LESSON 15

Format

- MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS

Answers to Practice Material

LESSON PREVIEW

In this lesson, we look at the methods for transcribing a mathematical expression that is too long to fit on the current line. This often occurs when line length is restricted due to the indented margins in displayed mathematical material. However, even a 40-cell line may not provide enough room for a particularly long expression. The examples in this lesson serve as a good review of other aspects of the Nemeth Code.

MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS

[NC 26.2]

15.1 Review

A mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. When there is insufficient space on the remainder of a line, the entire expression is brought down to the next line. One or both switch indicators can be placed on a separate line if, by doing so, the math will fit, undivided, on one line. Likewise, an identifier can be placed alone on the line if, by doing so, the math will fit, undivided, on the next line. Keeping the mathematical expression intact on one line is the priority.

In this lesson we discuss what to do when a mathematical expression is too long to be contained within the current margins – that is, when a division is unavoidable. First, here is a summary of items that must not be divided, and a review of runover rules already covered.

15.1.1 **Symbols to Keep Together.** The components of the following symbols must not be divided between braille lines.

a. A symbol of operation using plus and minus (Lesson 5)
b. A symbol of comparison compounded vertically or horizontally (Lesson 5)
c. A shape symbol with structural or interior modification (Lesson 11)
d. A keystroke construction (Lesson 11)
e. Superposed symbols (Lesson 13)
f. Tally marks belonging to the same group (Lesson 13)

15.1.2 **Expressions to Keep Together.** The following expressions must not be divided between braille lines, even if divided in print.

a. A hyphenated expression of which one component is mathematical. (Lesson 2)
b. An abbreviation and its related numeral or letter. (Lessons 3 and 4)
c. An enclosed list. (Lesson 4)
d. A fraction, a mixed number. (Lesson 8)
e. A shape symbol and its name (numeral, letter, or sequence of letters). (Lesson 11)
f. The components of an expression modified according to the five-step rule. (Lesson 12)
g. A function name (or its abbreviated form) and its argument. (Lesson 14)
h. A two-part function name. (Lesson 14)

15.1.3 **Runover Rules Already Studied.** When a long expression won't fit on the braille line within current margins, the following rules apply. The new line begins in the runover cell of the current format.

a. **Long Numeral** (Lesson 1) If a long numeral will not fit on one line, divide after a comma if a comma is present. A hyphen is inserted at the point of division. If the numeral does not contain a comma, the hyphen may be inserted after any digit. The numeric indicator is restated before the first digit of the continuation of the numeral on the next line.

b. **Enclosed List** (Lesson 4) If an enclosed list will not fit on one line, divide after a comma used to separate the items.

c. **Linked Expression** (Lesson 8) If a linked expression will not fit on one line, divide before a link. It is not necessary to divide at every link unless it is a nested linked expression. If the anchor or a link will not fit on one line, divide before an operation sign as well as before each link.

d. **Keystroke** (Lesson 11) If a keystroke string will not fit on one line, division may be made after any item in the keystroke string but not within a keystroke.

15.1.4 **Code Switch Indicators.** (Lessons 1 and 3) If both switch indicators will not fit on the same line as the math expression, the opening Nemeth Code indicator may go on the previous line. The Nemeth Code terminator and any related punctuation may be placed on the following line.

15.2 Mathematical Units

The braille transcriber has only 40 cells available on a line, at most – perhaps as few as 30 cells for the runover to a nested link displayed to a subdivision. We often encounter a mathematical expression that will not fit on the current line. The margins in place at the time should not be changed in order to accommodate a long expression. Instead, the math will need to be divided between lines.

Runover sites should be chosen carefully. Every attempt should be made to keep the following mathematical units intact.

- a link
- a fraction
- a numerator
- a denominator
- a mixed number
- a base and its exponent; a subscript and its related item
- a grouped expression
- a radical expression
- a modified expression

A long or complicated mathematical expression can be organized into a series of mathematical units by following the procedures presented in this section. When the transcriber applies these principles, the reader is able to mentally reassemble the expression. On the other hand, a poorly divided expression will hinder the reader's understanding of the mathematics. Shrewd application of these guidelines can be properly rendered even if the transcriber is unfamiliar with the particular mathematics.

15.2.1 **A Systematic Approach.** Follow the list below when choosing division sites, starting with Step i, also keeping in mind that items enclosed within grouping symbols should not be divided.

- i. Before a comparison sign on the baseline. (See Section 15.3)
- ii. Before an operation sign on the baseline. (See Section 15.4)
- iii. Before a mathematical unit. (See Section 15.5)
- iv. After a termination indicator. (See Section 15.6)

Special considerations affect the division of function notation, integral notation, and Sigma and Pi notation. (See Section 15.7)

In order to focus on the layout, the isolated examples with no narrative do not include code switch indicators. Unless otherwise noted, embedded material is assumed to be within a 3-1 paragraph and displayed material begins in cell 3 with runovers in cell 5.

15.3 Step i: Divide Before a Comparison Sign on the Baseline

Linked Expressions: As you learned in Lesson 8, when a linked expression will not fit on one braille line, a division is made at the link, before the sign of comparison. A few examples are shown here, as a review. You may wish to revisit the topic of linked expressions and nested linked expressions in Lesson 8. Key points are listed below. (For a closer look at the division of linked expressions, see Section 15.5.2.)

- The comparison sign at which the division is made must be on the baseline of writing.
- The print copy may divide after a comparison sign, but the braille transcription follows Nemeth rules and divides before the comparison sign. (Example 15-34)
- The link begins in the appropriate cell according to the current format. (Examples 15-1 and 15-2)
- If the expression contains more than one link, it is not necessary to divide at every link unless it is a nested linked expression. (Example 15-2)
- Even if the anchor consists of only one letter or number, if the link will not fit on the line with the anchor, the line is divided after the anchor. (Example 15-3)
- When a line begins with a sign of comparison, the transition to a new braille line terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines. (Example 15-4)

- In itemized formats, if an anchor will not fit on the line with its identifier but it fits on the next line starting in the runover cell, put it there in order to keep the anchor intact. The identifier will then be the only item on the first line. (Examples 15-5 and 15-6)
- Material within mathematical grouping signs is a unit and should not be divided. (Example 15-7)

Example 15-1

(two layouts)

$1{,}778 + 1{,}294 + 865 + 905 + 2574 + 485 + 100\ >\ 8{,}000$

Embedded:

```
⠼⠂⠠⠶⠶⠦⠬⠂⠠⠆⠔⠲⠬⠦⠖⠢⠬⠔⠴⠢⠬⠆⠢⠶⠲⠬⠲⠦⠢⠬⠂⠴⠴
⠨⠂ ⠼⠦⠠⠴⠴⠴
```

As an embedded expression, the link continues on the next line in the runover cell of the current format.

Displayed:

```
⠀⠀⠼⠂⠠⠶⠶⠦⠬⠂⠠⠆⠔⠲⠬⠦⠖⠢⠬⠔⠴⠢⠬⠆⠢⠶⠲⠬⠲⠦⠢⠬⠂⠴⠴
    ⠨⠂ ⠼⠦⠠⠴⠴⠴
```

As a displayed expression, the link begins on the next line, indented two cells from the anchor.

Example 15-2

(displayed)

To factor the expression $-2ab + a^2 + b^2$,
Jared wrote: $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$
Dom wrote: $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$
Both solutions are correct. Explain.

```
1    ⠠⠞⠕ ⠋⠁⠉⠞⠕⠗ ⠮ ⠑⠭⠏⠗⠑⠎⠨⠝
2  ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆ ⠸⠱⠂
3    ⠠⠚⠜⠫ ⠺⠗⠕⠞⠑⠒ ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆
4      ⠨⠅ ⠃⠘⠆⠐⠤⠆⠁⠃⠬⠁⠘⠆ ⠨⠅ ⠷⠃⠤⠁⠾⠘⠆ ⠸⠱
5    ⠠⠙⠕⠍ ⠺⠗⠕⠞⠑⠒ ⠸⠩ ⠤⠼⠆⠁⠃⠬⠁⠘⠆⠐⠬⠃⠘⠆
6      ⠨⠅ ⠁⠘⠆⠐⠤⠆⠁⠃⠬⠃⠘⠆ ⠨⠅ ⠷⠁⠤⠃⠾⠘⠆ ⠸⠱
7  ⠠⠃⠕⠹ ⠎⠕⠇⠥⠰⠝⠎ ⠜⠑ ⠉⠕⠗⠗⠑⠉⠞⠲ ⠠⠑⠭⠏⠇⠁⠔⠲
```

Lines 1-2: Paragraph begins in cell 3 with runovers in cell 1.

Lines 3-6: Two displayed expressions – each begins in cell 3, with runover in cell 5.
Lines 4 and 6: It is not necessary to divide the linked expression at every comparison sign.
Line 7: Paragraph continues in the runover cell (cell 1).

Example 15-3

(embedded)

The degree of the polynomial $y = 6x^3 - 2x^2y^1 + x^2y^2 - 3x^2y^2 + z$ is 4.

⠀⠀⠠⠮ ⠙⠑⠛⠗⠑⠑ ⠷ ⠮ ⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇ ⠸⠩ ⠽
⠀⠨⠅ ⠼⠖⠭⠘⠒⠐⠤⠆⠭⠘⠆⠐⠽⠘⠂⠐⠬⠭⠘⠆⠐⠽⠘⠆⠐⠤⠒⠭⠘⠆⠐⠽⠘⠆⠐⠬⠵
⠸⠱ ⠊⠎ ⠼⠙⠲

This anchor consists of the letter y and sits alone at the end of line 1 because it does not fit on the same line as its long link.

Example 15-4

(as a subdivision starting in cell 3)

13.
a. $16x^4 + 81y^4 = (4x^2)^2 + (9y^2)^2$

⠼⠂⠒⠸⠲
⠀⠀⠰⠁⠸⠲ ⠼⠂⠖⠭⠘⠲⠐⠬⠦⠂⠽⠘⠲
⠀⠀⠀⠀⠨⠅ ⠷⠲⠭⠘⠆⠐⠾⠘⠆⠐⠬⠷⠔⠽⠘⠆⠐⠾⠘⠆

The link begins in the current runover cell—cell 5, in this case. Transition to a new line following a sign of comparison returns the reader to the baseline.

Example 15-5

(a main division starting in cell 1)

13. $16x^4 + 81y^4 = (4x^2)^2 + (9y^2)^2$

⠼⠂⠒⠸⠲
⠀⠀⠼⠂⠖⠭⠘⠲⠐⠬⠦⠂⠽⠘⠲ ⠨⠅ ⠷⠲⠭⠘⠆⠐⠾⠘⠆⠐⠬⠷⠔⠽⠘⠆⠐⠾⠘⠆

The expression begins in the current runover cell—cell 3, in this case—because it does not fit on the line above with its idenifier.

Example 15-6

(displayed)

Dividing by $a^2(a^2-c^2)$,

$$\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1. \qquad (3.3)$$

1 ⠀⠀⠠⠙⠊⠧⠊⠙⠬ ⠃⠽ ⠸⠩ ⠁⠘⠆⠐⠷⠁⠘⠆⠐⠤⠉⠘⠆⠐⠾⠠
2 ⠀⠀⠷⠒⠨⠒⠾
3 ⠀⠀⠀⠀⠹⠭⠘⠆⠐⠌⠁⠘⠆⠐⠼⠬⠹⠽⠘⠆⠐⠌⠁⠘⠆⠐⠤⠉⠘⠆⠐⠼ ⠨⠅ ⠼⠂
4 ⠀⠀⠀⠀⠸⠱⠲

Line 1: The paragraph begins in cell 3. There is no runover.
Line 2: The displayed material begins in cell 3. The label stands alone because the math will not fit on this line. (Review Section 7.2 in Lesson 7 regarding displayed material with labels printed to the right.)
Line 3: The displayed expression will fit on one line in the runover position (cell 5).
Line 4: The Nemeth Code terminator and punctuation do not fit on the line above. They are placed in the runover position (cell 5).

15.3.1 A Sign of Comparison within a Mathematical Unit

a. **Keep Together: Grouped Expression.** Items enclosed within grouping signs should not be divided unless the entire grouped expression will not fit on the line.

Example 15-7

(embedded)

{x | x is an even integer} ⊈ {x | x is an odd integer}

⠨⠷⠭ ⠳ ⠭ ⠊⠎ ⠁⠝ ⠑⠧⠑⠝ ⠊⠝⠞⠑⠛⠑⠗⠨⠾
⠌⠸⠐⠅⠱ ⠨⠷⠭ ⠳ ⠭ ⠊⠎ ⠁⠝ ⠕⠙⠙ ⠊⠝⠞⠑⠛⠑⠗⠨⠾

Each grouped expression is kept intact on one line. only the comparison sign outside of the grouped expressions is a suitable division site.

b. **Keep Together: Other Mathematical Units.** A comparison sign within a mathematical unit such as a modifier, superscript, subscript, fraction, or radical expression is not a suitable division site unless the mathematical unit will not fit on the line.

Example 15-8

(embedded)

$$\sum_{i=0}^{n}\sum_{j=0}^{m} = \lim_{n\to\infty} \underline{\qquad}$$

⠐⠨⠠⠎⠩⠊⠀⠨⠅⠀⠼⠴⠣⠝⠻⠐⠨⠠⠎⠩⠚⠀⠨⠅⠀⠼⠴⠣⠍⠻⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠨⠅⠀⠐⠇⠊⠍⠩⠝⠀⠫⠕⠀⠠⠿⠻⠀⠤⠤⠤⠤

Each modifier is kept intact on one line. Only the comparison sign on the baseline of writing is a suitable division site.

Instructions: Keep the following points in mind as you select runover sites. (1) Keep mathematical units intact on one line, if possible; (2) before dividing an expression, try placing the switch indicators on a different line; (3) before dividing an expression, try placing the identifier on a different line; (4) if the entire expression will not fit on the line, divide before a comparison sign on the baseline.

PRACTICE 15A

A. Solve the linear inequalities.

- i. $(x+\frac{10}{3})(x+\frac{19}{3}) > (3x+\frac{46}{3})(\frac{x}{3}+1)$
- ii. $\frac{2x}{3} - 3 > \frac{16x}{21} - \frac{13}{3} - \frac{2x}{15}$
- iii. $(a-1)^2 - (a-7)(a-3) < 2a + 0.8$

B. $2 \times 423 = (2 \times 400) + (2 \times 20) + (2 \times 3) = 800 + 40 + 6 = 846$

Binomial coefficients get their name because they are the *coefficients* in the expansion of a *binomial:*

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

15.4 Step ii: Divide Before an Operation Sign on the Baseline

A new line may begin before a sign of operation on the baseline of writing. It is not necessary to divide at every operation sign. A fraction ine is not considered to be an operation sign in this regard. See Section 15.5.1.

Keep these rules in mind as you study the examples.

- Use as much of the line as possible before dividing the expression.
- The operation sign at which the division is made must be on the baseline of writing.
- The print copy may divide after an operation sign, but the braille transcription follows Nemeth rules and divides *before* the operation sign. (Example 15-17)
- Transition to a runover line does not take the place of a necessary baseline indicator. If an operation sign follows a superscript, begin the new line with the baseline indicator. (Example 15-11) Similarly, if an operation sign follows a subscript that has a subscript indicator, begin the new line with the baseline indicator. If the subscript does not use a subscript indicator, there will be no subsequent baseline indicator. Review Lesson 6 regarding nonuse of the subscript indicator. (Example 15-16)
- If the transition to a new line is made before a minus sign and a numeral, or before a minus sign and a decimal point and a numeral, a numeric indicator is required. (Example 15-10)
- Material within mathematical grouping signs is a unit and should not be divided. (Example 15-12)
- In a linked expression, if either anchor or link is divided, *each link* must begin on a new line. (Examples 15-14, 15-15, and 15-17)

Example 15-9

(embedded)

$1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2}$

⠼⠂⠸⠹⠢⠌⠖⠸⠼⠬⠔⠸⠹⠂⠌⠆⠸⠼⠬⠒⠸⠹⠂⠌⠂⠆⠸⠼⠬⠆⠸⠹⠂⠌⠲⠸⠼
⠬⠒⠸⠹⠂⠌⠆⠸⠼

Runover line (line 2): A numeral preceded by a plus sign does not require a numeric indicator even when the plus sign begins a braille line.

Example 15-10

(displayed)

$$2x^4 + 14x^3 - 15y^3 + 5x^2 - 4y^2 + 16x - 10y + 31$$

```
⠀⠀⠼⠆⠭⠘⠲⠐⠬⠂⠲⠭⠘⠒⠐⠤⠂⠢⠽⠘⠒⠐⠬⠢⠭⠘⠆⠐⠤⠲⠽⠘⠆⠐⠬⠂⠖⠭
    ⠤⠼⠂⠴⠽⠬⠒⠂
```

Runover line (line 2): A numeral preceded by a minus sign requires a numeric indicator when the minus sign begins a braille line.

Example 15-11

(displayed)

$$49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2$$

```
⠀⠀⠼⠲⠔⠭⠘⠶⠐⠽⠘⠖⠐⠤⠖⠒⠭⠘⠖⠐⠽⠘⠲⠐⠬⠢⠖⠭⠘⠢⠐⠽⠘⠢
    ⠐⠬⠖⠲⠭⠘⠲⠐⠽⠘⠆
```

The baseline indicator is the first symbol in the runover line of this divided expression.

15.4.1 A Sign of Operation within a Mathematical Unit

a. **Keep Together: Grouped Expression.** Items enclosed within grouping signs should not be divided unless the entire grouped expression will not fit on the line.

Example 15-12

(embedded)

Find a solution, using PEMDAS: $(2^2 + 3^3) - 2^5 \div 4 + (22 \div 2 - 2 \cdot 5) + (14 - 6 \div 6)$

```
  ⠠⠋⠔⠙ ⠁ ⠎⠕⠇⠥⠰⠝⠂ ⠥⠎⠬ ⠠⠠⠏⠑⠍⠙⠁⠎⠒
⠸⠩ ⠷⠆⠘⠆⠐⠬⠒⠘⠒⠐⠾⠤⠼⠆⠘⠢⠐⠨⠌⠲⠬⠷⠆⠆⠨⠌⠆⠤⠆⠡⠢⠾
⠬⠷⠂⠲⠤⠖⠨⠌⠖⠾ ⠸⠱
```

Each grouped expression is kept intact on one line. Only the operation sign outside of the grouped expressions is a suitable division site.

b. **Keep Together: Fractions and Other Mathematical Units.** An operation sign within a mathematical unit such as a modifier, superscript, subscript, fraction, or radical expression is not a suitable division site unless the mathematical unit will not fit on the line.

Example 15-13

(displayed)

$$\frac{1}{y^2-6y+8}+\frac{1}{y^2-16}-\frac{5}{y^2+2y-8}$$

⠀⠀⠹⠂⠌⠽⠘⠆⠐⠤⠖⠽⠬⠦⠼⠬⠹⠂⠌⠽⠘⠆⠐⠤⠂⠖⠼
⠀⠀⠀⠀⠤⠹⠢⠌⠽⠘⠆⠐⠬⠆⠽⠤⠦⠼

Each fraction is kept intact on one line. Only the operation signs on the baseline of writing are suitable division sites.

15.4.2 **Linked Expressions.** If an anchor or a link must be divided, further rules apply. If a division occurs within the anchor or its link, a division must always be made before the link (Step i, divide before a comparison sign on the baseline). Furthermore, in a linked expression with more than one link, division must occur before each link if the anchor or if any link requires division. The resulting transcription maintains an orderly representation of mathematical units. (For a closer look at the division of linked expressions, see Section 15.5.2.)

Example 15-14

(displayed)

$$20\frac{1}{6}=1\frac{5}{6}+9\frac{1}{2}+3\frac{1}{12}+2\frac{1}{4}+3\frac{1}{2}$$

⠀⠀⠼⠆⠴⠸⠹⠂⠌⠖⠸⠼
⠀⠀⠀⠀⠨⠅⠀⠼⠂⠸⠹⠢⠌⠖⠸⠼⠬⠔⠸⠹⠂⠌⠆⠸⠼⠬⠒⠸⠹⠂⠌⠂⠆⠸⠼
⠀⠀⠀⠀⠬⠆⠸⠹⠂⠌⠲⠸⠼⠬⠒⠸⠹⠂⠌⠆⠸⠼

Step i: The linked expression is divided before the equals sign. Step ii: The long link is divided before a plus sign.

Example 15-15

(itemized)

5. $144\ \text{ft}^2+17\ \text{ft}^2+112\ \text{ft}^2+15\ \text{ft}^2-131\ \text{ft}^2=N$

⠼⠢⠸⠲⠀⠼⠂⠲⠲⠀⠋⠞⠘⠆⠀⠬⠂⠶⠀⠋⠞⠘⠆⠀⠬⠂⠂⠆⠀⠋⠞⠘⠆
⠀⠀⠬⠂⠢⠀⠋⠞⠘⠆⠀⠤⠼⠂⠒⠂⠀⠋⠞⠘⠆
⠀⠀⠨⠅⠀⠠⠝

Working backwards, the linked expression is divided before the equals sign (Step i). The long anchor is divided before a plus sign (Step ii). The link must not be placed on line 2, even though it will fit, because Step ii has been applied—that is, because the anchor has been divided.

Example 15-16

(itemized)

1. Consider the following chemical equations.

   a. $CH_3CH_2CH_2COONa+CH_3OH \rightarrow CH_3CH_2CH_2COOCH_3+NaOH$

   b. …

```
1 ⠼⠁⠲ ⠠⠒⠎⠊⠙⠻ ⠮ ⠋⠕⠇⠇⠪⠬ ⠡⠑⠍⠊⠉⠁⠇ ⠑⠟⠥⠁⠰⠝⠎⠲ ⠸⠩
2   ⠰⠁⠸⠲⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
3     ⠠⠉⠠⠓⠒⠠⠉⠠⠓⠆⠠⠉⠠⠓⠆⠠⠉⠠⠕⠠⠕⠠⠝⠁⠬⠠⠉⠠⠓⠒⠠⠕⠠⠓
4     ⠫⠕ ⠠⠉⠠⠓⠒⠠⠉⠠⠓⠆⠠⠉⠠⠓⠆⠠⠉⠠⠕⠠⠕⠠⠉⠠⠓⠒⠀⠀⠀⠀⠀⠀⠀
5     ⠬⠠⠝⠁⠠⠕⠠⠓
6   ⠰⠃⠸⠲ ⠄⠄⠄ ⠸⠱
```

Lines 2-3: The anchor does not fit on the line with the identifier, but it will will fit entirely on the next line. It begins in the runover position for subdivisions, which is cell 5.
Lines 4-5: Line 4 begins with the comparison sign (right-pointing arrow). The link will not fit on one line. It is divided before the plus sign. Because the subscript 3 does not require a subscript indicator, there is no baseline indicator needed before the plus sign on line 5.

Example 15-17

(embedded)

Show how Margaret solved this problem by using fraction cubes. $1\frac{5}{6}+9\frac{1}{2}+3\frac{1}{12}+2\frac{1}{4}+3\frac{1}{2}=18\frac{26}{12}=20\frac{2}{12}=20\frac{1}{6}$. Can you express the result as a decimal?

```
1   ⠠⠩⠪ ⠓⠪ ⠠⠍⠜⠛⠜⠑⠞ ⠎⠕⠇⠧⠫ ⠹ ⠏⠗⠕⠃⠇⠑⠍ ⠃⠽ ⠥⠎⠬
2 ⠋⠗⠁⠉⠰⠝ ⠉⠥⠃⠑⠎⠲ ⠸⠩ ⠼⠂⠸⠹⠢⠌⠖⠸⠼⠬⠔⠸⠹⠂⠌⠆⠸⠼⠀⠀⠀⠀⠀
3 ⠬⠒⠸⠹⠂⠌⠂⠆⠸⠼⠬⠆⠸⠹⠂⠌⠲⠸⠼⠬⠒⠸⠹⠂⠌⠆⠸⠼⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
4 ⠨⠅ ⠼⠂⠦⠸⠹⠆⠖⠌⠂⠆⠸⠼
5 ⠨⠅ ⠼⠆⠴⠸⠹⠆⠌⠂⠆⠸⠼
6 ⠨⠅ ⠼⠆⠴⠸⠹⠂⠌⠖⠸⠼ ⠸⠱⠲ ⠠⠉ ⠽ ⠑⠭⠏⠗⠑⠎⠎ ⠮ ⠗⠑⠎⠥⠇⠞
7 ⠵ ⠁ ⠙⠑⠉⠊⠍⠁⠇⠦
```

Line 2: The anchor begins on line 2 and is divided before a plus sign.
Line 3: The anchor is completed on this line.
Lines 4-6: Because the anchor is divided, the expression is also divided before <u>each</u> link even if there is room on the line for more.
Line 6: The paragraph continues following the completion of the embedded expression.

Instructions: In addition to the tactics outlined with PRACTICE 15A, keep the following point in mind as you select runover sites. Apply Step i (divide before a comparison sign) before applying Step ii (divide before an operation sign).

PRACTICE 15B

A. $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 = 490°$

B. $2\frac{3}{4}\text{ yd} + 1\frac{3}{4}\text{ yd} + \frac{3}{4}\text{ yd} = 5\frac{1}{4}\text{ yd}$

C. $\sqrt{(x + a^2) + (y + a^2)} - \sqrt{(x - a^2) + (y - a^2)} = \pm 2a$

D. Sommer's routine can be represented by the following expression.

$$\text{Routine S:}\ \ 5 + 3 + 4(1 + (-1)) + (-3) + (-5) + 3\big(5 + (-2)\big) + 1$$

Draw a simple diagram to represent *Routine S*.

15.5 Step iii: Divide Before a Mathematical Unit

The need to apply this rule occurs most often when the line length is restricted due to indented margins applied to displayed material. By seeing an expression as a series of mathematical units, the transcriber can make wise decisions when a long expression must be divided. Prudent division sites allow the reader to mentally reassemble the expression in an orderly fashion.

15.5.1 **A Fraction is a Unit.** A fraction as a whole is a unit. Within a fraction, the numerator and the denominator are each a unit. If an expression requires division when a fraction is encountered, follow these guidelines.

- If the entire fraction will fit on one line, divide before the opening fraction indicator. If a baseline indicator is required before the fraction indicator, divide the expression before the baseline indicator.
- If the entire fraction will not fit on one line, divide before the fraction line. (Examples 15-18, 15-20, and 15-21) If a baseline indicator is required before the fraction line, divide the expression before the baseline indicator. (Example 15-19)
- If the numerator or denominator requires division, division must also be made before the fraction line. (Example 15-19)
- If the fraction is part of a mixed number, see Section 15.5.3.

Example 15-18

(displayed)

$$\frac{93{,}000 \times 0.0006 \times 2.0 \times 10.56}{32 \times 1.257 \times 10}$$

⠀⠀⠹⠔⠒⠠⠴⠴⠴⠈⠡⠴⠨⠴⠴⠴⠖⠈⠡⠆⠨⠴⠈⠡⠂⠴⠨⠢⠖⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀

⠌⠒⠆⠈⠡⠂⠨⠆⠢⠶⠈⠡⠂⠴⠼

The fraction will not fit on one line. Division is made before the fraction line.

Example 15-19

(displayed)

$$\frac{e^{1/n} + e^{2/n} + e^{3/n} + \ \dots + e^{(n-1)/n} + e^{n/n}}{n}$$

1 ⠀⠀⠹⠑⠘⠂⠸⠌⠝⠐⠬⠑⠘⠆⠸⠌⠝⠐⠬⠑⠘⠒⠸⠌⠝⠐⠬ ⠄⠄⠄⠀⠀⠀⠀⠀⠀⠀⠀⠀
2 ⠬⠑⠘⠷⠝⠤⠂⠾⠸⠌⠝⠐⠬⠑⠘⠝⠸⠌⠝⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
3 ⠐⠌⠝⠼

Line 3: The first division is made before the fraction line. A baseline indicator is needed before this fraction line because the last item in the numerator is a superscript. The baseline indicator begins the runover line.
Lines 1-2: The numerator will not fit on one line. It is divided before a sign of operation that is situated on the baseline within the numerator. Even though the denominator will fit on line 2, division is required before the fraction line.

15.5.2 **Dividing a Linked Expression, Step By Step.** If a Step iii division must be made within an anchor or within a link, work your way through the steps as follows.

a. First, divide before the link, starting the runover line with the comparison symbol (step i). If there is more than one link, start a new line with each link.

b. Within the anchor or link that will not fit on the line, divide before an operation sign on the baseline, if present (step ii).

c. Finally, divide before a mathematical unit (step iii).

Examples 15-20 and 15-21 demonstrate this approach using a fraction as the Step iii mathematical unit that must be divided.

Example 15-20

(embedded linked expression)

In this model, $\frac{\text{Fixed Cost}}{\text{Unit Revenue}-\text{Marginal Cost}} = P$

1 ⠠⠔ ⠹ ⠍⠕⠙⠑⠇⠂ ⠸⠩ ⠹⠠⠋⠊⠭⠑⠙ ⠠⠉⠕⠎⠞⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
2 ⠌⠠⠥⠝⠊⠞ ⠠⠗⠑⠧⠑⠝⠥⠑⠤⠠⠍⠁⠗⠛⠊⠝⠁⠇ ⠠⠉⠕⠎⠞⠼⠀⠀⠀⠀⠀⠀⠀⠀
3 ⠨⠅ ⠠⠏ ⠸⠱

This anchor (a fraction) needs to be divided. Working through the steps,
Step i (sign of comparison on the baseline): The first division is made before the link. This will become line 3, which begins with the comparison sign.
Step iii (mathematical unit): Since the fraction will not fit on one line, division is made before the fraction line. The denominator fits on line 2, beginning with the fraction line and ending with the closing fraction indicator.
(see next page)

Line 1: Note that, since the fraction will not fit on one line, the numerator is placed on the same line as the narrative.
Line 3: Even though the link (= P) will fit on line 2, division before the comparison sign is required (step i).

Example 15-21

(displayed linked expression)

$$x > \frac{93{,}000 \times 0.0006 \times 2.0 \times 10.56}{32 \times 1.257 \times 10} + 76$$

1 ⠀⠀⠭

2 ⠀⠀⠀⠀⠨⠂ ⠹⠔⠒⠠⠴⠴⠴⠈⠡⠴⠨⠴⠴⠴⠖⠈⠡⠆⠨⠴⠈⠡⠂⠴⠨⠢⠖

3 ⠌⠒⠆⠈⠡⠂⠨⠆⠢⠶⠈⠡⠂⠴⠼

4 ⠬⠶⠖

This link needs to be divided. Working through the steps,
Step i: The first division is made before the link. Line 2 begins with the comparison sign.
Step ii: The link is first divided before the sign of operation on the baseline—the plus sign. This will become line 4.
Step iii: The fraction must be divided. A division is made before the fraction line. Even though +76 will fit on line 3, it is placed on line 4 according to step ii division guidelines.

15.5.3 **A Mixed Number is a Unit.** Do not separate the fractional part of a mixed number from its whole number. Review the topic of mixed numbers in Lesson 8.

Example 15-22

(displayed linked expression)

$$7\frac{4}{9} + 13\frac{7}{12} + 9\frac{5}{18} = 7\frac{16}{36} + 13\frac{21}{36} + 18\frac{10}{36} = 38\frac{47}{36} = 38 + 1 + \frac{11}{36} = 39\frac{11}{36}$$

1 ⠼⠶⠸⠹⠲⠌⠔⠸⠼⠬⠂⠒⠸⠹⠶⠌⠂⠆⠸⠼⠬⠔⠸⠹⠢⠌⠂⠦⠸⠼

2 ⠨⠅ ⠼⠶⠸⠹⠂⠖⠌⠒⠖⠸⠼⠬⠂⠒⠸⠹⠆⠂⠌⠒⠖⠸⠼

3 ⠬⠂⠦⠸⠹⠂⠴⠌⠒⠖⠸⠼

4 ⠨⠅ ⠼⠒⠦⠸⠹⠲⠶⠌⠒⠖⠸⠼

5 ⠨⠅ ⠼⠒⠦⠬⠂⠬⠹⠂⠂⠌⠒⠖⠼

6 ⠨⠅ ⠼⠒⠔⠸⠹⠂⠂⠌⠒⠖⠸⠼ ⠸⠱

Line 1: The anchor fits on line 1, starting in cell 3.
Lines 2-3: The first link requires division. Each mixed number is a mathematical unit and so is not divided. Division is made before the second plus sign.
Lines 4-6: Each link must begin on a new line because one of the links is divided.

15.5.4 **A Base and its Exponent are a Unit.** The general rule is to keep a base and its exponent together on the same line, but if the unit must be divided, begin the new line with the superscript indicator.

If the exponent (superscript) does not fit in its entirety on the new line, apply further division rules within the superscript. If transition to a new braille line must be made within a superscript, the level in effect continues just as it would if the expression were not divided. (Example 15-25)

Similar rules apply to a subscript and its related item.

Example 15-23

(displayed linked expression)

1. Using the product of powers property to simplify the first term,

$$(6^2 6^3 x^{\frac{1}{2}-2} y^{\frac{1}{3}-3} + z)^3 = (6^{2+3} x^{\frac{1}{2}-2} y^{\frac{1}{3}-3} + z)^3.$$

1 ⠼⠁⠲ ⠠⠥⠎⠬ ⠮ ⠏⠗⠕⠙⠥⠉⠞ ⠷ ⠏⠪⠻⠎ ⠏⠗⠕⠏⠻⠞⠽ ⠞⠕
2 ⠀⠀⠎⠊⠍⠏⠇⠊⠋⠽ ⠮ ⠋⠌ ⠞⠻⠍⠂ ⠸⠩
3 ⠀⠀⠀⠀⠷⠖⠘⠆⠐⠖⠘⠒⠐⠭⠘⠹⠂⠌⠆⠼⠤⠆⠐⠽⠘⠹⠂⠌⠒⠼⠤⠒⠐⠬⠵⠾⠘⠒
4 ⠀⠀⠀⠀⠀⠀⠨⠅ ⠷⠖⠘⠆⠬⠒⠐⠭⠘⠹⠂⠌⠆⠼⠤⠆⠐⠽⠘⠹⠂⠌⠒⠼⠤⠒⠐⠬⠵⠾
5 ⠀⠀⠀⠀⠀⠀⠘⠒ ⠸⠱⠲

Line 3: The anchor fits on one line.
Line 4: The link must be divided. The expression between the parentheses (the base) fits on the line but the superscript (the exponent) does not.
Line 5: The superscript begins the runover line, starting with the superscript indicator.

Example 15-24

(embedded)

$$\frac{1}{2}\left[\frac{1}{2} sin\, 2u - u\right]_{A(u=0)}^{P(u=u)}$$

⠹⠂⠌⠆⠼⠈⠷⠹⠂⠌⠆⠼⠎⠊⠝ ⠼⠆⠥⠤⠥⠈⠾⠰⠠⠁⠷⠥ ⠰⠨⠅ ⠼⠴⠾
⠘⠠⠏⠷⠥ ⠘⠨⠅ ⠥⠾

The expression enclosed between brackets is a unit and so is not divided. The subscript is transcribed first, according to Nemeth rules for simultaneous superscripts and subscripts (Lesson 6). The subscript fits on the line but the superscript does not. The superscript begins the runover line, starting with the superscript indicator.

Example 15-25

(embedded)

$$x^{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\,\dots\,+\frac{1}{n}}$$

⠭⠘⠂⠬⠹⠂⠌⠆⠼⠬⠹⠂⠌⠒⠼⠬⠹⠂⠌⠲⠼⠬⠹⠂⠌⠢⠼⠬⠹⠂⠌⠖⠼⠬ ⠄⠄⠄
⠬⠹⠂⠌⠝⠼

The superscript level initiated on line 1 continues on line 2 without need for restatement, as it would were division not necessary. (Review Section 6.12.4 in Lesson 6 regarding an ellipsis in the superscript position.)

Instructions: Apply the tactics outlined with PRACTICE 15A and PRACTICE 15B as you select runover sites. Note that the ellipsis in item c. is on the baseline of writing. Review Section 8.22.4 in Lesson 8 regarding the layout of problem 2.

PRACTICE 15C

1. Compute and/or simplify.

a. $$\frac{\left(\frac{3}{2}\right)\times\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{1\times 2\times 3}$$

b. $$\frac{\frac{\mathrm{d}x}{\mathrm{d}t}\,\frac{\mathrm{d}^2y}{\mathrm{d}t^2}-\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\,\frac{\mathrm{d}y}{\mathrm{d}t}}{(\mathrm{d}x/\mathrm{d}t)^3}$$

c. $(\pm)a_{1i_1}a_{2i_2}a_{3i_3}a_{4i_4}\dots a_{ni_n}$

2. $$\frac{x^2+2}{x(2x^2+1)^2}=\frac{A}{x}+\frac{Bx+C}{2x^2+1}+\frac{Dx+E}{(2x^2+1\,)^2}$$

$$=\frac{A\left(4x^4+4x^2+1\right)+B\left(2x^4+x^2\right)+C\left(2x^3+x\right)+Dx^2+Ex}{x(2x^2+1)^2}$$

15.5.5 A Grouped Expression is a Unit

a. **A Series of Unspaced Grouped Expressions.** If consecutive groupings do not fit on the line, division may be made between groupings, beginning a new line with the left grouping symbol of the next factor.

Example 15-26

(embedded)

Multiply these six terms: $(x + 5)(2x - 1)(x^2 + 2x + 1)\ (x^2 + 3x + 4)(4 - x)(5x + 7)$.

⠠⠍⠥⠇⠞⠊⠏⠇⠽ ⠘⠮ ⠎⠊⠭ ⠞⠻⠍⠎⠒ ⠸⠩ ⠷⠭⠬⠢⠾⠷⠆⠭⠤⠂⠾
⠷⠭⠘⠆⠐⠬⠆⠭⠬⠂⠾⠷⠭⠘⠆⠐⠬⠒⠭⠬⠲⠾⠷⠲⠤⠭⠾⠷⠢⠭⠬⠶⠾ ⠸⠱⠲

Division is made between terms that are enclosed between parentheses.

b. **A Grouped Expression Will Not Fit on the Line.** If a grouped expression will not fit on one line, follow the guidelines in this lesson to determine the best place to divide it. If the grouped expression is an enclosed list, divide after a comma.

Example 15-27

(displayed linked expression)

$$(49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2) \div 7x^3y^2 = ?$$

1 ⠀⠀⠷⠲⠔⠭⠘⠶⠐⠽⠘⠖⠐⠤⠖⠒⠭⠘⠖⠐⠽⠘⠲⠐⠬⠢⠖⠭⠘⠢⠐⠽⠘⠢⠀⠀⠀⠀⠀⠀
2 ⠐⠬⠖⠲⠭⠘⠲⠐⠽⠘⠆⠐⠾
3 ⠨⠌⠶⠭⠘⠒⠐⠽⠘⠆
4 ⠨⠅ ⠿

Line 1: The entire grouped expression will not fit on one line. Division is made before the baseline indicator associated with a plus sign.
Line 3: Although the rest of the anchor will fit on line 2, division is made before the operation sign (division symbol) according to Step ii.
Line 4: A new line must begin with the equals sign because Step i (dividing before a comparison sign) must be applied when the anchor is divided.

Example 15-28

(displayed linked expression)

$$y = \left(-8,\ -4,\ \frac{1}{2},\ 3,\ 8,\ 10,\ 20\frac{3}{4},\ 100 \right)$$

⠀⠀⠽
⠨⠅ ⠷⠤⠦⠠ ⠤⠲⠠ ⠹⠂⠌⠆⠼⠠ ⠒⠠ ⠦⠠ ⠂⠴⠠⠀⠀⠀⠀⠀⠀⠀⠀
⠆⠴⠸⠹⠒⠌⠲⠸⠼⠠ ⠂⠴⠴⠾

The first division is made before the comparison sign (Step i). The link is an enclosed list and will not fit on one line. Division is made after a comma. See Lesson 4 for further rules regarding enclosed lists.

PRACTICE 15D

1. Multiply these four polynomials: $(4x^9y)(7x^8y^2)(5x^7y^3)(6x^6y^4)$.
2. $\pm\left(\dfrac{a^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{1}{3}} + b^{\frac{2}{3}}}{3}\right)^{\frac{3}{2}}$
3. $(16x^4 + 8x^3y + 4x^2y^2 + 2xy^3 + y^4)(2x - y) = 32x^5 - y^5$

15.6 Step iv: Divide After a Termination Indicator

If no suitable division site can be found within a long string of expressions, and if a termination indicator is present, begin a new line after the termination indicator.

15.6.1 **A Radical Expression is a Unit.** Division may be made after the termination indicator that ends a radical expression. If the entire radical expression will not fit on the line, apply division strategies to the radicand.

Example 15-29

(displayed)

$$\sqrt[5]{32a^5b^5c^5d^5}\ \sqrt[5]{2abc^2d^3e^2}$$

```
⠀⠀⠣⠢⠜⠒⠆⠁⠘⠢⠐⠃⠘⠢⠐⠉⠘⠢⠐⠙⠘⠢⠐⠻
    ⠣⠢⠜⠆⠁⠃⠉⠘⠆⠐⠙⠘⠒⠐⠑⠘⠆⠐⠻
```

Division is made after the termination indicator of the first radical expression.

Example 15-30

(itemized)

2. $\sqrt[3]{x^1}\ \sqrt[6]{x^2}\ \sqrt[12]{x^3}\ \sqrt[24]{x^4}\ \sqrt[48]{x^5}\ \ldots$

```
⠼⠆⠸⠲⠀⠀⠣⠒⠜⠭⠘⠂⠐⠻⠣⠖⠜⠭⠘⠆⠐⠻⠣⠂⠆⠜⠭⠘⠒⠐⠻⠣⠆⠲⠜⠭⠘⠲⠐⠻
  ⠣⠲⠦⠜⠭⠘⠢⠐⠻⠀⠄⠄⠄
```

Division is made after the termination indicator of the fourth radical expression.

15.6.2 **A Modified Expression is a Unit.** Division may be made after the termination indicator that ends an expression which is modified according to the five-step rule of modification. (Lesson 12). If the entire modified expression will not fit on the current line, it is acceptable to divide before the directly-over or the directly-under indicator. The next section discusses strategies for dividing longer modified expressions.

Note: In item 2, the expression is displayed.

PRACTICE 15E

1. $\sqrt[3]{x^1}\,\sqrt[6]{x^2}\,\sqrt[12]{x^3}\,\sqrt[24]{x^4}\,\sqrt[48]{x^5}\ldots$
2. The general solution for one root of the cubic equation is

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\,.$$

15.7 Function Notation, Integral Notation, Sigma Notation, and Pi Notation

We hope you will bear with us as we conclude this lesson with some rather complicated-looking notation which is commonly encountered in the study of calculus. The Nemeth Code gives basic guidelines for choosing runover sites in a long mathematical expression. In this lesson manual, we look more specifically at the topic as it applies to functions, integral, Sigma, and Pi notation. Frequently, the transcriber needs to choose between which mathematical unit to keep intact and which unit to divide in order to present such notation clearly.

There often is more than one valid layout. In this section, we have collected advice from mathematicians to help in your decision making. Please note that these strategies are not explicitly discussed in the Nemeth Code.

15.7.1 **A Function and its Argument are a Unit.** The space between a function and the following expression (the "argument") is not a suitable division site. If the entire expression will not fit on the line, apply division strategies to the argument. Sometimes, this will mean dividing within a mathematical unit.

Example 15-31

In the logarithmic number system, LNS $= -\log\left[2\sin\frac{\theta}{2}\cdot e^{-i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)}\right]$.

⠀⠀⠠⠔ ⠮ ⠇⠕⠛⠜⠊⠹⠍⠊⠉ ⠝⠥⠍⠃⠻ ⠎⠽⠌⠑⠍⠂ ⠸⠩ ⠠⠠⠇⠝⠎
⠨⠅ ⠤⠇⠕⠛ ⠈⠷⠆⠎⠊⠝ ⠹⠨⠹⠌⠆⠼
⠡⠑⠘⠤⠊⠷⠹⠨⠏⠌⠆⠼⠤⠹⠨⠹⠌⠆⠼⠾⠐⠈⠾ ⠸⠱⠲

LNS is the abbreviation for "logarithmic number system" and so uses the double capitalization indicator. Even though the grouped expression will fit on one line, division is made within the argument, not at the space after the abbreviated function name.

PRACTICE 15F

1. Daylight …
   a. The number of minutes of daylight for any location at 60° N latitude is modeled by the function below, where $d =$ day of the year.

$$m(d) = 390\sin\left[\frac{2\pi}{365.25}(d-80)\right] + 738$$

15.7.2 **Integral Notation.** It is helpful to understand that this mathematical unit is comprised of the entire expression from the integral symbol to the dx, dy, etc. (the "differential"). To a mathematician, it is important to keep the integral and its associated modifiers or superscript/subscript intact, and also to begin the associated expression which follows the integral on the same line, if possible. Additionally, it is desirable not to leave the differential dangling alone on a line.

When a choice must be made, it is advisable to apply division strategies to the expression between the integral and the differential. This may mean dividing within a mathematical unit such as a fraction or a grouped expression. This approach is illustrated showing the same example using two types of integral notation—printing the limits as a superscript and subscript (Example 15-32), and printing the limits as modifiers (Example 15-33).

Example 15-32

$$\int_{-\infty}^{\infty} \frac{\frac{a(x)}{b(x)}}{\frac{c(x)}{f(x)}} dx$$

Embedded: The 40-cell expression will fit on one line.

⠮⠰⠤⠠⠿⠘⠿⠐⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

See Section 13.1.8.a in Lesson 13 regarding simultaneous superscripts and subscripts with the slanted integral symbol.

Using displayed margins starting in cell 5, the expression will no longer fit on one line. Two options are illustrated.

Displayed: Option 1, dividing before dx (not recommended).

⠀⠀⠀⠀⠮⠰⠤⠠⠿⠘⠿⠐⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼
⠙⠭

Although this interpretation keeps the fraction intact, dx is now in an undesirable location, sitting alone on the next line.

Displayed: Option 2, dividing the fraction (recommended).

⠀⠀⠀⠀⠮⠰⠤⠠⠿⠘⠿⠐⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼
⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

By dividing before the complex fraction line, the integral notation is on the same line as the beginning of its related expression.

Example 15-33

$$\int_{-\infty}^{\infty} \frac{\frac{a(x)}{b(x)}}{\frac{c(x)}{f(x)}}\, dx$$

Embedded: The 40-cell expression will fit on one line.

⠐⠮⠩⠤⠠⠿⠣⠠⠿⠻⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

Using displayed margins, the expression will no longer fit on one line. Three options are illustrated.

Displayed: Option 1, dividing after the termination indicator (less desirable).

⠀⠀⠐⠮⠩⠤⠠⠿⠣⠠⠿⠻
⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

The Nemeth Code recommends dividing after the termination indicator of the modified integral. However, this separates the integral from the related expression which follows it. The mathematicians we consulted prefer Option 2.

Displayed: Option 2, dividing the fraction (recommended).

⠀⠀⠐⠮⠩⠤⠠⠿⠣⠠⠿⠻⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼
⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

By dividing before the complex fraction line, the integral symbol is on the same line as the beginning of its related expression.

Displayed: Option 3, dividing before dx (not recommended).

⠀⠀⠐⠮⠩⠤⠠⠿⠣⠠⠿⠻⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼
⠙⠭

The entire fraction will fit on the first line, but dx is now in an undesirable location, sitting alone on the next line.

Instructions: Make your division decision based on the mathematician's preference. (The unusual letter in the last numerator is a lowercase Greek zeta.)

PRACTICE 15G

The solutions involving the direct functions

$$\int \frac{\log^2(z+1)}{z}\,\mathrm{d}z = \log(-z)\log^2(z+1) + 2\mathrm{Li}_2(z+1)\log(z+1) - 2\mathrm{Li}_3(z+1)$$

and

$$\int_0^\infty \frac{\log(t+1)\log\left(1+\frac{1}{t^2}\right)}{t}\,\mathrm{d}t = C\pi - \frac{3\zeta(3)}{8}$$

can be found in Chapter 7.

15.7.3 **Sigma and Pi Notation.** To a mathematician, it is important to keep the Sigma or Pi and its associated modifiers or superscript/subscript intact, and also to begin the associated expression which follows (the "argument") on the same line, if possible. When a choice must be made, it is advisable to apply division strategies to the argument. This may mean dividing within a mathematical unit such as a fraction or a grouped expression.

Example 15-34

$$\prod_{i=1}^{2}\prod_{j=4}^{6}(3ij) = \prod_{i=1}^{2}\big((3i\cdot 4)(3i\cdot 5)(3i\cdot 6)\big)$$

Displayed: Option 1, dividing after the termination indicator.

⠀⠀⠐⠨⠠⠏⠩⠊ ⠨⠅ ⠼⠂⠣⠆⠻⠐⠨⠠⠏⠩⠚ ⠨⠅ ⠼⠲⠣⠖⠻⠷⠒⠊⠚⠾
    ⠨⠅ ⠐⠨⠠⠏⠩⠊ ⠨⠅ ⠼⠂⠣⠆⠻
    ⠷⠷⠒⠊⠡⠲⠾⠷⠒⠊⠡⠢⠾⠷⠒⠊⠡⠖⠾⠾

The link must be divided. The Nemeth Code recommends dividing after the termination indicator of the modification. Although this keeps the grouped expression intact, it separates the Pi notation from its related expression. The mathematicians we consulted prefer Option 2.

Displayed: Option 2, dividing between grouped factors.

⠀⠀⠐⠨⠠⠏⠩⠊ ⠨⠅ ⠼⠂⠣⠆⠻⠐⠨⠠⠏⠩⠚ ⠨⠅ ⠼⠲⠣⠖⠻⠷⠒⠊⠚⠾
    ⠨⠅ ⠐⠨⠠⠏⠩⠊ ⠨⠅ ⠼⠂⠣⠆⠻⠷⠷⠒⠊⠡⠲⠾⠷⠒⠊⠡⠢⠾
    ⠷⠒⠊⠡⠖⠾⠾

The link must be divided. To keep the argument starting on the same line as the Pi notation, division is made between factors inside the larger grouping signs.

Example 15-35

$$d(\vec{x},\vec{y}) =$$

$$\sum_{\substack{Z_{xy}\in\vec{Z}_{xy}\\ \forall x\in\vec{x}\\ \forall y\in\vec{y}}} f\left(Z_{xy}\right)$$

Displayed: Option 1, dividing after the termination indicator.

```
1  ⠀⠀⠙⠷⠐⠭⠣⠫⠕⠻⠠ ⠐⠽⠣⠫⠕⠻⠾
2      ⠨⠅ ⠐⠨⠠⠎⠩⠠⠵⠰⠭⠽ ⠈⠑ ⠐⠠⠵⠣⠫⠕⠻⠰⠭⠽
3      ⠐⠩⠩⠈⠯⠭ ⠈⠑ ⠐⠭⠣⠫⠕⠻⠩⠩⠩⠈⠯⠽ ⠈⠑ ⠐⠽⠣⠫⠕⠻⠻
4      ⠋⠷⠠⠵⠰⠭⠽⠐⠾
```

Line 1: The anchor fits on one line, in cell 3.
Line 2: The link begins in cell 5. The entire modified expression will not fit on the line. Division is made before the second order directly-under indicator, including the baseline indicator associated with it.
Line 3: There is not enough room on this line to complete the link. The Nemeth Code recommends dividing after the termination indicator of the modification.
Line 4: The argument is alone on this line. The mathematicians we consulted prefer Option 2.

Displayed: Option 2, dividing before a directly-under indicator.

```
1  ⠀⠀⠙⠷⠐⠭⠣⠫⠕⠻⠠ ⠐⠽⠣⠫⠕⠻⠾
2      ⠨⠅ ⠐⠨⠠⠎⠩⠠⠵⠰⠭⠽ ⠈⠑ ⠐⠠⠵⠣⠫⠕⠻⠰⠭⠽
3      ⠐⠩⠩⠈⠯⠭ ⠈⠑ ⠐⠭⠣⠫⠕⠻
4      ⠩⠩⠩⠈⠯⠽ ⠈⠑ ⠐⠽⠣⠫⠕⠻⠻⠋⠷⠠⠵⠰⠭⠽⠐⠾
```

Line 1: The anchor fits on one line, in cell 3.
Line 2: The link begins in cell 5. The entire modified expression will not fit on the line. Division is made before the second order directly-under indicator, including the baseline indicator associated with it.
Line 3: There is not enough room on this line to complete the link. It is undesirable to leave the argument alone on the next line, so division is made before the third order directly-under indicator.

Instructions: Transcribe this expression as if it were embedded within narrative, beginning with an opening Nemeth Code switch indicator in cell 1. Make your division decision based on the mathematician's preference.

PRACTICE 15H

$$\sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n} \frac{z^n}{n!}$$

15.7.4 **Notation Combinations.** As with any formatting choice, consistency in your treatment of runovers in complicated mathematical expressions will give your reader the advantage of knowing what to expect. By keeping in mind the strategies learned in this lesson, your transcription will be well done.

Example 15-36

$$\int_a^b f\left(x\right)dx = \lim_{n\to\infty}\sum_{i=1}^{n}\Delta x\cdot f\left(x_i\right)$$

Displayed: Option 1, dividing before the Sigma notation.

```
1  ⠀⠀⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭
2      ⠨⠅ ⠐⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻
3      ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠂⠣⠝⠻⠨⠠⠙⠭⠡⠋⠷⠭⠰⠊⠐⠾
```

Line 1: The anchor fits on one line.
Line 2: Division is made before the Sigma notation, although this puts the function's argument on a different line.
Line 3: The entire Sigma notation fits on one line.

Displayed: Option 2, dividing before the multiplication dot.

```
1  ⠀⠀⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭
2      ⠨⠅ ⠐⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠂⠣⠝⠻⠨⠠⠙⠭
3      ⠡⠋⠷⠭⠰⠊⠐⠾
```

Line 1: The anchor fits on one line.
Line 2: The function's argument begins on this line. In the Sigma notation, the argument is divided before the multiplication dot.
Line 2: Recall from Lesson 14 that, when a function name carries a modifier, the required space after the function name follows the termination of the modifier.

Instructions: Make your division decision based on the mathematician's preference.

PRACTICE 15I

1. The rectangle rule for numerical integration (The Midpoint Rule) is inspired by the use of Riemann sums.

$$\frac{b-a}{n}\sum_{i=0}^{n-1} f(a + i\frac{b-a}{n}) \approx \int_a^b f(x)dx$$

SUMMARY

If a mathematical expression must be divided, the following strategies and rules were explored in this lesson.

- When a linked expression will not fit on one line, division is made before the comparison sign that begins the link.
- When a link will not fit on one line, division is made before an operation sign within the link.
- In a linked expression, if either the anchor or any link must be divided, a division must also be made before each link.
- Fractions are kept intact by dividing before an opening fraction indicator. If a fraction must be divided, division is made before the fraction line. A mixed number should not be divided.
- Transition to a runover line does not take the place of a necessary baseline indicator. The baseline indicator will be the first symbol on the new line.
- When an item and its exponent or subscript are too long to fit on a single braille line, division is made before a change-of-level indicator.
- The space between a function and its argument is not a suitable division site. If the entire expression will not fit on the line, division strategies are applied to the argument.
- Items within grouping symbols should not be divided. If consecutive groupings do not fit on the line, a division may be made between groupings.
- If a grouped expression will not fit on one line, division strategies are applied within the grouping symbols. If the grouped expression is an enclosed list, division is made after a comma.
- Division may be made after a mathematical termination indicator such as termination of a radical expression or termination of a modified expression.
- When an embedded math expression must be divided, it may begin on the current line provided division is made in accordance with the principles defined in this section.
- When a displayed math expression must be divided, all runovers begin two cells to the right of the original display cell unless it is a nested linked expression in which case a second indent level is applied.

Submit Exercise 15 to your instructor.

BLANK PAGE

ANSWERS TO PRACTICE MATERIAL

PRACTICE 15A

```
1   ⠠⠁⠲ ⠠⠎⠕⠇⠧⠑ ⠮ ⠇⠔⠑⠜ ⠔⠑⠟⠥⠁⠇⠊⠞⠊⠑⠎⠲ ⠸⠩
2     ⠰⠊⠸⠲ ⠷⠭⠬⠹⠂⠴⠌⠒⠼⠾⠷⠭⠬⠹⠂⠔⠌⠒⠼⠾
3       ⠨⠂ ⠷⠒⠭⠬⠹⠲⠖⠌⠒⠼⠾⠷⠹⠭⠌⠒⠼⠬⠂⠾
4     ⠰⠊⠊⠸⠲
5       ⠹⠆⠭⠌⠒⠼⠤⠒ ⠨⠂ ⠹⠂⠖⠭⠌⠆⠂⠼⠤⠹⠂⠒⠌⠒⠼⠤⠹⠆⠭⠌⠂⠢⠼
6     ⠰⠊⠊⠊⠸⠲ ⠷⠁⠤⠂⠾⠘⠆⠐⠤⠷⠁⠤⠶⠾⠷⠁⠤⠒⠾ ⠐⠅ ⠼⠆⠁⠬⠴⠨⠦
7   ⠰⠠⠃⠸⠲ ⠼⠆⠈⠡⠲⠆⠒ ⠨⠅ ⠷⠆⠈⠡⠲⠴⠴⠾⠬⠷⠆⠈⠡⠆⠴⠾⠬⠷⠆⠈⠡⠒⠾
8     ⠨⠅ ⠼⠦⠴⠴⠬⠲⠴⠬⠖ ⠨⠅ ⠼⠦⠲⠖ ⠸⠱
9   ⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
10    ⠘⠂⠠⠃⠔⠕⠍⠊⠁⠇ ⠘⠂⠉⠕⠑⠖⠊⠉⠊⠢⠞⠎ ⠛⠑⠞ ⠸⠮ ⠐⠝ ⠆⠉
11  ⠮⠽ ⠜⠑ ⠮ ⠨⠂⠉⠕⠑⠖⠊⠉⠊⠢⠞⠎ ⠔ ⠮ ⠑⠭⠏⠁⠝⠨⠝ ⠷ ⠁
12  ⠨⠂⠃⠔⠕⠍⠊⠁⠇⠒ ⠸⠩
13    ⠷⠭⠬⠽⠾⠘⠝
14      ⠨⠅ ⠐⠨⠠⠎⠩⠁ ⠨⠅ ⠼⠴⠣⠝⠻⠷⠝⠩⠅⠾⠭⠘⠅⠐⠽⠘⠝⠤⠅ ⠸⠱⠲
```

Line 2: The anchor fits on the line with the identifier.
Line 3: The link begins in the runover cell (cell 5).
Line 5: This expression will fit on one line if it begins on the line below its identifier.
Line 7: The anchor and the first link fit on the line with the identifier.
Line 8: The remaning two links fit on one line, beginning in the runover cell (cell 3).
Line 9: A blank line precedes the change in format from itemized material to a narrative paragraph.
Line 14: The runover line begins with the equals sign that begins the link. The equals sign in the modifier is not a suitable division site because a modifier is a mathematical unit.

PRACTICE 15B

```
1   ⠸⠩
2   ⠰⠠⠁⠸⠲ ⠫⠪ ⠼⠂⠬⠫⠪ ⠼⠆⠬⠫⠪ ⠼⠒⠬⠫⠪ ⠼⠲⠬⠫⠪ ⠼⠢
3     ⠬⠫⠪ ⠼⠖⠬⠫⠪ ⠼⠶
4     ⠨⠅ ⠼⠲⠔⠴⠘⠨⠡
5   ⠰⠠⠃⠸⠲
6     ⠼⠆⠸⠹⠒⠌⠲⠸⠼ ⠽⠙ ⠬⠂⠸⠹⠒⠌⠲⠸⠼ ⠽⠙ ⠬⠹⠒⠌⠲⠼ ⠽⠙
7     ⠨⠅ ⠼⠢⠸⠹⠂⠌⠲⠸⠼ ⠽⠙
8   ⠰⠠⠉⠸⠲ ⠜⠷⠭⠬⠁⠘⠆⠐⠾⠬⠷⠽⠬⠁⠘⠆⠐⠾⠻
9     ⠤⠜⠷⠭⠤⠁⠘⠆⠐⠾⠬⠷⠽⠤⠁⠘⠆⠐⠾⠻
10    ⠨⠅ ⠬⠤⠆⠁ ⠸⠱
11  ⠰⠠⠙⠲ ⠠⠎⠕⠍⠍⠻⠄⠎ ⠗⠳⠞⠔⠑ ⠉ ⠆ ⠗⠑⠏⠗⠑⠎⠢⠞⠫ ⠃⠽ ⠮
12    ⠋⠕⠇⠇⠪⠬ ⠑⠭⠏⠗⠑⠎⠨⠝⠲
13      ⠠⠗⠳⠞⠔⠑ ⠰⠠⠎⠒ ⠸⠩ ⠼⠢⠬⠒⠬⠲⠷⠂⠬⠷⠤⠂⠾⠾⠬⠷⠤⠒⠾
14        ⠬⠷⠤⠢⠾⠬⠒⠷⠢⠬⠷⠤⠆⠾⠾⠬⠂ ⠸⠱
15    ⠠⠙⠗⠁⠺ ⠁ ⠎⠊⠍⠏⠇⠑ ⠙⠊⠁⠛⠗⠁⠍ ⠞⠕ ⠗⠑⠏⠗⠑⠎⠢⠞
16    ⠨⠂ ⠠⠗⠳⠞⠔⠑ ⠨⠆⠰⠠⠎⠲
```

Line 4: Since a division is made in the anchor, a division must also be made before the link.
Line 6: The anchor will fit on one line if it starts in the runover position of this itemized problem (cell 3). The only necessary division is made before the link.
Line 8: Since the anchor will not fit on one line in this itemized format (it uses 39 cells), it begins on line 8 until an appropriate division site is encountered.
Line 9: Division is made before the operation sign between the radical expressions.
Line 10: Since a division is made in the anchor, a division must also be made before the link.
Line 13: Since the displayed expression will not fit on one line (it uses 36 cells), it begins on line 13 until an appropriate division site is encountered..
Line 14: The runover line begins with an operation sign that is not part of an expression enclosed between parentheses.

PRACTICE 15C

1 ⠼⠁⠲ ⠠⠉⠕⠍⠏⠥⠞⠑ ⠯⠸⠌⠕⠗ ⠎⠊⠍⠏⠇⠊⠋⠽⠲ ⠸⠩
2 ⠰⠁⠸⠲ ⠠⠹⠷⠹⠒⠌⠆⠼⠾⠈⠡⠷⠹⠂⠌⠆⠼⠾⠈⠡⠷⠤⠹⠂⠌⠆⠼⠾
3 ⠠⠌⠂⠈⠡⠆⠈⠡⠒⠠⠼
4 ⠰⠃⠸⠲ ⠠⠹⠹⠙⠭⠌⠙⠞⠼⠹⠙⠘⠆⠐⠽⠌⠙⠞⠘⠆⠐⠼
5 ⠤⠹⠙⠘⠆⠐⠭⠌⠙⠞⠘⠆⠐⠼⠹⠙⠽⠌⠙⠞⠼
6 ⠠⠌⠷⠙⠭⠸⠌⠙⠞⠾⠘⠒⠐⠠⠼
7 ⠰⠉⠸⠲ ⠷⠬⠤⠾⠁⠰⠂⠊⠰⠰⠂⠐⠁⠰⠆⠊⠰⠰⠆⠐⠁⠰⠒⠊⠰⠰⠒
8 ⠐⠁⠰⠲⠊⠰⠰⠲ ⠐⠄⠄⠄ ⠁⠰⠝⠊⠰⠰⠝
9 ⠼⠆⠸⠲ ⠹⠭⠘⠆⠐⠬⠆⠌⠭⠷⠆⠭⠘⠆⠐⠬⠂⠾⠘⠆⠐⠼
10 ⠨⠅ ⠹⠠⠁⠌⠭⠼⠬⠹⠠⠃⠭⠬⠠⠉⠌⠆⠭⠘⠆⠐⠬⠂⠼
11 ⠬⠹⠠⠙⠭⠬⠠⠑⠌⠷⠇⠭⠘⠆⠐⠬⠂⠾⠘⠆⠐⠼
12 ⠨⠅ ⠹⠠⠁⠷⠲⠭⠘⠲⠐⠬⠲⠭⠘⠆⠐⠬⠂⠾⠬⠠⠃⠷⠆⠭⠘⠲⠐⠬⠭⠘⠆⠐⠾
13 ⠬⠠⠉⠷⠆⠭⠘⠒⠐⠬⠭⠾⠬⠠⠙⠭⠘⠆⠐⠬⠠⠑⠭
14 ⠌⠭⠷⠆⠭⠘⠆⠐⠬⠂⠾⠘⠆⠐⠼ ⠸⠱

Line 4: The numerator will not fit on one braille line. It begins on this line.
Line 5: A division is made before the minus sign.
Line 6: Since the numerator is divided, a division is also required before the fraction line. (Review complex fractions in Lesson 8.)
Lines 7-8: Each base is on the same braille line as its subscript.
Line 8: A baseline indicator begins the runover line.
Line 8: Review the rules regarding an ellipsis on the baseline following a subscript in Lesson 6.
Lines 9-14. See Section 8.22.b in Lesson 8 regarding margin rules for an itemized nested linked expression with no narrative.
Line 9: The anchor begins on the same line as the identifier.
Line 10: The link begins in cell 3, indented two cells from the identifier on line 9. The first two fractions fit on this line. Division is made before a sign of operation on the baseline.
Line 11: The runover begins in cell 5 for this nested linked expression.
Line 12: The second link begins on a new line. Addends A and B fit on this line.
Line 13: Addends C and D complete the numerator. The line begins with a sign of operation on the baseline within the numerator.
Line 14: The denominator fits on one line. The line begins with the fraction line.

PRACTICE 15D

```
1   ⠼⠁⠲ ⠠⠍⠥⠇⠞⠊⠏⠇⠽ ⠘⠮ ⠋⠳⠗ ⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇⠎⠒
2     ⠸⠩ ⠷⠲⠭⠘⠔⠐⠽⠾⠷⠶⠭⠘⠦⠐⠽⠘⠆⠐⠾⠷⠢⠭⠘⠶⠐⠽⠘⠒⠐⠾
3     ⠷⠖⠭⠘⠖⠐⠽⠘⠲⠐⠾⠸⠲
4   ⠼⠆⠸⠲
5     ⠬⠤⠷⠹⠁⠘⠹⠆⠌⠒⠼⠐⠬⠁⠘⠹⠆⠌⠒⠼⠐⠃⠘⠹⠂⠌⠒⠼⠐⠬⠃⠘⠹⠆⠌⠒⠼
6     ⠐⠌⠒⠼⠾⠘⠹⠒⠌⠆⠼
7   ⠼⠒⠸⠲
8     ⠷⠂⠖⠭⠘⠲⠐⠬⠦⠭⠘⠒⠐⠽⠬⠲⠭⠘⠆⠐⠽⠘⠆⠐⠬⠆⠭⠽⠘⠒⠐⠬⠽⠘⠲⠐⠾
9     ⠷⠆⠭⠤⠽⠾
10    ⠨⠅ ⠼⠒⠆⠭⠘⠢⠐⠤⠽⠘⠢ ⠸⠱
```

Lines 4-6: The grouped expression (the fraction) must be divided. The numerator will fit on one line when you start the expression on the line after the identifier (line 5), beginning in the runover cell (cell 3).

Line 6: The simple fraction is divided at the fraction line. The line begins with its associated baseline indicator.

Line 8: The first grouping will fit on one line if it starts on the line following the identifier.

Line 10: Because the anchor is divided, division must also be made before the link.

PRACTICE 15E

```
⠼⠁⠲ ⠸⠩ ⠣⠒⠜⠭⠘⠂⠐⠻⠣⠖⠜⠭⠘⠆⠐⠻⠣⠂⠆⠜⠭⠘⠒⠐⠻
  ⠣⠆⠲⠜⠭⠘⠲⠐⠻⠣⠲⠦⠜⠭⠘⠢⠐⠻ ⠄⠄⠄ ⠸⠱
⠼⠃⠲ ⠠⠮ ⠛⠢⠻⠁⠇ ⠎⠕⠇⠥⠰⠝ ⠿ ⠐⠕ ⠗⠕⠕⠞ ⠷ ⠮ ⠉⠥⠃⠊⠉
  ⠑⠟⠥⠁⠰⠝ ⠊⠎ ⠸⠩
    ⠭
      ⠨⠅ ⠣⠒⠜⠤⠹⠟⠌⠆⠼
      ⠬⠨⠜⠹⠟⠘⠆⠐⠌⠲⠼⠬⠹⠏⠘⠒⠐⠌⠆⠶⠼⠨⠻⠻
      ⠬⠣⠒⠜⠤⠹⠟⠌⠆⠼⠤⠨⠜⠹⠟⠘⠆⠐⠌⠲⠼⠬⠹⠏⠘⠒⠐⠌⠆⠶⠼⠨⠻⠻
      ⠸⠱⠲
```

Line 1: Division is made after the termination indicator, before the fourth radical expression.
Line 2: The runover line begins with the index-of-radical indicator.
Lines 3-4: The second itemized narrative begins in cell 1 with runovers in cell 3.
Line 5: The anchor consists only of the letter x (cell 5).
Line 6: The link begins with an equals sign in cell 7. Although the first radical expression will fit entirely on the next line, the equals sign cannot be the only symbol on line 6, so a division must be made within the radical expression. Division is made before the first plus sign in the radicand. The second plus sign is not a suitable division site because the nested radicand is a mathematical unit.
Line 7: The nested radical fits on this line. Division is made after the two termination indicators.
Line 8: This line begins with the plus sign before the second radical expression (Step ii). This radical expression will fit on one line, but the Nemeth Code terminator and the final period do not, so they are placed on line 9.

PRACTICE 15F

```
1  ⠼⠁⠲ ⠠⠐⠙⠇⠊⠣⠞ ⠲⠲⠲
2    ⠁⠲ ⠠⠮ ⠝⠥⠍⠃⠻ ⠷ ⠍⠔⠥⠞⠑⠎ ⠷ ⠐⠙⠇⠊⠣⠞ ⠿ ⠁⠝⠽
3      ⠇⠕⠉⠁⠰⠝ ⠁⠞ ⠸⠩ ⠼⠖⠴⠘⠨⠡ ⠰⠠⠝ ⠸⠱ ⠇⠁⠞⠊⠞⠥⠙⠑
4      ⠊⠎ ⠍⠕⠙⠑⠇⠫ ⠃⠽ ⠮ ⠋⠥⠝⠉⠰⠝ ⠆⠇⠂ ⠐⠱
5      ⠸⠩ ⠙ ⠨⠅ ⠙⠁⠽ ⠕⠋ ⠞⠓⠑ ⠽⠑⠁⠗⠲
6        ⠍⠷⠙⠾
7          ⠨⠅ ⠼⠒⠔⠴⠎⠊⠝ ⠈⠷⠹⠆⠨⠏⠌⠒⠖⠢⠨⠆⠢⠼
8          ⠷⠙⠤⠦⠴⠾⠈⠾
9          ⠬⠶⠒⠦ ⠸⠱
```

Line 5: The entire statement "d = day of the year" is mathematical. The words are uncontracted and are punctuated without the use of a punctuation indicator.
Lines 6-9: The displayed expression begins in cell 7, with runovers in cell 9.
Lines 7-8: The space between the function and its argument is not a suitable division site. The bracketed expression must be divided. Division is made before the left parenthesis in order to keep the expression grouped within the inner parentheses intact.
Line 9: Division is made before the plus sign, according to Step ii.

PRACTICE 15G

```
1    ⠠⠮ ⠎⠕⠇⠥⠰⠝⠎ ⠔⠧⠕⠇⠧⠬ ⠮ ⠙⠊⠗⠑⠉⠞ ⠋⠥⠝⠉⠰⠝⠎ ⠸⠩
2    ⠮⠹⠇⠕⠛⠘⠆ ⠷⠵⠬⠂⠾⠌⠵⠼⠙⠵
3      ⠨⠅ ⠇⠕⠛ ⠷⠤⠵⠾⠇⠕⠛⠘⠆ ⠷⠵⠬⠂⠾
4      ⠬⠆⠠⠇⠊⠆⠷⠵⠬⠂⠾⠇⠕⠛ ⠷⠵⠬⠂⠾⠤⠇⠠⠇⠊⠒⠷⠵⠬⠂⠾
5  ⠠⠄⠯
6    ⠮⠰⠴⠘⠠⠿⠐⠠⠹⠇⠕⠛ ⠷⠞⠬⠂⠾⠇⠕⠛ ⠷⠂⠬⠹⠂⠌⠞⠘⠆⠐⠼⠾
7      ⠠⠌⠞⠠⠼⠙⠞
8      ⠨⠅ ⠠⠉⠨⠏⠤⠹⠒⠨⠵⠷⠒⠾⠌⠦⠼ ⠸⠱
9  ⠋⠨⠙ ⠢ ⠠⠡⠁⠏⠞⠻ ⠼⠛⠲
```

This is one paragraph interrupted twice by displayed material.
Line 1: The paragraph begins in cell 3.
Line 2: The displayed expressions begin in cell 3.
Line 3: The link begins in the runover cell for displayed material (cell 5). Division is made before the plus sign (Step ii).
Line 4: It is not necessary to divide before the second operation sign (the minus sign).
Line 5: The paragraph continues in the runover cell for narrative (cell 1).
Line 6: The displayed expression begins in cell 3. The notation from the integral symbol to dt will not fit on one line. A decision needs to be made whether to (a) divide before the opening fraction indicator in order to keep the fraction unit together or (b) begin the fraction on this line, dividing at the complex fraction line. Choice (b) was transcribed, in order to prevent dt from being left alone on line 7.
Line 7: This line begins with the complex fraction line and ends with the dt that finishes the integral notation.
Line 8: A new line begins with the final link (Step i), placed in the runover cell, cell 5.
Line 9: The paragraph concludes.

PRACTICE 15H

```
1  ⠸⠩ ⠐⠨⠠⠎⠩⠝ ⠨⠅ ⠼⠴⠣⠠⠿⠻⠹⠷⠁⠾⠰⠝⠐⠷⠃⠾⠰⠝⠐⠌⠷⠉⠾⠰⠝⠐⠼
2  ⠹⠵⠘⠝⠐⠌⠝⠯⠼ ⠸⠱
```

The expression is divided before the second fraction. This keeps the Sigma notation intact with the beginning of its argument.

PRACTICE 15I

```
1 ⠼⠁⠲ ⠠⠮ ⠗⠑⠉⠞⠁⠝⠛⠇⠑ ⠗⠥⠇⠑ ⠿ ⠝⠥⠍⠻⠊⠉⠁⠇
2   ⠔⠞⠑⠛⠗⠁⠰⠝ ⠐⠣⠠⠮ ⠠⠍⠊⠙⠏⠕⠔⠞ ⠠⠗⠥⠇⠑⠐⠜ ⠊⠎
3   ⠔⠎⠏⠊⠗⠫ ⠃⠽ ⠮ ⠥⠎⠑ ⠷ ⠠⠗⠊⠑⠍⠁⠝⠝ ⠎⠥⠍⠎⠲ ⠸⠩
4     ⠹⠃⠤⠁⠌⠝⠼
5       ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠴⠣⠝⠤⠂⠻⠋⠷⠁⠬⠊⠹⠃⠤⠁⠌⠝⠼⠾
6       ⠈⠱⠈⠱ ⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭ ⠸⠱
```

Line 4: The anchor will not fit on one line. Division is made after the first closing fraction indicator.

Line 5: The Sigma notation and its argument fit on one line. No further division decisions are necessary.

Line 6: The link is placed on a new line, starting with the double tilde.