the same rules taught in this lesson. Note that abbreviated function names are printed in regular type.

| Function Name | Abbreviated Form |
|---|---|
| amplitude | amp |
| antilogarithm | antilog |
| arc | arc |
| argument | arg |
| cologarithm | colog |
| cosine | cos |
| hyperbolic cosine | cosh |
| cotangent | cot |
| hyperbolic cotangent | coth |
| coversine | covers |
| cosecant | csc |
| hyperbolic cosecant | csch |
| cotangent | ctn |
| hyperbolic cotangent | ctnh |
| determinant | det |
| error function | erf |
| exponential | exp |
| exsecant | exsec |
| gradient | grad |
| haversine | hav |
| imaginary part | im |
| infimum | inf |
| limit | lim |
| upper limit | $\overline{\mathrm{lim}}$ or $\overline{\mathrm{limit}}$ |
| lower limit | $\underline{\mathrm{lim}}$ or $\underline{\mathrm{limit}}$ |
| natural logarithm | ln |
| logarithm | log |
| maximum | max |
| minimum | min |
| modulo | mod |
| real part | re |
| secant | sec |

| | |
|---|---|
| hyperbolic secant | sech |
| sine | sin |
| hyperbolic sine | sinh |
| supremum | sup |
| tangent | tan |
| hyperbolic tangent | tanh |
| versine | vers |

## 14.2 Code Switching and Punctuation

14.2.1 **Function Names in UEB.** A fully spelled-out function name which is not associated with a math term is transcribed in UEB. Appropriate contractions and punctuation are used.

*Example 14-1*

Consider the Law of Sines.

⠠⠒⠎⠊⠙⠻ ⠮ ⠠⠇⠁⠺ ⠷ ⠠⠎⠔⠑⠎⠲

*"Sines" is not associated with a mathematical item and so is transcribed in UEB, using appropriate contractions.*

*Example 14-2*

Some trigonometric functions are sine, cosine, and tangent.

⠠⠐⠎ ⠞⠗⠊⠛⠕⠝⠕⠍⠑⠞⠗⠊⠉ ⠋⠥⠝⠉⠰⠝⠎ ⠜⠑ ⠎⠔⠑⠂
⠉⠕⠎⠔⠑⠂ ⠯ ⠞⠁⠝⠛⠢⠞⠲

14.2.2 **Function Names in Nemeth.** A fully spelled-out function name which is associated with a term or which otherwise occurs in mathematical context is transcribed in Nemeth, without contractions. A fully spelled-out function name used in conjunction with an abbreviated function name is a mathematical term and a switch to Nemeth is required.

| | | |
|---|---|---|
| ⋙ | sine $\alpha$ | ⠎⠊⠝⠑ ⠨⠁ |
| ⋙ | logsine | ⠇⠕⠛⠎⠊⠝⠑ |
| ⋙ | arc sin | ⠁⠗⠉ ⠎⠊⠝ |

*Example 14-3*

What is the meaning of logsine?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠍⠂⠝⠬ ⠷ ⠸⠩ ⠇⠕⠛⠎⠊⠝⠑ ⠸⠱⠦

*"log" is an abbreviated function name, therefore "logsine" requires a switch to Nemeth.*

14.2.3 **Abbreviated Function Names.** The abbreviated forms of function names are mathematical items and are transcribed following the rules of the Nemeth Code. An abbreviated function name is punctuated mathematically inside the switches.

*Example 14-4*

Some trigonometric functions are sin, cos, and tan.

⠠⠐⠎ ⠞⠗⠊⠛⠕⠝⠕⠍⠑⠞⠗⠊⠉ ⠋⠥⠝⠉⠰⠝⠎ ⠜⠑ ⠸⠩ ⠎⠊⠝⠠
⠉⠕⠎⠠ ⠠⠄⠯ ⠞⠁⠝ ⠸⠱⠲

*The abbreviated function names are punctuated in mathematical mode.*

*Example 14-5*

The abbreviated form of "logarithm" is "log".

⠠⠮ ⠁⠆⠗⠑⠧⠊⠁⠞⠫ ⠿⠍ ⠷ ⠦⠇⠕⠛⠜⠊⠹⠍⠴ ⠊⠎
⠸⠩ ⠦⠇⠕⠛⠸⠴ ⠸⠱⠲

*The fully spelled-out function name is in UEB. Its abbreviated form is in Nemeth. A punctuation indicator is required before the closing quotation mark because an abbreviated function name is a mathematical term.*

*Example 14-6*

The inverse sine function is written $\sin^{-1}$ .

⠠⠮ ⠔⠧⠻⠎⠑ ⠎⠔⠑ ⠋⠥⠝⠉⠰⠝ ⠊⠎ ⠺⠗⠊⠞⠞⠢
⠸⠩ ⠎⠊⠝⠘⠤⠂ ⠸⠱⠲

14.2.4 **"Arc" in Context.** "Arc" can be a function name, an abbreviated function name, or a word referring to a curve.

*Example 14-7*

What is the arc sine function?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠜⠉ ⠎⠔⠑ ⠋⠥⠝⠉⠰⠝⠦

*The two-part function name "arc sine" is transcribed in UEB because "sine" is fully spelled-out.*

*Example 14-8*

What is the arc sin function?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠸⠩ ⠁⠗⠉ ⠎⠊⠝ ⠸⠱ ⠋⠥⠝⠉⠰⠝⠦

*The two-part function name "arc sin" is transcribed in Nemeth because it contains the abbreviated function name "sin".*

*Example 14-9*

ACB is a major arc in Circle O.

⠸⠩ ⠠⠁⠠⠉⠠⠃ ⠸⠱ ⠊⠎ ⠁ ⠍⠁⠚⠕⠗ ⠜⠉ ⠔⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠠⠉⠊⠗⠉⠇⠑ ⠠⠕⠲

*The word "arc" is referring to the curve ACB and so is transcribed in UEB.*

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**PRACTICE 14A**

1. "sin θ" is pronounced "sine theta".
2. "Arcsin" is the "inverse sine".
3. cosine 45° = 0.7071 or $1/\sqrt{2}$
4. The logsine function is related to the logcosine function by $S_n = 2C_n$.

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## 14.3 Spacing of Function Names

Within a mathematical expression, the following spacing rules are observed. These rules apply regardless of the spacing used in the print copy.

a. No space comes before a function name unless it follows a sign of comparison or other symbol that requires spacing.

b. A space is required after a function name or its inverse (the space follows the superscript). There is one exception – see Section 14.3.2.

| | |
|---|---|
| ⋙ cos 20° | ⠉⠕⠎ ⠼⠆⠴⠘⠨⠡ |
| ⋙ 3 cos 20° | ⠼⠒⠉⠕⠎ ⠼⠆⠴⠘⠨⠡ |
| ⋙ sin θ | ⠎⠊⠝ ⠨⠹ |
| ⋙ $i$ sin θ | ⠊⠎⠊⠝ ⠨⠹ |
| ⋙ tan $(x)$ | ⠞⠁⠝ ⠷⠭⠾ |
| ⋙ $\tan^{-1}(x)$ | ⠞⠁⠝⠘⠤⠂ ⠷⠭⠾ |
| ⋙ sine $\alpha$ − sine $\beta$ | ⠎⠊⠝⠑ ⠨⠁⠤⠎⠊⠝⠑ ⠨⠃ |
| ⋙ $f(x) = \sin\,(x)$ | ⠋⠷⠭⠾ ⠨⠅ ⠎⠊⠝ ⠷⠭⠾ |

*Example 14-10*

For any angle $\theta$, $\sin(\theta + 360°) = \sin\theta$ and $\cos(\theta + 360°) = \cos\theta$.

⠠⠿ ⠁⠝⠽ ⠁⠝⠛⠇⠑ ⠸⠩ ⠨⠹⠠
⠎⠊⠝ ⠷⠨⠹⠬⠒⠖⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠎⠊⠝ ⠨⠹ ⠠⠄⠯
⠉⠕⠎ ⠷⠨⠹⠬⠒⠖⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠉⠕⠎ ⠨⠹ ⠸⠱⠲

*In print, there is no space before each opening parenthesis. In braille, a space is required following each function name.*

*Example 14-11*

sin(35°) = Opposite/Hypotenuse

⠸⠩
⠎⠊⠝ ⠷⠒⠢⠘⠨⠡⠐⠾ ⠨⠅ ⠠⠕⠏⠏⠕⠎⠊⠞⠑⠸⠌⠠⠓⠽⠏⠕⠞⠑⠝⠥⠎⠑
⠸⠱

*In print, there is no space between sin and (35°). In braille, a space is required following the function name.*

14.3.1 **Spacing with Operation Symbols.** In braille, an operation symbol is usually unspaced from the symbols which precede and follow it. However, when a function name is followed by an operation symbol, a space is required.

⫸ $\tan\cdot\sin$

⠞⠁⠝ ⠡⠎⠊⠝

*"tan" is followed by a space. A space is not required before "sin".*

⫸ $1/\cos-\cos=\tan\cdot\sin$

⠼⠂⠸⠌⠉⠕⠎ ⠤⠉⠕⠎ ⠨⠅ ⠞⠁⠝ ⠡⠎⠊⠝

*Each function name is followed by a space.*

14.3.2 **Spacing with Indicators.** A space is not inserted between a function name and an indicator which applies to it.

⫸ $\frac{1}{\cos}$ ⠹⠂⠌⠉⠕⠎⠼

*Example 14-12*

**Reciprocal Functions:** $\frac{1}{\cos}-\cos=\tan\cdot\sin$

⠘⠂⠠⠗⠑⠉⠊⠏⠗⠕⠉⠁⠇ ⠘⠂⠠⠋⠥⠝⠉⠰⠝⠎⠒
⠸⠩ ⠹⠂⠌⠉⠕⠎⠼⠤⠉⠕⠎ ⠨⠅ ⠞⠁⠝ ⠡⠎⠊⠝ ⠸⠱

*The abbreviated function name "cos" in the denominator is unspaced from the closing fraction indicator. The expression continues, following other spacing rules of the Nemeth Code.*

*Example 14-13*

$\tan^{-1}\beta=\arctan\beta$

⠸⠩ ⠞⠁⠝⠘⠤⠂ ⠨⠃ ⠨⠅ ⠁⠗⠉⠞⠁⠝ ⠨⠃ ⠸⠱

*The abbreviated function name "tan" is unspaced from the superscript indicator.*

14.3.3 **Examples.** Examine the spacing in the examples below.

*Examples are in Nemeth. The code switch indicators are omitted from the simbraille.*

(1) $\cos\theta=\frac{1}{\sin\theta}$

⠉⠕⠎ ⠨⠹ ⠨⠅ ⠹⠂⠌⠎⠊⠝ ⠨⠹⠼

(2) $\sin(\theta + 90°) = \cos\theta$

⠎⠊⠝ ⠷⠨⠹⠬⠔⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠉⠕⠎ ⠨⠹

(3) $y = 3\tan 2x$

⠽ ⠨⠅ ⠼⠒⠞⠁⠝ ⠼⠆⠭

(4) $a^2 = 2ac\cos\beta + c^2$

⠁⠘⠆ ⠨⠅ ⠼⠆⠁⠉⠉⠕⠎ ⠨⠃⠬⠉⠘⠆

(5) $\sin(\alpha + \beta) + \sin(\alpha - \beta)$

⠎⠊⠝ ⠷⠨⠁⠬⠨⠃⠾⠬⠎⠊⠝ ⠷⠨⠁⠤⠨⠃⠾

(6) $y' = x\cos\phi - y\sin\phi$

⠽⠄ ⠨⠅ ⠭⠉⠕⠎ ⠨⠋⠤⠽⠎⠊⠝ ⠨⠋

(7) $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} = C$

⠼⠆⠜⠭⠻⠎⠊⠝ ⠜⠭⠻⠬⠆⠉⠕⠎ ⠜⠭⠻ ⠨⠅ ⠠⠉

(8) $6\sin 2A\cos 4A$

⠼⠖⠎⠊⠝ ⠼⠆⠠⠁⠉⠕⠎ ⠼⠲⠠⠁

(9) $\cos 203°\csc 203°$

⠉⠕⠎ ⠼⠆⠴⠒⠘⠨⠡⠐⠉⠎⠉ ⠼⠆⠴⠒⠘⠨⠡

(10) $\dfrac{2\sin\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}}$

⠠⠹⠆⠎⠊⠝ ⠹⠨⠁⠌⠆⠼⠠⠌⠆⠉⠕⠎ ⠹⠨⠁⠌⠆⠼⠠⠼

(11) $\ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

⠇⠝ ⠳⠞⠁⠝ ⠷⠹⠨⠏⠌⠲⠼⠬⠹⠭⠌⠆⠼⠾⠳⠬⠠⠉

*"ln" is the abbreviated form of "natural logarithm". Refer to the table at the beginning of this lesson.*

## PRACTICE 14B

(1) $\sin\theta/\cos\theta$

(2) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

(3) $\dfrac{\tan 90°}{\cot 90°}$

(4) $r[3\cos\theta + 4\sin\theta] = 5$

(5) $7(\cos 20° + i\sin 20°)$

(6) $\frac{1}{2}\ln|\sec 2t + \tan 2t| + C$

(7) $a\sin\frac{x}{a}\cdot\frac{1}{a} = \sin\frac{x}{a}$

14.3.4 **Spacing with Consecutive Function Names.** A space is required between two or more consecutive function names unless they are clearly unspaced in the print text. When there is doubt concerning the presence of a space between function names, a space should be inserted.

⫸ $y = \text{arc}\,\sin(x)$ ⠽ ⠨⠅ ⠁⠗⠉ ⠎⠊⠝ ⠷⠭⠾

*A space is required before (x) even though no space appears there in print. See Section 14.3.b.*

*Example 14-14*

What is $\cos \arctan(-1)$?

⠠⠱⠁⠞ ⠊⠎ ⠸⠩ ⠉⠕⠎ ⠁⠗⠉⠞⠁⠝ ⠷⠤⠂⠾ ⠸⠱⠦

14.3.5 **Examples.** Study the following examples.

*Examples are in Nemeth. The code switch indicators are omitted from the simbraille.*

(1) $n = \log \sin 50°27'$

⠝ ⠨⠅ ⠇⠕⠛ ⠎⠊⠝ ⠼⠢⠴⠘⠨⠡⠐⠆⠶⠄

(2) $\cos\left[2\,\text{Arc}\,\csc\left(-\frac{7}{5}\right)\right]$

⠉⠕⠎ ⠈⠷⠆⠠⠁⠗⠉ ⠉⠎⠉ ⠷⠤⠹⠶⠌⠢⠼⠾⠈⠾

(3) $\cos\left(\text{arc}\,\text{Tan}\,x + \frac{\pi}{3}\right)$

⠉⠕⠎ ⠷⠁⠗⠉ ⠠⠞⠁⠝ ⠭⠬⠹⠨⠏⠌⠒⠼⠾

(4) $\text{Arctan}\,x + \text{Arccot}\,x = \frac{1}{2}\pi$

⠠⠁⠗⠉⠞⠁⠝ ⠭⠬⠠⠁⠗⠉⠉⠕⠞ ⠭ ⠨⠅ ⠹⠂⠌⠆⠼⠨⠏

## 14.4 Nonuse of the English-letter Indicator

The English-letter indicator is not used with an English letter or a Roman numeral following a function name.

⫸ $\sin x$ ⠎⠊⠝ ⠭

⫸ $\text{sine}\ x$ ⠎⠊⠝⠑ ⠭

⫸ cot I ⠉⠕⠞ ⠠⠊

14.4.1 **Examples.** Examine the English letters and the spacing in the following examples.

*Examples are in Nemeth. The code switch indicators are omitted from the simbraille.*

(1) $\sin x+y$

⠎⠊⠝ ⠭⠬⠽

(2) $\text{ctn} - \text{A} = -\text{ctn A}$

⠉⠞⠝ ⠤⠠⠁ ⠨⠅ ⠤⠉⠞⠝ ⠠⠁

(3) $y = 2\sin x + \sin 2x$

⠽ ⠨⠅ ⠼⠆⠎⠊⠝ ⠭⠬⠎⠊⠝ ⠼⠆⠭

(4) $y = \sqrt{\cot x}$

⠽ ⠨⠅ ⠜⠉⠕⠞ ⠭⠻

(5) $\{\sin x \mid \sin x + 2 \leq +1\}$

⠨⠷⠎⠊⠝ ⠭ ⠳ ⠎⠊⠝ ⠭⠬⠆ ⠐⠅⠱ ⠬⠂⠨⠾

(6) $y = \ln|\tan x|$

⠽ ⠨⠅ ⠇⠝ ⠳⠞⠁⠝ ⠭⠳

## 14.5 Keep Together

A function name and the sign which follows it (known as the "argument") is regarded as a single mathematical item and therefore should not be divided between braille lines. Also, a two-part function name should not be divided between braille lines. These rules also apply in UEB context.

*Example 14-15*

If θ = 51° is the angle between vectors, determine sin θ and cos θ.

⠠⠊⠋ ⠸⠩ ⠨⠹ ⠨⠅ ⠼⠑⠂⠘⠨⠡ ⠸⠱ ⠊⠎ ⠮ ⠁⠝⠛⠇⠑ ⠆⠞
⠧⠑⠉⠞⠕⠗⠎⠂ ⠙⠑⠞⠻⠍⠔⠑ ⠸⠩ ⠎⠊⠝ ⠨⠹ ⠠⠄⠯⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠉⠕⠎ ⠨⠹ ⠸⠱⠲

*"cos θ" is not divided between lines even though "cos" fits on the previous line.*

*Example 14-16*

**Inverse Functions** The inverse function $\tan^{-1} x$ may also be called the arc tangent of x, or arctan $x$.

⠘⠂⠠⠔⠧⠻⠎⠑ ⠘⠂⠠⠋⠥⠝⠉⠰⠝⠎ ⠠⠮ ⠔⠧⠻⠎⠑ ⠋⠥⠝⠉⠰⠝
⠸⠩ ⠞⠁⠝⠘⠤⠂ ⠭ ⠸⠱ ⠍⠁⠽ ⠁⠇ ⠆ ⠉⠁⠇⠇⠫ ⠮
⠜⠉ ⠞⠁⠝⠛⠢⠞ ⠷ ⠰⠭⠂ ⠕⠗ ⠸⠩ ⠁⠗⠉⠞⠁⠝ ⠭ ⠸⠱⠲

*"arc tangent" is not divided between lines even though "arc" fits on the previous line.*

## 14.6 Clarification—Function Names in an Enclosed List

A function name and the item which follows it are regarded as a single item. Although the numeric indicator is not used at the beginning of an item in an enclosed list, it must be used before a numeral (or decimal point and a numeral) following a function name.

⋙ (2 sin 30°, 3 cos 60°)

⠷⠆⠎⠊⠝ ⠼⠒⠴⠘⠨⠡⠠ ⠒⠉⠕⠎ ⠼⠖⠴⠘⠨⠡⠐⠾

---

**PRACTICE 14C**

(A) sine $x$ − sine $y$

(B) $2 \sin x + 3 \cos y$

(C) $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$

(D) The logarithm of sine 18° is written log sin 18°.

(E) $\cos 225° = -\sqrt{\frac{1+\cos 450°}{2}}$

(F) ArcTan[x, y] gives the arc tangent of $\frac{y}{x}$, taking into account in which quadrant the point $(x, y)$ lies.

(G) The arc tangent of the complex number q is written "ArcTan[q]".

(H) Consider the ordered pair (cos .8000, 2 cos .8000).

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## 14.7 Superscripts and Subscripts

When a function name carries a superscript or a subscript, the required space follows the superscript or subscript. A letter, numeral, or other mathematical expression following this space assumes the same level as the function name.

*In the following three samples, letters "θ" and "x" are printed on the baseline of writing and are printed without a leading space.*

⋙ $\sin^2\theta$ ⠎⠊⠝⠘⠆ ⠨⠹

⋙ $\sin^2x$ ⠎⠊⠝⠘⠆ ⠭

⋙ $\mathrm{sine}^2x$ ⠎⠊⠝⠑⠘⠆ ⠭

### Example 14-17

The coordinates are the cosine and sine, so we conclude $\sin^2\theta + \cos^2\theta = 1$.

⠠⠮ ⠉⠕⠕⠗⠙⠔⠁⠞⠑⠎ ⠜⠑ ⠮ ⠉⠕⠎⠔⠑ ⠯ ⠎⠔⠑⠂ ⠎ ⠺⠑
⠒⠉⠇⠥⠙⠑ ⠸⠩ ⠎⠊⠝⠘⠆ ⠨⠹⠬⠉⠕⠎⠘⠆ ⠨⠹ ⠨⠅ ⠼⠂ ⠸⠱⠲

*The required space follows the superscript in sin$^2$ and cos$^2$. There is no space following the plus sign.*

### Example 14-18

Verify that $1 - \cos\frac{2\pi}{3} = 2\sin^2\frac{4\pi}{3}$.

⠠⠧⠻⠊⠋⠽ ⠞ ⠸⠩⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠼⠂⠤⠉⠕⠎ ⠹⠆⠨⠏⠌⠒⠼ ⠨⠅ ⠼⠆⠎⠊⠝⠘⠆ ⠹⠲⠨⠏⠌⠒⠼ ⠸⠱⠲

*The required space follows "cos" and "sin$^2$". There is no space following the minus sign.*

14.7.1 **Examples.** Examine the spacing in the following examples.

*Examples are in Nemeth. The code switch indicators are omitted from the simbraille.*

(1) $\sin^2 A + \cos^2(B + A)$

⠎⠊⠝⠘⠆ ⠠⠁⠬⠉⠕⠎⠘⠆ ⠷⠠⠃⠬⠠⠁⠾

(2) $(1 - \sin^2 x)^2\cos^2 x$

⠷⠂⠤⠎⠊⠝⠘⠆ ⠭⠾⠘⠆⠐⠉⠕⠎⠘⠆ ⠭

(3) $\sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta} - 1$

⠎⠊⠝⠘⠆⠀⠨⠹⠈⠡⠹⠉⠕⠎⠘⠆⠀⠨⠹⠌⠎⠊⠝⠘⠆⠀⠨⠹⠼⠤⠂

(4) $\frac{1-\frac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$

⠠⠹⠂⠤⠹⠎⠊⠝⠘⠆⠀⠭⠌⠉⠕⠎⠘⠆⠀⠭⠼⠠⠌⠎⠑⠉⠘⠆⠀⠭⠠⠼

14.7.2 **Use/Nonuse of the Subscript Indicator.** When an abbreviated function name carries a numeric subscript on the first level below the baseline of writing, a subscript indicator is not used.

⫸ $\log_3 81 = 4$ ⠇⠕⠛⠒⠀⠼⠦⠂⠀⠨⠅⠀⠼⠲

*The numeral "3" is printed at the subscript level.*

A subscript indicator is required in all other circumstances.

⫸ $\text{logarithm}_3\ 81 = 4$

⠇⠕⠛⠁⠗⠊⠞⠓⠍⠰⠒⠀⠼⠦⠂⠀⠨⠅⠀⠼⠲

*A subscript indicator is required because the subscript applies to a word.*

⫸ $\log_b N \cdot \log_a b$ ⠇⠕⠛⠰⠃⠀⠠⠝⠡⠇⠕⠛⠰⠁⠀⠃

*A subscript indicator is required because each subscript is a letter.*

⫸ $\log_{2e} x = -1.4$ ⠇⠕⠛⠰⠆⠑⠀⠭⠀⠨⠅⠀⠤⠼⠂⠨⠲

*A subscript indicator is required because the subscript contains a letter.*

14.7.3 **Function Names Within a Superscript or a Subscript.** When a function name occurs within a superscript or subscript, the required space following it maintains the level at which the function name appears. A restatement of the level indicator is not needed.

⫸ $e^{\sin x}$ ⠑⠘⠎⠊⠝⠀⠭

*"sin x" is in the superscript position.*

⫸ $y = e^{\cos^2 x}$ ⠽⠀⠨⠅⠀⠑⠘⠉⠕⠎⠘⠘⠆⠀⠭

*"$\cos^2 x$" is in the superscript position.*

14.7.4 **Examples.** Study the following examples.

*Examples are in Nemeth. The code switch indicators are omitted from the simbraille.*

(1) $y = e^{\sin x}$ ⠽⠀⠨⠅⠀⠑⠘⠎⠊⠝⠀⠭

(2) $y = e^{\text{sine}\,x}$ ⠽⠀⠨⠅⠀⠑⠘⠎⠊⠝⠑⠀⠭

(3) $y = (\sin x)^{\tan x}$ ⠽⠀⠨⠅⠀⠷⠎⠊⠝⠀⠭⠾⠘⠞⠁⠝⠀⠭

(4) $e^{\ln x - 2\ln y}$ ⠑⠘⠇⠝⠀⠭⠤⠆⠇⠝⠀⠽

(5) $\mathrm{a}^{\mathrm{g(x)log_a f(x)}}$ ⠁⠘⠛⠷⠭⠾⠇⠕⠛⠘⠰⠁⠀⠋⠷⠭⠾

(6) $3^{\log_3 9}$ ⠼⠒⠘⠇⠕⠛⠘⠰⠒⠀⠼⠔

*Recall from Lesson 6 that a subscript indicator is required in superscript and subscript combinations. The super/sub indicator shows a numeric subscript in the superscript position.*

(7) $3^{\log_3 7} + 2^{\log_2 5}$

⠼⠒⠘⠇⠕⠛⠘⠰⠒⠀⠼⠶⠐⠬⠆⠘⠇⠕⠛⠘⠰⠆⠀⠼⠢

*Same note as (6), above.*

(8) $a^{\log_a x} = x$

⠁⠘⠇⠕⠛⠘⠰⠁⠀⠭⠀⠨⠅⠀⠭

*Recall from Lesson 6 that the space before a comparison sign returns the reader to the baseline.*

(9) $e^{\sin x = a} > y$

⠑⠘⠎⠊⠝⠀⠭⠀⠘⠨⠅⠀⠁⠀⠨⠂⠀⠽

*Recall from Lesson 6 that when a comparison sign occurs within a superscript, the level is restated before the comparison sign.*

*Example 14-19*

The behavior described by the following relationship is called **Wien's displacement law.**

$$\lambda_{\mathrm{max}}T = 2.898 \times 10^{-3}\ \mathrm{m} \cdot \mathrm{K}$$

```
1      ⠠⠮ ⠆⠓⠁⠧⠊⠕⠗ ⠙⠑⠎⠉⠗⠊⠃⠫ ⠃⠽ ⠮ ⠋⠕⠇⠇⠪⠬
2    ⠗⠑⠇⠁⠰⠝⠩⠊⠏ ⠊⠎ ⠉⠁⠇⠇⠫ ⠘⠶⠠⠺⠊⠢⠄⠎ ⠲⠏⠇⠁⠉⠑⠰⠞⠀⠀⠀⠀
3    ⠇⠁⠺⠲ ⠸⠩
4      ⠨⠇⠰⠍⠁⠭⠐⠠⠞ ⠨⠅ ⠼⠆⠨⠦⠔⠦⠈⠡⠂⠴⠘⠤⠒ ⠰⠍⠡⠰⠠⠅ ⠸⠱
```

*Line 3: The switch to Nemeth terminates the UEB bold passage.*
*Line 4: Since both Nemeth switches do not fit on this line, the opening switch is placed on line 3.*
*The multiplication dot which comes between the two-part abbreviation is unspaced.*
*Review Section 4.20.1 in Lesson 4.*

---

## PRACTICE 14D

1. $\log_n .125 = -.6$
2. $\mathrm{antilog}_{\mathrm{a}}\, \mathrm{x} = \mathrm{N}$
3. $\log_{.0543}\mathrm{x} = -.8$
4. $\cot^{-1} x + \frac{\pi}{2} - \tan^{-1} x$
5. $\sin^2 90° + \cos^2 90° = 1$
6. $e^{x+\ln x}$
7. $e^{\sin x} + e^{\sin y}$
8. $2^{\sec x} = y$

---

## 14.8 Modifiers

Modified function names are transcribed according to the five-step rule for the transcription of modified expressions introduced in Lesson 12. When a function name carries a modifier, the required space after the function name follows the termination of the modifier.

⋙ $\lim\limits_{x\to a}$ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠁⠻

⋙ $\lim\limits_{x\to a} f(x) = 1$

⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠁⠻ ⠋⠷⠭⠾ ⠨⠅ ⠼⠂

14.8.1 **Examples.** Study these additional examples.

*Examples are in Nemeth. The code switch indicators are omitted from the simbraille.*

(1) $\lim\limits_{x\uparrow 4}(x-4)^{-1}$

⠐⠇⠊⠍⠩⠭ ⠫⠣⠒⠒⠕ ⠼⠲⠻ ⠷⠭⠤⠲⠾⠘⠤⠂

(2) $\lim\limits_{\theta\to\theta_0}(\tan\theta)$

⠐⠇⠊⠍⠩⠨⠹ ⠫⠕ ⠨⠹⠴⠻ ⠷⠞⠁⠝ ⠨⠹⠾

14.8.2 **Special Case—Upper Limit and Lower Limit.** The symbols in the box below denote "upper limit" or "lower limit". The horizontal bar directly over or under "lim" or "limit" is not treated as a modifier.

| | | |
|---|---|---|
| ⠣⠇⠊⠍ | `upper limit` | $\overline{\text{lim}}$ |
| ⠣⠇⠊⠍⠊⠞ | `upper limit` | $\overline{\text{limit}}$ |

⋙ $\overline{\lim\limits_{n\to\infty}} f_n(x)$

⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

⋙ $\overline{\text{limit}}\limits_{n\to\infty} f_n(x)$

⠐⠣⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

|  |  |  |
|---|---|---|
| ⠩⠇⠊⠍ | lower limit | $\underline{\text{lim}}$ |
| ⠩⠇⠊⠍⠊⠞ | lower limit | $\underline{\text{limit}}$ |

⋙ $\underset{n\to\infty}{\underline{\lim}}\ f_n(x)$

⠐⠩⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

⋙ $\underset{n\to\infty}{\underline{\text{limit}}}\ f_n(x)$

⠐⠩⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

---

## PRACTICE 14E

1. Find $\lim\limits_{x\to 0.6} 2^{25x^2-10x-1}$.
2. Formulate a precise definition for $\lim\limits_{x\downarrow -\infty} f(x) = L$.
3. If $\underset{n\to\infty}{\overline{\lim}}\, a_n = A$ and $\underset{n\to\infty}{\overline{\lim}}\, b_n = B$, must it be true that $\underset{n\to\infty}{\overline{\lim}}\, (a_n + b_n) = A + B$ ?
4. Find $\underset{n\to\infty}{\overline{\text{limit}}}\, a_n$ when $a_n = (-1)_n$.
5. $\lim\limits_{x\to 0} \csc x \ln\,(1 + x)$

---

You may wish to revisit the Review of Format for Spatial Arrangements in Lesson 10. *Note: In the examples that do not contain narrative, code switch indicators are omitted and blank lines are implied.*

## *SQUARE ROOT DIVISION*

### [NC Rule 25.6]

### 14.9 Review of Terminology

Radical expressions were presented in Lesson 8. When an answer is shown, a spatial arrangement is required. Here are the names of the parts of a radical expression.

$$\begin{array}{rl} 12 & \textit{root} \\ \sqrt{144} & \textit{radicand} \end{array}$$

The line above the radicand is the vinculum. √ is the radical sign.

### 14.10 Spatial Arrangement for Square Root Problems

In the spatially arranged radical expression, the vinculum is transcribed as a separation line. The first cell of the vinculum is placed directly above the radical symbol. The last cell of the vinculum extends one cell beyond the radicand.

```
⠒⠒⠒
⠜          Radical (with Vinculum)    √‾‾
```

*Example 14-20*

The square root of 144 is 12, and is written as follows.

$$\begin{array}{r} 12 \\ \sqrt{144} \end{array}$$

```
1   ⠠⠮ ⠎⠟⠥⠜⠑ ⠗⠕⠕⠞ ⠷ ⠼⠁⠙⠙ ⠊⠎ ⠼⠁⠃⠂ ⠯ ⠊⠎
2 ⠺⠗⠊⠞⠞⠢ ⠵ ⠋⠕⠇⠇⠪⠎⠲ ⠸⠩
3 ⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
4     ⠂⠆
5 ⠀⠀⠒⠒⠒⠒⠒
6 ⠀⠀⠜⠂⠲⠲
7 ⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
8 ⠸⠱
```

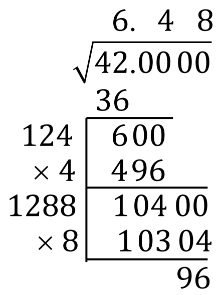
*Lines 5-6: The leftmost braille cell of the displayed spatial expression is in cell 3 (displayed to 3-1 narrative).*

a. **Solving a Square Root Problem.** A square root arrangement is similar to a division arrangement except that no divisor is present. (Review alignment and spacing rules for long division in Lesson 10.) The layout and alignment are adapted to resemble the print arrangement as closely as possible. Separation lines may be different lengths and may be indented. Follow print.

The vertical line that separates the parts of the problem is represented by dots 456. Numbers to the left of this line are unspaced from the line. Follow print regarding the column in which the vertical line is placed. This may mean placing it in the same column as a digit. (See Example 14-22.)

*Example 14-21*

```
 1         ⠖⠨ ⠲  ⠦
 2     ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
 3     ⠜⠲⠆⠨⠴⠴ ⠴⠴
 4      ⠒⠖
 5     ⠒⠒⠒⠒⠒⠒⠒
 6  ⠂⠆⠲⠸ ⠖ ⠴⠴
 7  ⠈⠡⠲⠸ ⠲ ⠔⠖
 8     ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
 9 ⠂⠆⠦⠦⠸ ⠂ ⠴⠲ ⠴⠴
10  ⠈⠡⠦⠸ ⠂ ⠴⠒ ⠴⠲
11     ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
12            ⠔⠖
```



*All lines: Spacing between digits matches print in order to attain proper vertical alignment.*

*Line 2: The vinculum begins in the cell above the radical symbol and ends one cell beyond the rightmost character in the entire arrangement.*

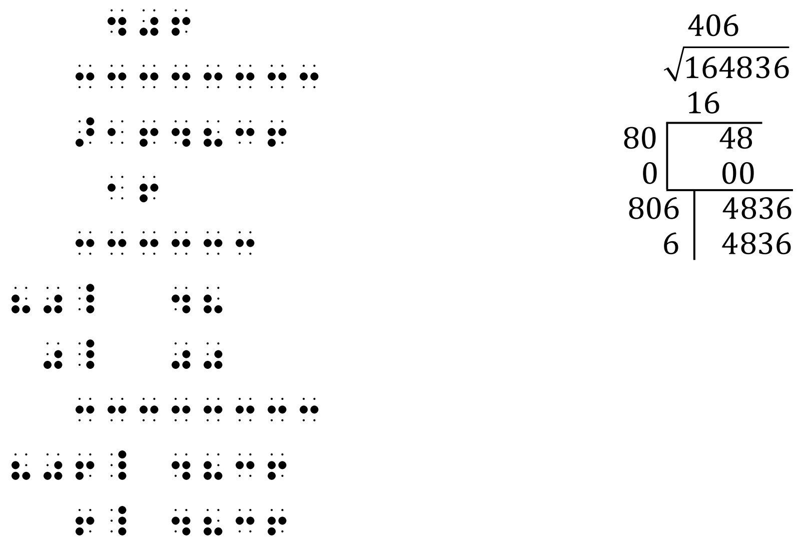
*Line 5: This separation line begins in the same cell as the vertical line below it and ends one cell to the right of the rightmost digit on the line below, as printed.*

*Lines 8 and 11: These separation lines begin in the same cell as the vertical line and end one cell to the right of the rightmost digit in order to align with the rightmost cell of the entire arrangement.*

*Lines 6, 7, 9, 10: The vertical line in this problem aligns below the radical symbol, as printed. It is unspaced from the number to its left. Note that the vertical line is interrupted by the separation lines on lines 5, 8, and 11.*

*Lines 7 and 10: The multiplication cross is unspaced from the multiplier.*

*Example 14-22*

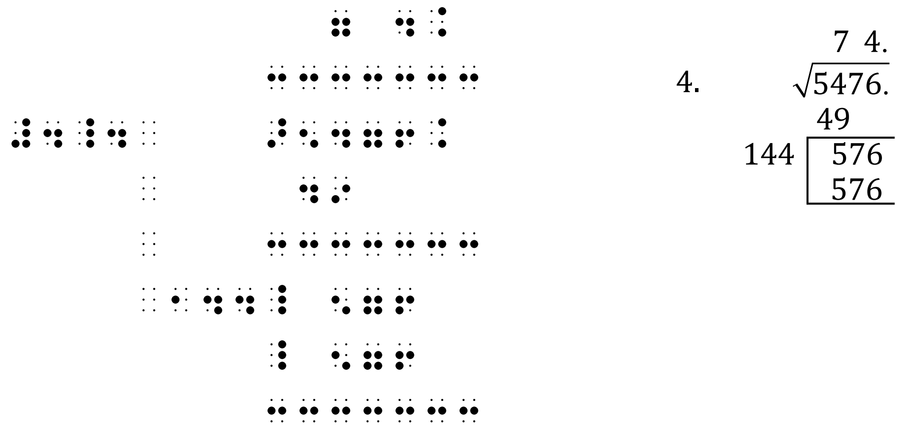


*Line 5: The separation line is shorter than the others, as printed.*
*Lines 6-7: This vertical line is aligned beneath the radical symbol, as printed. It is unspaced from the number to its left.*
*Lines 9-10: This vertical line is situated in the same column as some digits. It is unspaced from the number to its left.*

### 14.11 Placement of Identifiers with Spatial Radical Expressions

An identifier, if present, is placed on the line with the radicand. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

*Example 14-23*



## PRACTICE 14F

(A)

$$
\begin{array}{r|l}
\multicolumn{1}{r}{} & 5.\ \ 7\ \ \ 4 \\
\multicolumn{1}{r}{} & \sqrt{33.00\,00} \\
\multicolumn{1}{r}{} & 25 \\ \cline{2-2}
107 & 800 \\
\times 7 & 749 \\ \cline{2-2}
1144 & 5100 \\
\times 4 & 4576 \\ \cline{2-2}
\multicolumn{1}{r}{} & \phantom{0}524
\end{array}
$$

## *OTHER PRINT LAYOUTS SHOWING DIVISION*

In the next two layouts, note that the rules regarding placement of the vertical line differ from each other, and also differ from the vertical line rules in a square root problem in Section 14.10.a. Before transcribing, analyze the print and refer to the appropriate rules.

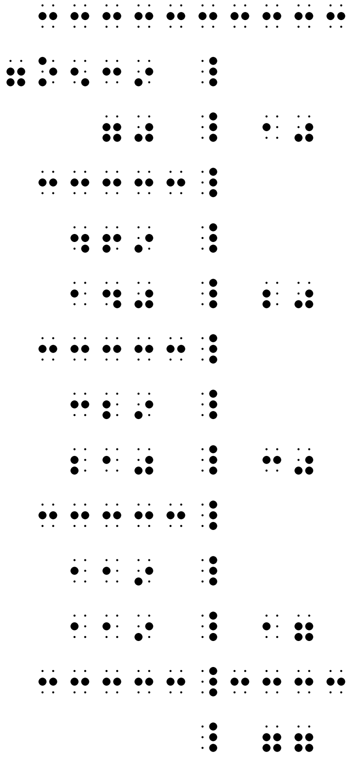
### 14.12 Partial Quotients [NC Rule 25.5.8]

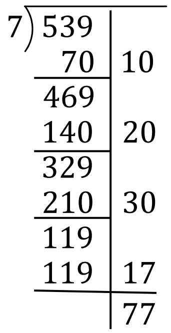
*Print Observations:* This layout shows partial quotients printed to the right of the division problem. A vertical line separates the partial quotients from the rest of the problem. The partial quotients may or may not be aligned by place value.

*Braille Rules:* The vertical line may be either drawn as a tactile graphic or it may be represented by dots 456. The partial quotients are aligned as printed. Space is left between the vertical line and any digit preceding or following it. More than one space may be needed if the partial quotients are aligned by place value.

No space is inserted between a separation line and the vertical line. If a vertical line and a horizontal line cross, the vertical line is kept intact, as shown in Example 14-24.

*Example 14-24*





*Notice the comparative lengths of the separation lines as well as their vertical alignment.*

*Instructions:* Review Section 10.13.6.d in Lesson 10 regarding alignment of the minus signs.

**PRACTICE 14G**

$$
\begin{array}{r|l}
132 & \\
6\overline{)792} & \\
-600 & 100 \\
\cline{1-1}
192 & \\
-\ 60 & 10 \\
\cline{1-1}
132 & \\
-\ 60 & 10 \\
\cline{1-1}
72 & \\
-\ 60 & 10 \\
\cline{1-1}
12 & \\
-\ 12 & 2 \\
\cline{1-1}
0 &
\end{array}
$$

### 14.13 Synthetic Division [NC Rule 25.7]

Synthetic division is a method of showing division of polynomials. There is not a standard print layout. The transcription replicates the print design, following alignment rules discussed in this section and using the separation line and vertical line of the Nemeth Code. Here is a sample of one possible layout of a synthetic division problem.

$$\begin{array}{r|rrrr} +2 & 1 & -3 & +4 & +5 \\ & & +2 & -2 & +4 \\ \hline & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} \end{array}$$

The parts to this problem are labeled as follows.

$$\begin{array}{lr|rrrrl} \textit{synthetic divisor} & +2 & 1 & -3 & +4 & +5 & \textit{synthetic dividend} \\ & & & +2 & -2 & +4 & \textit{synthetic product} \\ \cline{3-6} \textit{synthetic quotient} & \multicolumn{1}{r}{} & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} & \textit{synthetic remainder} \end{array}$$

14.13.1 **Alignment and Spacing.** In the examples which follow, look carefully at the vertical alignment. The numerals in the synthetic dividend, product, and quotient are aligned by place value. Symbols of operation are vertically aligned. (See Example 14-29.) At least one blank cell is present between adjacent columns.

14.13.2 **Vertical Line and Separation Line.** Dots 456 represent the vertical line that comes between the synthetic divisor and the division arrangement. This vertical line begins on the line with the dividend and ends on the line with the product. No space is left between the vertical line and the dividend or divisor. Another vertical line may appear between the synthetic quotient and the synthetic remainder. It is transcribed below a blank column.

The separation line (dots 25) begins directly under the vertical line at one end and terminates one cell beyond the overall arrangement at the other end.

*Example 14-25*

$$\begin{array}{r|rrrr} +2 & 1 & -3 & +4 & +5 \\ & & +2 & -2 & +4 \\ \hline & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} \end{array}$$

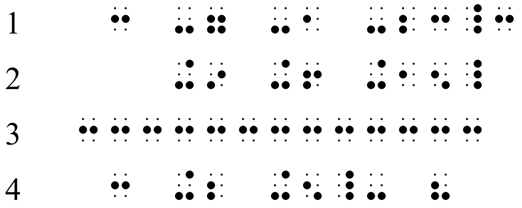
```
⠬⠆⠸⠂ ⠤⠒ ⠬⠲ ⠬⠢
  ⠸  ⠬⠆ ⠤⠆ ⠬⠲
  ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
   ⠂ ⠤⠂ ⠬⠆⠸⠬⠔
```

*Note the vertical alignment of the numerals and the operation symbols, the spacing of the vertical line, and the length of the separation line.*

14.13.3 **Another Print Style—Synthetic Divisor on the Right.** If the synthetic divisor is printed to the right of the overall problem, the same layout is followed in braille. Follow the alignment and spacing rules outlined in Sections 14.13.1 and 14.13.2, particularly noting that at least one blank cell must be left between adjacent columns. The vertical lines are unspaced from the dividend and the divisor, as well as from the quotient and the remainder.

*Example 14-26*

$$\begin{array}{rrrr|l} 3 & -7 & -1 & -23 & 3 \\ & +9 & +6 & +15 & \\ \cline{1-4} 3 & +2 & +5 & \multicolumn{1}{|r}{-8} & \end{array}$$

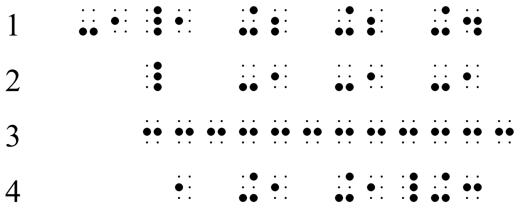


14.13.4 **Another Print Style—Boxed Synthetic Divisor.** If the synthetic divisor appears boxed on two sides, the boxing is omitted in braille. A vertical line is inserted between the divisor and the dividend in order to differentiate the divisor from the rest of the arrangement. Follow the alignment and spacing rules outlined in Sections 14.13.1 and 14.13.2.

Example 14-27 shows the synthetic divisor at the left; Example 14-28 shows the synthetic divisor at the right.

*Example 14-27*

$$\begin{array}{r|rrrr} \underline{-1} & 1 & +2 & +2 & +4 \\ \multicolumn{1}{r}{} & & -1 & -1 & -1 \\ \cline{2-5} \multicolumn{1}{r}{} & 1 & +1 & +1 & \multicolumn{1}{|r}{+3} \end{array}$$



*Example 14-28*

$$\begin{array}{rrrrl} 1 & +2 & +2 & +4 & \underline{|-2} \\ & -2 & +0 & -4 & \\ \cline{1-4} 1 & +0 & +2 & +0 & \end{array}$$

```
 ⠂ ⠬⠆ ⠬⠆ ⠬⠲⠸⠤⠆
   ⠤⠆ ⠬⠴ ⠤⠲⠸
⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
 ⠂ ⠬⠴ ⠬⠆ ⠬⠴
```

*Note that this example has no remainder.*

14.13.5 **Placement of Identifiers with Synthetic Division.** An identifier, if present, is placed on the line with the dividend (the top line of the arrangement, in this case). One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

*Example 14-29*

197. $\begin{array}{lrrrr} \underline{|+2} & 1 & +6 & -1 & -30 \\ & & +2 & +16 & +30 \\ \cline{2-5} \end{array}$

```
⠼⠂⠔⠶⠸⠲ ⠬⠆⠸⠂ ⠬⠖ ⠤ ⠂ ⠤⠒⠴
         ⠸  ⠬⠆ ⠬⠂⠖ ⠬⠒⠴
         ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
```

*Print Observation: The 2-sided box is to the left and beneath this synthetic divisor. In braille, a vertical line is placed between the divisor and the dividend.*
*Braille Observation: Following alignment rules, the operation symbols with –1 and +16 are vertically aligned even though they are not aligned in print.*

## PRACTICE 14H

**Dividing Polynomials:** Divide $(3x^4 + 12x^3 - 5x^2 - 18x + 8) \div (x + 4)$

$$\begin{array}{r|rrrrr} \underline{-4} & 3 & 12 & -5 & -18 & 8 \\ & & -12 & 0 & 20 & -8 \\ \hline & 3 & 0 & -5 & 2 & \multicolumn{1}{|r}{0} \end{array}$$

*Answer:* $3x^2 - 5x - 2$

---

*For further practice, see Addendum 1—Reading Practice.*

---

Submit Exercise 14 to your instructor.

# ANSWERS TO PRACTICE MATERIAL

## PRACTICE 14A

1 ⠼⠁⠲ ⠸⠩ ⠦⠎⠊⠝ ⠨⠹⠸⠴ ⠸⠱ ⠊⠎ ⠏⠗⠕⠝⠳⠝⠉⠫ ⠦⠎⠔⠑

2 ⠮⠞⠁⠴⠲

3 ⠼⠃⠲ ⠸⠩ ⠦⠠⠁⠗⠉⠎⠊⠝⠸⠴ ⠸⠱ ⠊⠎ ⠮ ⠦⠔⠧⠿⠎⠑ ⠎⠔⠑⠴⠲

4 ⠸⠩

5 ⠼⠒⠸⠲ ⠉⠕⠎⠊⠝⠑ ⠼⠲⠢⠘⠨⠡ ⠨⠅ ⠼⠴⠨⠶⠴⠶⠂ ⠠⠄⠕⠗

6 ⠼⠂⠸⠌⠜⠆

7 ⠼⠲⠸⠲ ⠠⠄⠠⠮ ⠇⠕⠛⠎⠊⠝⠑ ⠸⠱ ⠋⠥⠝⠉⠰⠝ ⠊⠎ ⠗⠑⠇⠁⠞⠫ ⠞⠕

8 ⠮ ⠸⠩ ⠇⠕⠛⠉⠕⠎⠊⠝⠑ ⠸⠱ ⠋⠥⠝⠉⠰⠝ ⠃⠽

9 ⠸⠩ ⠠⠎⠰⠝ ⠨⠅ ⠼⠆⠠⠉⠰⠝ ⠸⠱⠲

*Line 5: The word "cosine" is part of the math equality and so is inside the switches and is uncontracted.*

*Line 6: The radical sign with no vinculum does not use a terminator. (See Section 8.12.a in Lesson 8)*

*Lines 7 and 8: "log" is an abbreviated function name, therefore "logsine" requires a switch to Nemeth.*

## PRACTICE 14B

1 ⠸⠩

2 ⠷⠂⠾ ⠎⠊⠝ ⠨⠹⠸⠌⠉⠕⠎ ⠨⠹

3 ⠷⠆⠾ ⠎⠊⠝ ⠼⠆⠨⠁ ⠨⠅ ⠼⠆⠎⠊⠝ ⠨⠁⠉⠕⠎ ⠨⠁

4 ⠷⠒⠾ ⠹⠞⠁⠝ ⠼⠔⠴⠘⠨⠡⠐⠌⠉⠕⠞ ⠼⠔⠴⠘⠨⠡⠐⠼

5 ⠷⠲⠾ ⠗⠈⠷⠒⠉⠕⠎ ⠨⠹⠬⠲⠎⠊⠝ ⠨⠹⠈⠾ ⠨⠅ ⠼⠢

6 ⠷⠢⠾ ⠼⠶⠷⠉⠕⠎ ⠼⠆⠴⠘⠨⠡⠐⠬⠊⠎⠊⠝ ⠼⠆⠴⠘⠨⠡⠐⠾

7 ⠷⠖⠾ ⠹⠂⠌⠆⠼⠇⠝ ⠳⠎⠑⠉ ⠼⠆⠞⠬⠞⠁⠝ ⠼⠆⠞⠳⠬⠠⠉

8 ⠷⠶⠾ ⠁⠎⠊⠝ ⠹⠭⠌⠁⠼⠡⠹⠂⠌⠁⠼ ⠨⠅ ⠎⠊⠝ ⠹⠭⠌⠁⠼ ⠸⠱

## PRACTICE 14C

⠸⠩
⠷⠠⠁⠾ ⠎⠊⠝⠑ ⠭⠤⠎⠊⠝⠑ ⠽
⠷⠠⠃⠾ ⠼⠆⠎⠊⠝ ⠭⠬⠒⠉⠕⠎ ⠽
⠷⠠⠉⠾ ⠹⠂⠬⠉⠕⠎ ⠭⠌⠎⠊⠝ ⠭⠼⠬⠹⠎⠊⠝ ⠭⠌⠂⠬⠉⠕⠎ ⠭⠼ ⠸⠱
⠐⠣⠰⠠⠙⠐⠜ ⠠⠮ ⠇⠕⠛⠜⠊⠹⠍ ⠷ ⠸⠩ ⠎⠊⠝⠑ ⠼⠂⠦⠘⠨⠡ ⠸⠱
⠀⠀⠊⠎ ⠺⠗⠊⠞⠞⠢ ⠸⠩ ⠇⠕⠛ ⠎⠊⠝ ⠼⠂⠦⠘⠨⠡⠸⠲
⠷⠠⠑⠾
⠀⠀⠉⠕⠎ ⠼⠆⠆⠢⠘⠨⠡ ⠨⠅ ⠤⠜⠹⠂⠬⠉⠕⠎ ⠼⠲⠢⠴⠘⠨⠡⠐⠌⠆⠼⠻
⠷⠠⠋⠾ ⠠⠁⠗⠉⠠⠞⠁⠝ ⠈⠷⠭⠠ ⠽⠈⠾ ⠸⠱ ⠛⠊⠧⠑⠎ ⠮
⠀⠀⠜⠉ ⠞⠁⠝⠛⠢⠞ ⠷ ⠸⠩ ⠹⠽⠌⠭⠼ ⠸⠱⠂ ⠞⠁⠅⠬ ⠔⠞⠕ ⠁⠒⠨⠞
⠀⠀⠔ ⠱ ⠟⠥⠁⠙⠗⠁⠝⠞ ⠮ ⠏⠕⠔⠞ ⠸⠩ ⠷⠭⠠ ⠽⠾ ⠸⠱ ⠇⠊⠑⠎⠲
⠐⠣⠰⠠⠛⠐⠜ ⠠⠮ ⠜⠉ ⠞⠁⠝⠛⠢⠞ ⠷ ⠮ ⠉⠕⠍⠏⠇⠑⠭ ⠝⠥⠍⠃⠻
⠀⠀⠰⠟ ⠊⠎ ⠺⠗⠊⠞⠞⠢ ⠸⠩ ⠦⠠⠁⠗⠉⠠⠞⠁⠝⠈⠷⠟⠈⠾⠸⠴ ⠸⠱⠲
⠐⠣⠰⠠⠓⠐⠜ ⠠⠒⠎⠊⠙⠻ ⠮ ⠕⠗⠙⠻⠫ ⠏⠁⠊⠗
⠀⠀⠸⠩ ⠷⠉⠕⠎ ⠼⠨⠦⠴⠴⠴⠠ ⠆⠉⠕⠎ ⠼⠨⠦⠴⠴⠴⠾ ⠸⠱⠲

## PRACTICE 14D

⠸⠩
⠼⠂⠸⠲ ⠇⠕⠛⠰⠝ ⠼⠨⠂⠆⠢ ⠨⠅ ⠤⠼⠨⠖
⠼⠆⠸⠲ ⠁⠝⠞⠊⠇⠕⠛⠰⠁ ⠭ ⠨⠅ ⠠⠝
⠼⠒⠸⠲ ⠇⠕⠛⠨⠴⠢⠲⠒ ⠭ ⠨⠅ ⠤⠼⠨⠦
⠼⠲⠸⠲ ⠉⠕⠞⠘⠤⠂ ⠭⠬⠹⠨⠏⠌⠆⠼⠤⠞⠁⠝⠘⠤⠂ ⠭
⠼⠢⠸⠲ ⠎⠊⠝⠘⠆ ⠼⠔⠴⠘⠨⠡⠐⠬⠉⠕⠎⠘⠆ ⠼⠔⠴⠘⠨⠡ ⠨⠅ ⠼⠂
⠼⠖⠸⠲ ⠑⠘⠭⠬⠇⠝ ⠭
⠼⠶⠸⠲ ⠑⠘⠎⠊⠝ ⠭⠐⠬⠑⠘⠎⠊⠝ ⠽
⠼⠦⠸⠲ ⠼⠆⠘⠎⠑⠉ ⠭ ⠨⠅ ⠽ ⠸⠱

## PRACTICE 14E

1 ⠼⠁⠲ ⠠⠋⠔⠙ ⠸⠩
2 ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠼⠴⠨⠖⠻ ⠼⠆⠘⠆⠢⠭⠘⠘⠆⠘⠤⠂⠴⠭⠤⠂ ⠸⠱⠲
3 ⠼⠃⠲ ⠠⠿⠍⠥⠇⠁⠞⠑ ⠁ ⠏⠗⠑⠉⠊⠎⠑ ⠙⠑⠋⠔⠊⠰⠝ ⠿
4 ⠸⠩ ⠐⠇⠊⠍⠩⠭ ⠫⠩⠒⠒⠕ ⠤⠠⠿⠻ ⠋⠷⠭⠾ ⠨⠅ ⠠⠇⠸⠲
5 ⠼⠒⠸⠲ ⠠⠄⠠⠊⠋ ⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠁⠰⠝ ⠨⠅ ⠠⠁ ⠠⠄⠯
6 ⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠃⠰⠝ ⠨⠅ ⠠⠃ ⠸⠱⠂ ⠍⠌ ⠭ ⠆
7 ⠞⠗⠥⠑ ⠞ ⠸⠩
8 ⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠷⠁⠰⠝⠐⠬⠃⠰⠝⠐⠾ ⠨⠅ ⠠⠁⠬⠠⠃⠸⠦
9 ⠼⠲⠸⠲ ⠠⠄⠠⠋⠔⠙ ⠐⠣⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠁⠰⠝ ⠠⠄⠱⠢
10 ⠁⠰⠝ ⠨⠅ ⠷⠤⠂⠾⠰⠝⠸⠲
11 ⠼⠢⠸⠲ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠼⠴⠻ ⠉⠎⠉ ⠭⠇⠝ ⠷⠂⠬⠭⠾ ⠸⠱

*Line 2: The embedded expression will fit on one line but not with both switches so the opening switch is on line 1.*
*Line 8: The embedded expression will fit on one line but not with both switches so the opening switch is on line 7.*

# PRACTICE 14F

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⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
          ⠢⠨ ⠶  ⠲
        ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠷⠠⠁⠾    ⠜⠒⠒⠨⠴⠴ ⠴⠴
         ⠆⠢
        ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
     ⠂⠴⠶⠸ ⠦ ⠴⠴
     ⠈⠡⠶⠸ ⠶ ⠲⠔
        ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
      ⠂⠂⠲⠲⠸ ⠢⠂ ⠴⠴
       ⠈⠡⠲⠸ ⠲⠢ ⠶⠖
          ⠒⠒⠒⠒⠒⠒⠒⠒
             ⠢ ⠆⠲
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
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## PRACTICE 14G

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⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠀⠀⠂⠒⠆
⠀⠒⠒⠒⠒⠒⠒
⠖⠕⠶⠔⠆⠀⠸
⠀⠤⠖⠴⠴⠀⠸⠀⠂⠴⠴
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠂⠔⠆⠀⠸
⠀⠤⠀⠖⠴⠀⠸⠀⠀⠂⠴
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠂⠒⠆⠀⠸
⠀⠤⠀⠖⠴⠀⠸⠀⠀⠂⠴
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠀⠶⠆⠀⠸
⠀⠀⠤⠖⠴⠀⠸⠀⠀⠂⠴
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠀⠂⠆⠀⠸
⠀⠀⠤⠂⠆⠀⠸⠀⠀⠀⠆
⠀⠒⠒⠒⠒⠒⠸
⠀⠀⠀⠀⠴⠀⠸
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠸⠱
```

## PRACTICE 14H

```
⠀⠀⠘⠂⠠⠙⠊⠧⠊⠙⠬⠀⠘⠂⠠⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇⠎⠒⠀⠠⠙⠊⠧⠊⠙⠑
⠸⠩⠀⠷⠒⠭⠘⠲⠐⠬⠂⠆⠭⠘⠒⠐⠤⠢⠭⠘⠆⠐⠤⠂⠦⠭⠬⠦⠾⠨⠌⠷⠭⠬⠲⠾
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠀⠀⠤⠲⠸⠒⠀⠀⠂⠆⠀⠤⠢⠀⠤⠂⠦⠀⠀⠦
⠀⠀⠀⠀⠸⠀⠀⠤⠂⠆⠀⠀⠴⠀⠀⠆⠴⠀⠤⠦
⠀⠀⠀⠀⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠀⠀⠀⠀⠀⠒⠀⠀⠀⠴⠀⠤⠢⠀⠀⠀⠆⠸⠀⠴
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠀⠀⠠⠄⠨⠂⠠⠁⠝⠎⠺⠻⠒⠀⠼⠒⠭⠘⠆⠐⠤⠢⠭⠤⠆⠀⠀⠸⠱
```

*Line 4: The leftmost braille cell of the displayed spatial expression is in cell 3 (displayed to 3-1 narrative).*
*Line 7: The remainder does not touch the vertical line because a space must be inserted in order to align with the numbers above the separation line.*