LESSON 17

- SYSTEM OF EQUATIONS
  - Enlarged Signs of Grouping
- DETERMINANTS AND MATRICES

Answers to Practice Material

LESSON PREVIEW

Enlarged grouping signs are often encountered in the topics studied in this lesson: systems of equations and arrays (matrices and determinants). Format rules are given for these spatial arrangements, including further considerations regarding commentary.

SYSTEM OF EQUATIONS

[NC 25.9]

17.1 Definition and Recognition

A system of equations, sometimes called "simultaneous equations," is a collection of two or more equations which share variables. The student identifies the value of each variable by "solving the system." You can recognize a system by noticing the arrangement of equations on two or more lines. If the system consists of two equations, there will be two variables to solve (typically x and y); if the system consists of three equations, there will usually be three variables to solve (typically x, y, and z). The equations may or may not be joined by an enlarged grouping sign.

Here is a sample of a system of two equations using variables x and y.

$$4x - y = 10$$
$$2x = 12 - 3y$$

17.2 Transcription Rules for Systems of Equations

A system of equations is a spatial arrangement and is transcribed as follows.

(a) One blank line is left above and below the system. Placement of code switch indicators follows the general rules for spatial arrangements.

(b) Use and nonuse of the numeric indicator follows Nemeth rules for nonspatial material, even though this is a spatial arrangement. (See also subitem (d), below.)

(c) Alignment is maintained only if terms and symbols are aligned in print. If terms are aligned on one side of the equals sign but not on the other side, follow print. Alignment is disregarded if only the equals signs are aligned.

(d) When a space is inserted within the equation for the purpose of maintaining alignment, a numeric indicator is not used.

The same rules apply to a system of inequalities. (See Example 17-4).

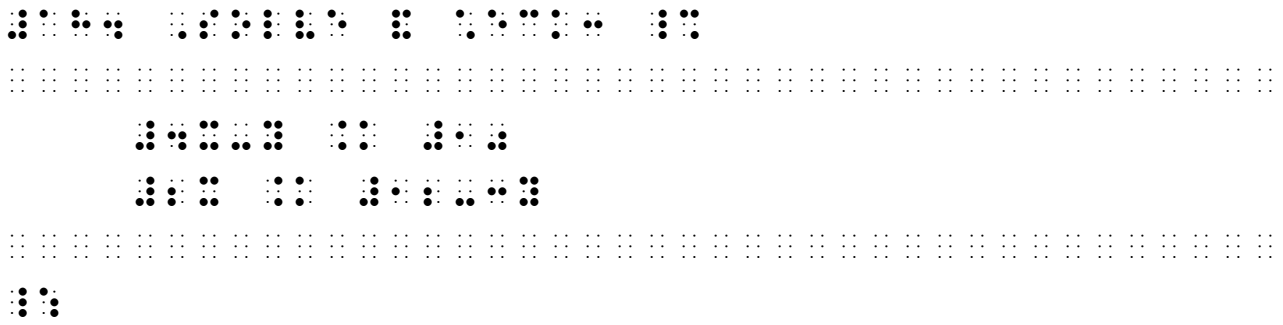
Note that, if a separation line is present, it is a spatially arranged addition problem and Nemeth rules for spatial addition are followed. (See Lesson 9.)

Example 17-1

18. Solve and check.

$$4x - y = 10$$
$$2x = 12 - 3y$$

Print observation: This system has two variables (x, y). There are two equations. The terms are not vertically aligned.



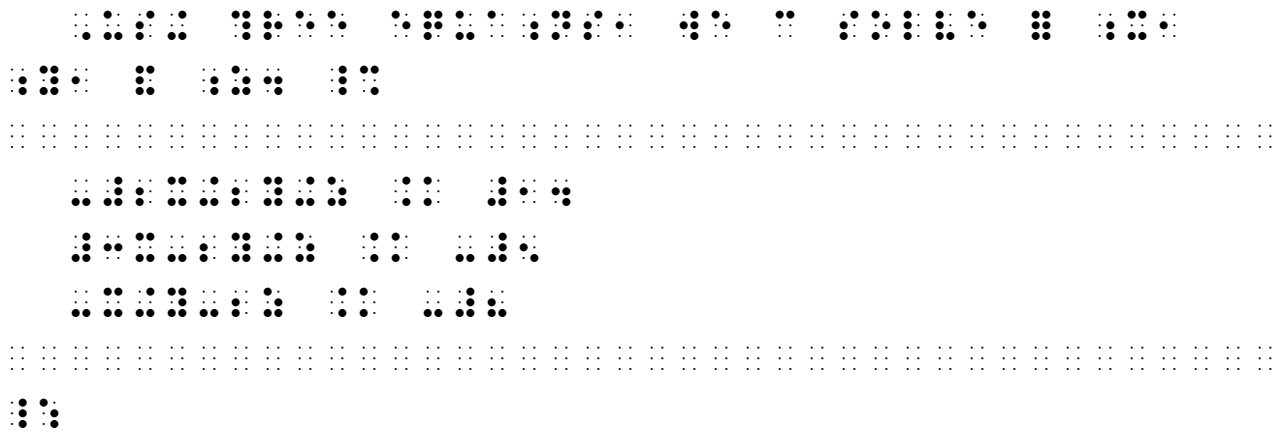
Braille observation: A blank line is inserted before and after the system (17.2 (a)). Each equation begins in cell 5, displayed to itemized material. Numeric indicators are used as needed according to Nemeth rules for nonspatial material (17.2 (b)).

Example 17-2

Using three equations, we can solve for x, y, and z.

$$\begin{aligned} -2x + 2y + z &= 14 \\ 3x - 2y + z &= -5 \\ -x + y - 2z &= -8 \end{aligned}$$

Print observation: This system has three variables (x, y, z). There are three equations. The equals signs are vertically aligned but the terms on either side are not.

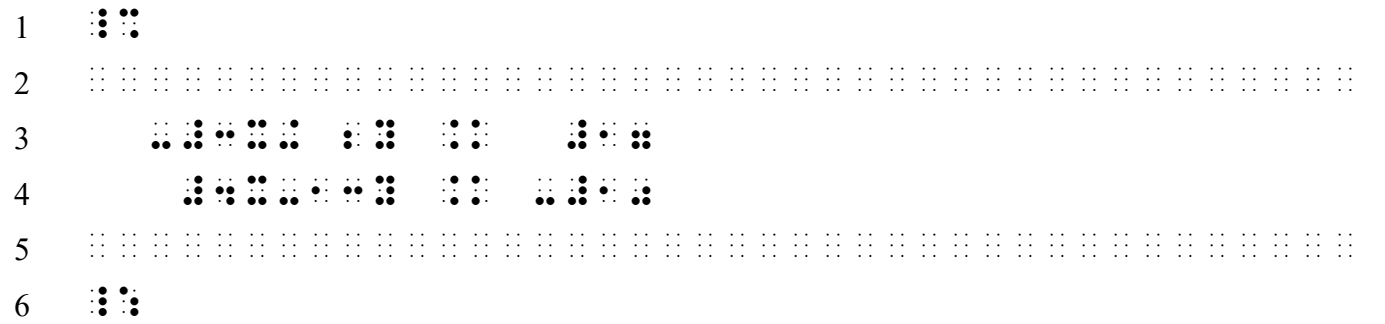


Braille observation: A blank line is inserted before and after the system (17.2 (a)). Each equation begins in cell 3, displayed to 3-1 narrative. Alignment of the equals signs is disregarded (17.2 (c)).

Example 17-3

$$-3x + 2y = 17$$
$$4x - 13y = -10$$

Print observation: The terms on both sides of the equals signs are vertically aligned.

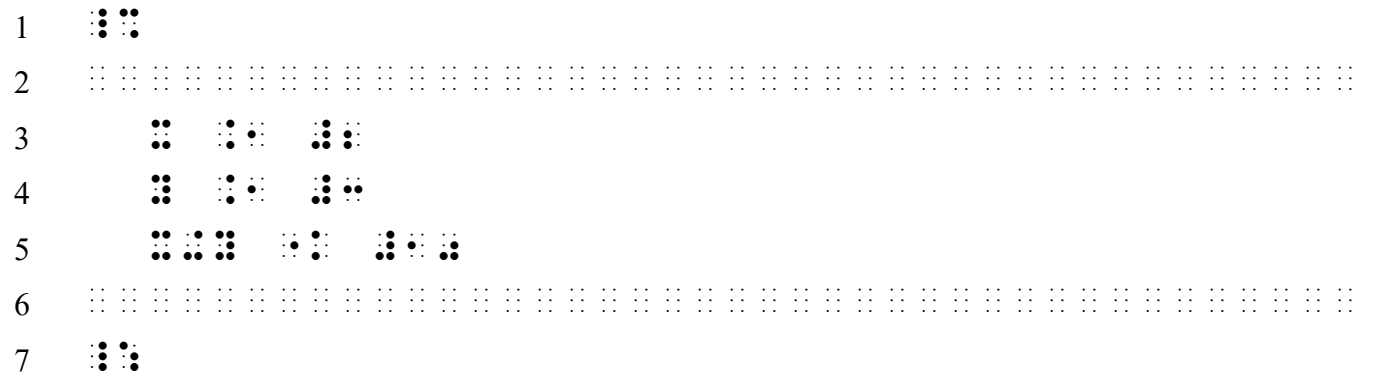


Braille observation: The leftmost symbol is placed in the appropriate cell for displayed material. Spaces are inserted in order to maintain alignment as printed (17.2 (c)). A numeric indicator is not used before the numeral "2" even though it is preceded by a space because when a space is inserted in order to achieve alignment, no numeric indicator is inserted (17.2 (d)). This rule does not apply to the first numeral to the right of the equals sign.

Example 17-4

$$x > 2$$
$$y > 3$$
$$x + y < 10$$

Print observation: Three inequalities are printed on three lines. Terms are not aligned.



Braille observation: A blank line precedes and follows the arrangement. Each inequality begins in the same cell.

PRACTICE 17A

1. Solve and check:

$$2x + 3y = 2$$
$$8x - 4z = 3$$
$$3y - 8z = -1$$

2. Solve:

$$2x - 5y + 6z = 11$$
$$3x - 2y + 3z = 9$$
$$2x + 4y - 9z = -3$$

3. Add:

$$\begin{array}{r} 3x - y = 7 \\ 2x + y = 8 \\ \hline 5x + 0 = 15 \end{array}$$

Enlarged Signs of Grouping

[NC 19.6]

17.3 A Unified Expression

The Nemeth Code calls an arrangement a "unified expression" when equations are grouped together using an enlarged grouping symbol. Here is a print sample of a unified system of equations, using a left enlarged brace.

$$\begin{cases} x = y \\ 5x - y = 4 \end{cases}$$

17.4 Transcription Rules for Enlarged Signs of Grouping

(a) If only the left or right grouping sign is shown in print, only that symbol is shown in braille.

(b) Enlarged grouping symbols are transcribed on each line of the unified expression and are vertically aligned.

(c) When terms are not aligned in print, each line of the unified expression begins in the same cell. When there is a left enlarged grouping symbol, each expression will begin in the cell which immediately follows the left enlarged grouping symbol.

(d) When the expressions require vertical alignment, at least one item must begin in the cell which immediately follows the left enlarged grouping symbol. The numeric indicator is not required before the first numeric character following a left grouping symbol.

(e) At least one item ends in the cell which immediately precedes the right enlarged grouping symbol.

17.4.1 **Left Enlarged Brace.** The left enlarged brace curves and points to the left in print. Notice that the enlarged braille symbol is formed by inserting a dot 6 before the second cell of the normal brace symbol.

⠸⠣	Left Brace (normal size)	{
⠸⠸⠣	Left Enlarged Brace (covering two or more lines)	

Example 17-5

$$\begin{cases} x = y \\ 5x - y = 4 \end{cases}$$

Print observation: A left enlarged brace groups the unified system of equations. Terms are not vertically aligned.

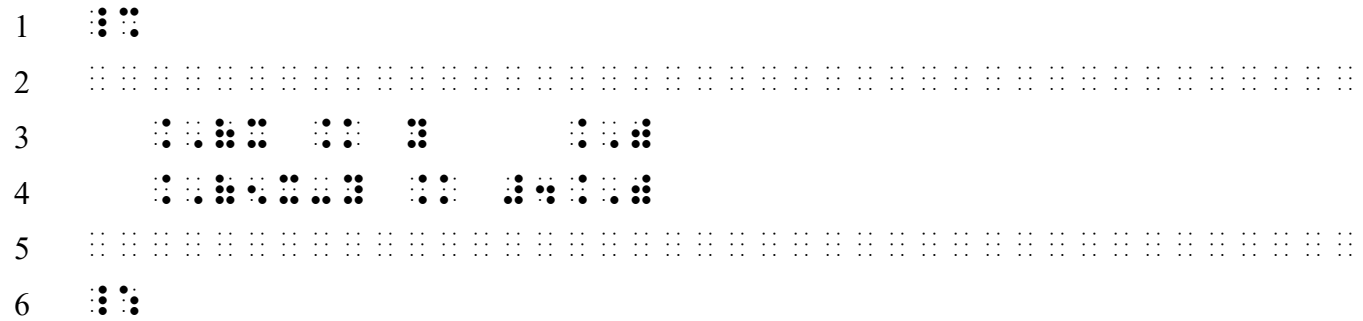
1 ⠸⠣

2 ⠸⠸⠀⠸⠸⠀⠸⠸⠀

The system shown in Example 17-5 is shown again in Example 17-6 with a right enlarged brace added and with each expression centered within the braces.

Example 17-6

$$\left\{\begin{array}{c} x = y \\ 5x - y = 4 \end{array}\right\}$$



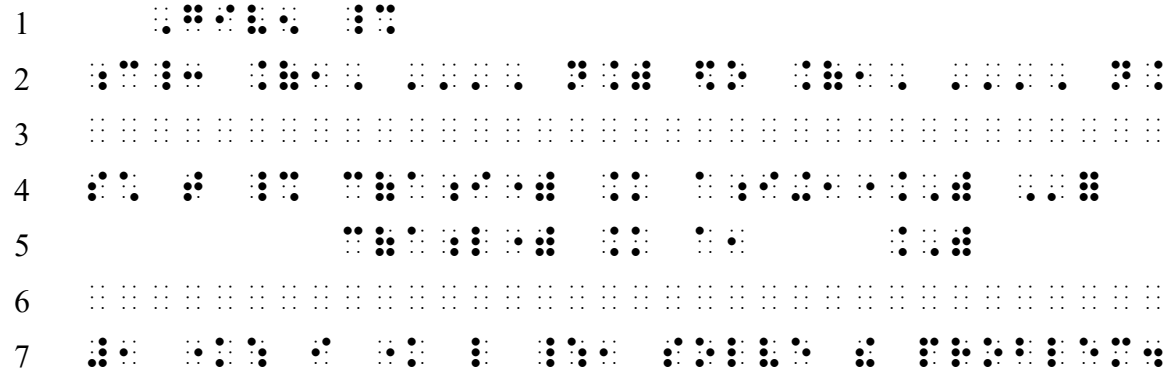
Each expression begins in the cell next to the left grouping symbol. The right grouping symbols are vertically aligned, starting in the cell next to the widest equation.

17.5 Embedded Vertical Groupings

Enlarged grouping signs are used in other structures as well, not only with systems of equations. If the grouping is embedded within narrative, the required blank lines before and after the arrangement become part of the paragraph. The surrounding text is placed on the top line of the arrangement only – before and/or after the math as it fits in context of the narrative. Text continues in the runover cell of the paragraph after the second required blank line. The opening Nemeth Code indicator and the Nemeth Code terminator are placed only on the top line of the embedded arrangement. The switch indicators apply to the whole arrangement.

Example 17-7

Given $c\colon \{1, \ldots, n\} \to \{1, \ldots, n\}$ such that $\left.\begin{array}{l} c(a_i) = a_{i+1} \\ c(a_l) = a_1 \end{array}\right\}$ for $1 \le i < l$, solve the problem.



Lines 1-2: The paragraph begins in cell 3. The runover is in cell 1.
Line 3: A blank precedes the spatial arrangement that is embedded in the paragraph.
Line 4: The narrative continues, in the runover cell of the paragraph. The opening switch is placed on this line only. The narrative continues after the top right enlarged grouping symbol.
Line 5: The second line of the embedded spatial arrangement is aligned with the line above, starting in the same cell and with the right grouping symbols aligned.
Line 6: A blank follows the spatial arrangement that lies within the paragraph.
Line 7: The paragraph continues.

PRACTICE 17B

After solving for x and y, we determine that the system $\begin{cases} 2x-3y=17 \\ 3x+2y=6 \end{cases}$ has the solution set $\{(4,-3)\}$. Name the solution set for this system.

$$\begin{cases} x+2y=6 \\ 2x-y=7 \end{cases}$$

17.6 Enlarged Parentheses

Notice that the enlarged braille parentheses are formed by inserting a dot 6 before the normal parenthesis symbol.

⠶	Left Parenthesis (normal)	(
⡇⢸	Left Enlarged Parenthesis (covering two or more lines)	
⠶	Right Parenthesis (normal)	)
⡇⢸	Right Enlarged Parenthesis (covering two or more lines)	

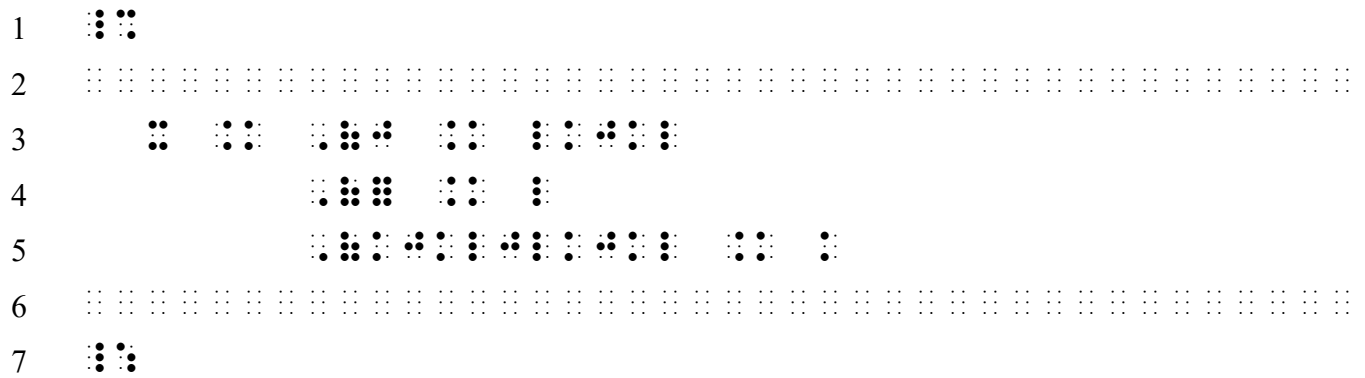
17.7 Placement of Symbols

Symbols which appear outside of the enlarged grouping symbol and which apply to the arrangement are placed on the top line of the arrangement, even if the items are centered in print.

Example 17-8

$$x = \begin{cases} j = lkjkl \\ = l \\ kjkljlkjkl = k \end{cases}$$

Print observation: A three-line system is grouped on the left with an enlarged parenthesis. The anchor, x, is centered with respect to the enlarged grouping symbol to its right. The equals signs in the grouped arrangement are aligned, but the terms are not.



Braille observation: The anchor is aligned with the top line of the arrangement. Each enlarged grouping symbol begins in the same cell. The grouped items are left adjusted because the terms are not aligned. A general omission symbol denotes the blank space before the middle equals sign. English-letter indicators are not needed for the single letters x, j, l, and k because they are next to a sign of comparison. Code switch indicators do not interfere with the spatial arrangement.

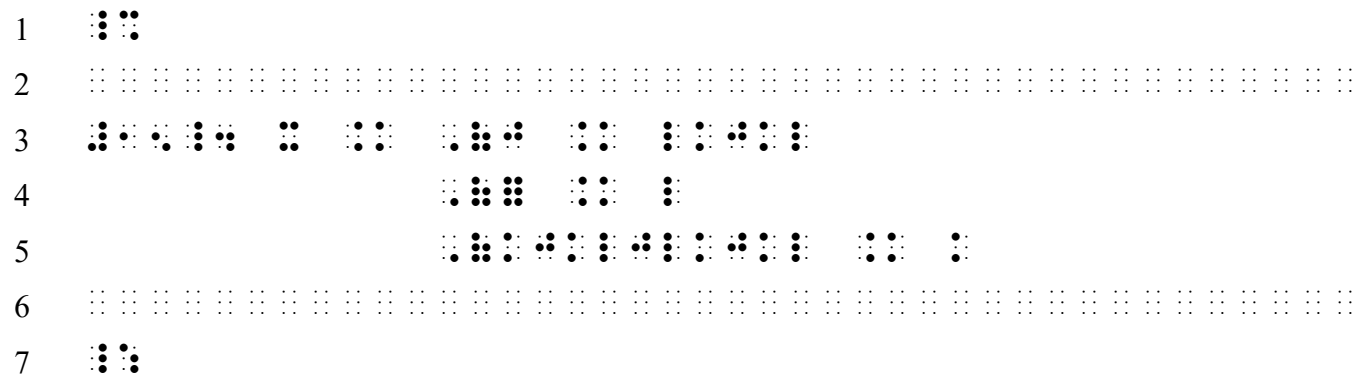
17.8 Placement of Identifiers and Punctuation

Identifiers and punctuation which appear outside of the enlarged grouping symbol and which apply to the arrangement are placed on the top line of the arrangement, even if the items are centered in print. Example 17-9 uses the same expression from Example 17-8, now preceded by an identifier. An itemized spatial arrangement is not considered to be embedded. The code switch indicators are placed outside of the arrangement.

Example 17-9

15. $$x = \begin{cases} j = lkjkl \\ = l \\ kjkljlkjkl = k \end{cases}$$

Print observation: The item number and the anchor are centered with respect to the associated enlarged grouping symbol to the right.

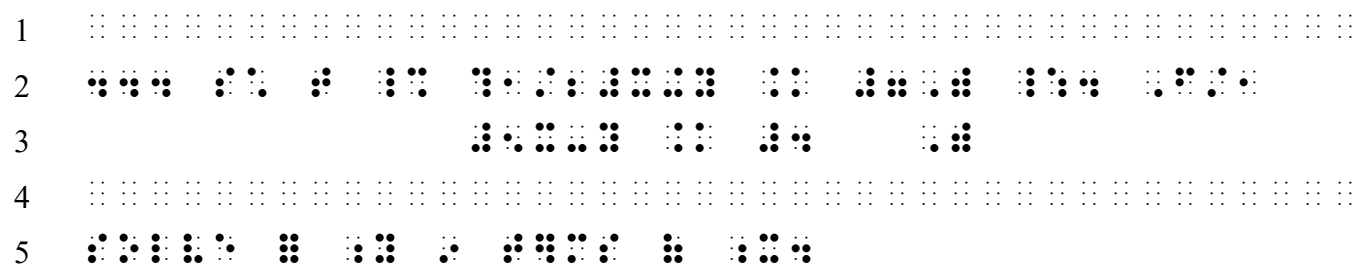


Braille observation: Item number and anchor are aligned with the top line of the arrangement. The code switch indicators are placed outside of the arrangement.

Example 17-10

... such that $\begin{matrix} \frac{1}{2}x + y = 7 \\ 5x - y = 4 \end{matrix}\Bigg)$. First, solve for y in terms of x.

Print observation: A vertical arrangement is embedded within the narrative. It is grouped on the right with a right enlarged parenthesis. A period follows the enlarged parenthesis, centered to the arrangement. The narrative continues on the same line.



Braille observation:
Line 1: A blank line precedes the line with the embedded spatial arrangement.
Line 2: The top line of the arrangement begins on the main line of the text. The code switch indicators are also placed on this line. The paragraph continues after the period.
Line 3: The second line of the arrangement is placed here. The right enlarged parenthesis is aligned with the identical symbol on the line above. No code switch indicators are placed in this line.
Line 4: A blank line follows the embedded spatial arrangement.
Line 5: The paragraph text continues.

17.9 Nested Grouping Symbols

Recall from Lesson 2 that nested grouping symbols may be printed in different sizes in order to visually distinguish the nested symbols, but they are not transcribed as enlarged symbols if they apply to only one line of characters.

⋙ $\mathrm{p}\left(q\left(f\left(g\left(h\left(x\right)\right)\right)\right)\right)$

⠏⠷⠟⠷⠋⠷⠛⠷⠓⠷⠭⠾⠾⠾⠾⠾

Similarly, grouping signs which enclose taller print constructions such as fractions, modified expressions, integrals, or binomial coefficients (12.7.1) are transcribed as normal grouping symbols because they apply to only one line of braille characters.

⋙ $\left(\frac{3}{5}\right)^3 = \frac{27}{125} = 0.216$

⠷⠹⠒⠌⠢⠼⠾⠘⠒ ⠨⠅ ⠹⠆⠶⠌⠂⠆⠢⠼ ⠨⠅ ⠼⠴⠨⠆⠂⠖

⋙ $\left(\sum_k \binom{n}{k}\right)$ ⠷⠐⠨⠠⠎⠩⠅⠻⠷⠝⠩⠅⠾⠾

Review: Notice the two types of question marks found in this Practice. In item A, each question mark is a modifier, transcribed as a punctuation mark placed directly above the equals sign. (See Lesson 12.) Each question mark in item B denotes an omitted item in an enclosed list, and so is transcribed as an omission symbol. (See Lesson 1).

PRACTICE 17C

A) Verify (YES or NO):

$$2\left(-\frac{4}{3}\right) - 1 \stackrel{?}{=} 4\left(-\frac{4}{3}\right) + 9$$

$$\left(-\frac{8}{3}\right) - 1 \stackrel{?}{=} \left(-\frac{16}{3}\right) + 9$$

$$-8 - 3 \stackrel{?}{=} -16 + 27$$

$$-11 = 11. \qquad \text{NO}$$

B) $\begin{pmatrix} \frac{x}{3} - \frac{y}{2} = 2 \\ 5x + 3y = 51 \end{pmatrix} = (?, ?)$

17.10 Conditions or Commentary Printed Next to a Spatial Arrangement [NC 19.8, 25.9, 25.10.1]

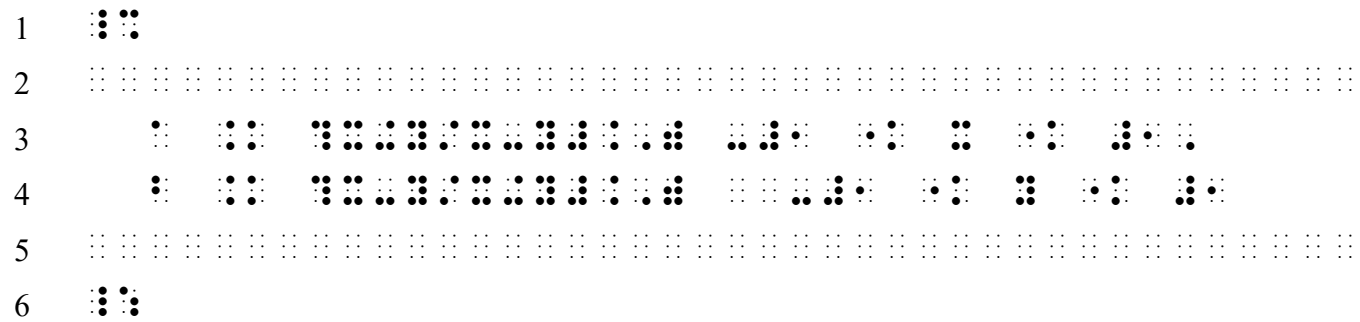
The format for instructional commentary presented in Lesson 16 is for comments that alternate with linear math problems. Now we will look at a condition or comment which applies to more than one line of a spatial arrangement. A large grouping sign may be present in print, which will alert you to this layout.

17.10.1 **Alignment and Runovers.** The comment or condition begins on the first line of the arrangement, even if it is centered to the arrangement in print. Runovers are indented two cells from the cell in which the comment begins.

Example 17-11

$$\left.\begin{aligned} a &= \frac{x+y}{x-y} \\ b &= \frac{x-y}{x+y} \end{aligned}\right\} \quad -1 < x < 1, -1 < y < 1$$

Print observation: A right enlarged brace groups two equations. Conditions for x and y are printed to the right of the brace.



Braille observation: Assuming this is displayed to narrative, each equation begins in cell 3. The comment begins to the right of the first (top) enlarged brace. The runover is indented two cells from the beginning of the comment.

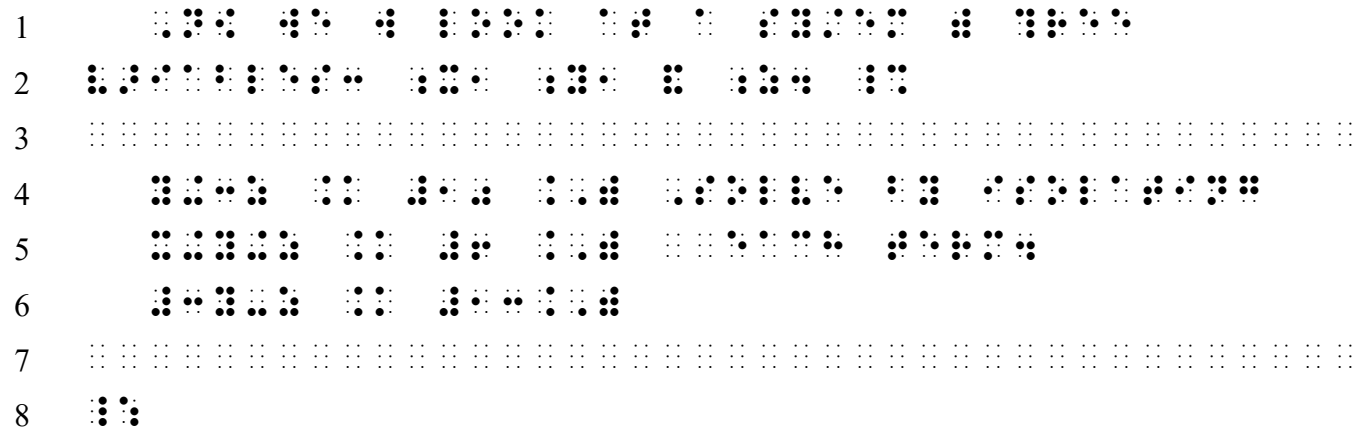
17.10.2 **When Comments are Words.** The entire spatial portion is transcribed in Nemeth. When the comment contains narrative, the words are uncontracted, even though the words may not be part of a mathematical expression.

Example 17-12

Now we will look at a system with three variables: x, y, and z.

$$\left.\begin{aligned} y + 3z &= 10 \\ x + y + z &= 6 \\ 3y - z &= 13 \end{aligned}\right\}$$ *Solve by isolating each term.*

Print Observation: Three equations are grouped with a right enlarged brace. A comment is printed to the right of the middle equation. The comment is narrative text.



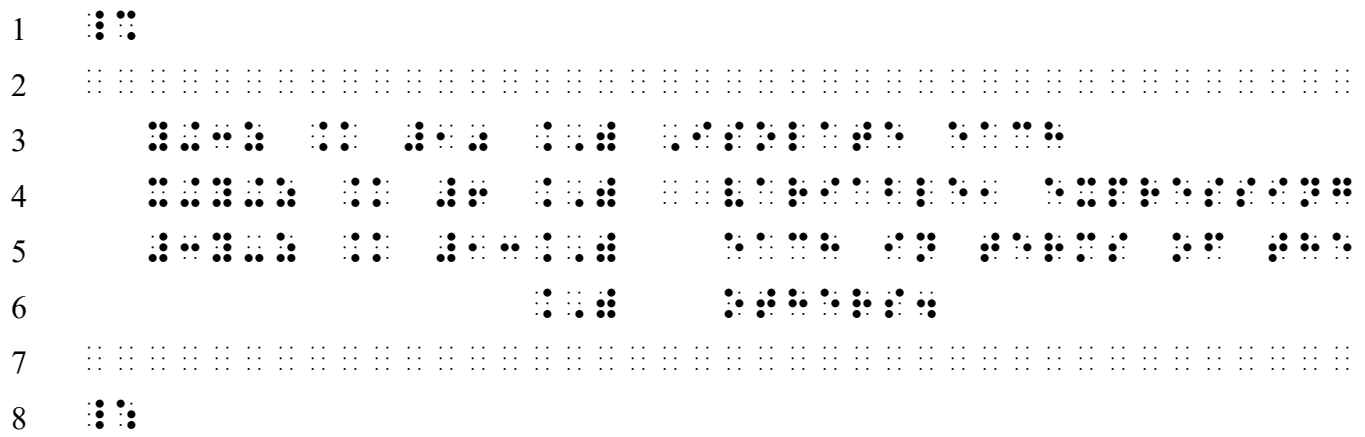
Braille observation:
Line 4: The comment begins on the top line of the arrangement. The comment is uncontracted.
Line 5: The runover is indented two cells from the first cell of the comment.
Line 7: A blank line follows the system.
Line 8: The Nemeth Code terminator is placed in cell 1.

17.10.3 **A Long Comment.** If the comment requires more lines than the spatial arrangement does, the enlarged grouping symbol is extended to cover the explanation.

Example 17-13

$$\left.\begin{aligned} y+3z&=10\\ x+y+z&=6\\ 3y-z&=13 \end{aligned}\right\}$$ *Isolate each variable, expressing each in terms of the others.*

Print Observation: This is similar to Example 17-12, but the comment is longer.



Braille observation:
Line 3: The comment begins on the top line of the arrangement.
Lines 4-6: The runovers are indented two cells from the first cell of the comment text.
Line 6: Four lines are required to complete the comment, so another right enlarged grouping symbol is transcribed.
Line 7: A blank line follows the spatial arrangement.
Line 8: The Nemeth Code terminator is placed in cell 1.

a. **An Alternate Option.** If there is little room beside the math for the comment, it may be placed before or after the math arrangement, at the transcriber's discretion. A transcriber's note must explain that the comments apply to the spatial arrangement. (No example given.)

17.10.4 **Transcriber-Inserted Grouping Symbol [NC 19.8, 25.10.1].** When a comment refers to more than one print line but no enlarged grouping sign is printed, the implied grouping is indicated by using the transcriber-inserted grouping symbol. Such a comment will be transcribed to the right of the arrangement, regardless of its position in print. This symbol begins with dots (6, 3) but, in context, will not be misread as a single-word switch indicator due to the vertical aspect.


```
Transcriber-Inserted Grouping Symbol
(on two or more lines)
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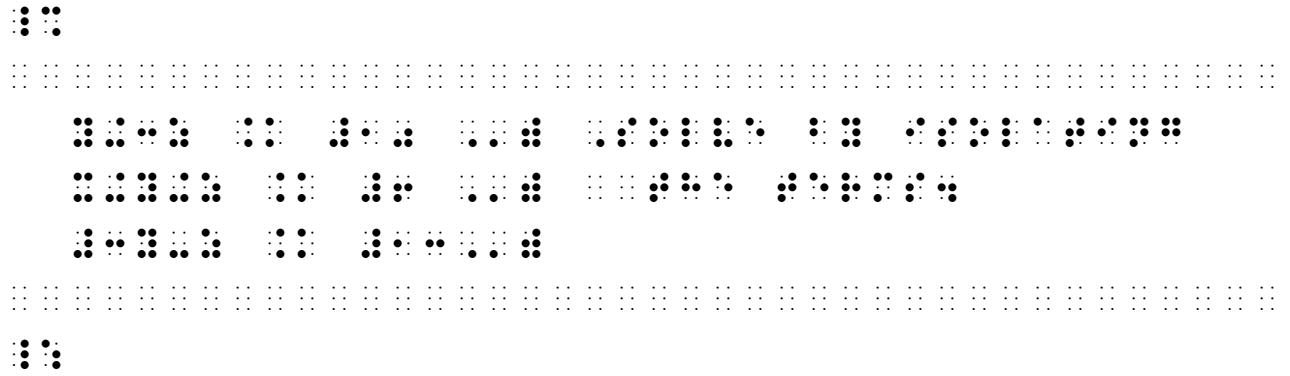
Apply the same formatting rules as you would for printed enlarged grouping symbols, discussed in Sections 17.10.1-17.10.3. If the comment takes fewer lines than the arrangement

to which it refers, a transcriber-inserted grouping symbol is still placed on every line of the arrangement.

Example 17-14

$$\begin{aligned} y + 3z &= 10 \\ x + y + z &= 6 \\ 3y - z &= 13 \end{aligned}$$ *Solve by isolating the terms.*

Print observation: A comment is printed to the right of a 3-line system. The comment applies to all three lines of the system.



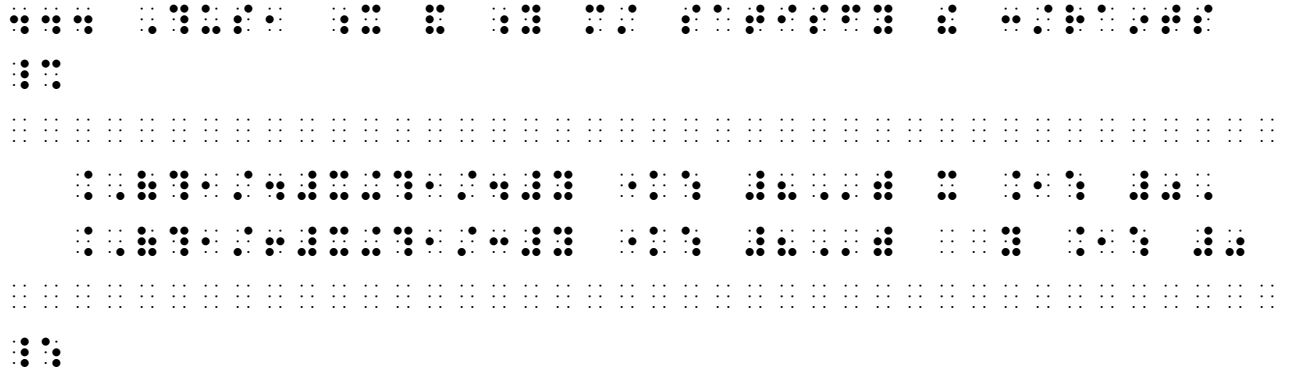
Braille observation: The transcriber-inserted grouping symbol is placed on all three lines of the system, even though the comment takes only two lines.

Example 17-15

... Thus, x and y must satisfy the constraints

$$\begin{cases} \frac{1}{4}x + \frac{1}{4}y \le 8 \\ \frac{1}{6}x + \frac{1}{3}y \le 8 \end{cases} \qquad x \ge 0,\ y \ge 0$$

Print observation: The arrangement is grouped on the left using a left enlarged brace. Conditions are printed to the right, separated from the arrangement by spacing.



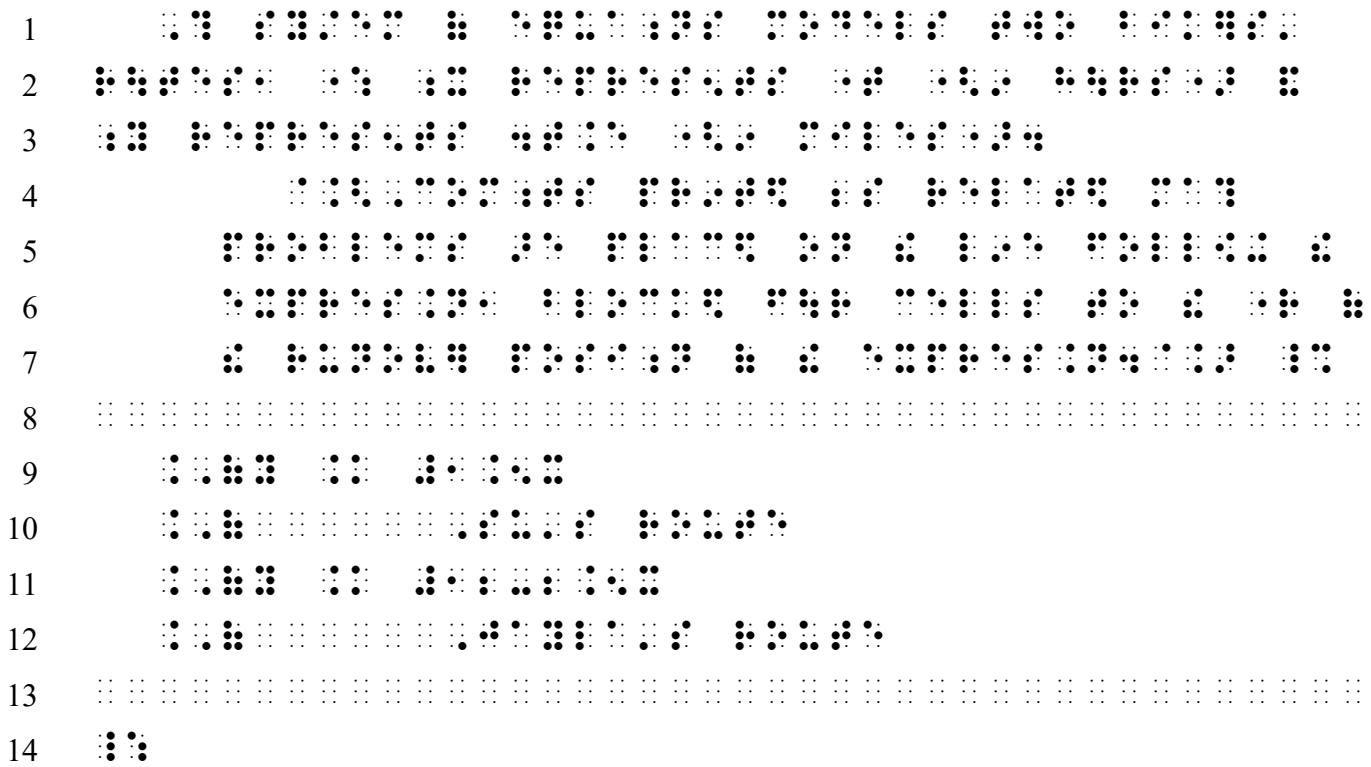
Braille observation: The left enlarged brace is transcribed as printed, beginning in cell 3 (displayed to narrative). The transcriber-inserted grouping symbol is placed between the arrangement and the conditions. The Nemeth Code terminator is placed in cell 1 after the blank line.

17.10.5 **Line-by-Line Commentary.** When comments refer to individual lines in a grouped expression, the method presented in Lesson 16 is applied, indenting the comments four cells to the right of the of the runover position of the line to which they apply. Because Nemeth enlarged grouping symbols are used, the comments are uncontracted. A transcriber's note is required to explain the layout.

Example 17-16

This system of equations models two bikers' routes, where x represents time (in hours) and y represents distance (in miles).

$$\begin{cases} y = 1.5x & \text{Su's route} \\ y = 12 - 2.5x & \text{Jayla's route} \end{cases}$$



Lines 4-7: The transcriber's note reads as follows. "Comments printed beside related math problems are placed on the line following the expression, blocked four cells to the right of the runover position of the expression."
Lines 8 and 13: A blank line precedes and follows the spatially-arranged material.

PRACTICE 17D

Find x and y in terms of a and b.

1. $\begin{cases} ax + by = 0 \\ x + y = 1 \end{cases} \quad (a \neq b)$

2. $\begin{cases} ax + by = 0 \\ a^2x + b^2y = 1 \end{cases} \quad (a \neq 0,\ b \neq 0,\ a \neq b)$

3. $\left.\begin{aligned} a &= \dfrac{x+y}{x-y} \\ b &= \dfrac{x-y}{x+y} \end{aligned}\right\} \quad -1 < x < 1,\ -1 < y < 1$

4. Solve.

$$\left.\begin{aligned} 11z - 4z + 2z &= 27 \\ 7z + 2z &= 27 \\ 9z &= 27 \\ z &= 3 \end{aligned}\right\} \quad \textit{Combine like terms.}$$

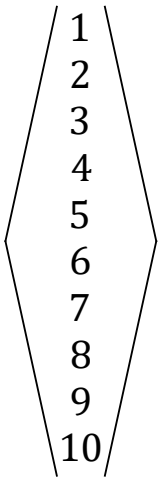
17.11 More Enlarged Signs of Grouping [NC Rule 19]

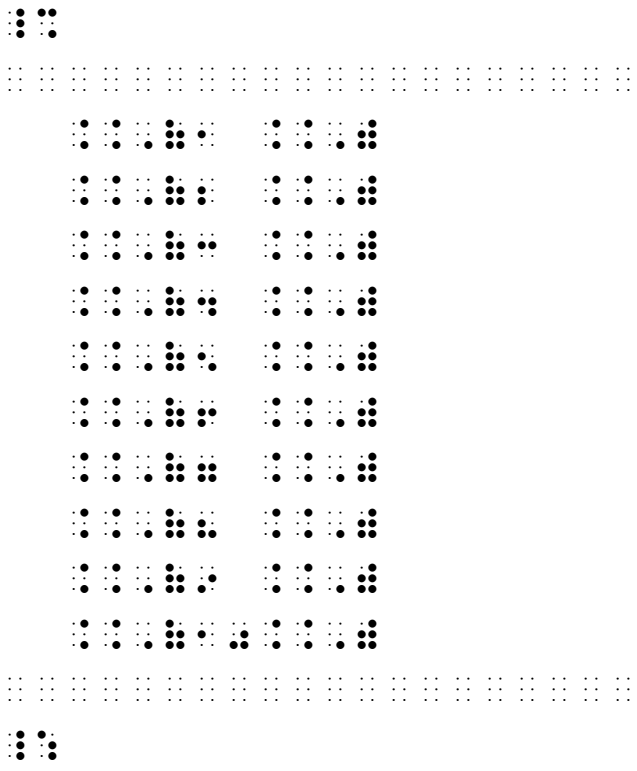
In addition to the enlarged braces and parentheses, the Nemeth Code provides symbols for six other enlarged signs of grouping. Notice that each enlarged braille symbol is formed by inserting a dot 6 before the ⠸, ⠼, or ⠳ symbol of the normal-sized grouping symbol.

Vertical Bar		
⠳	Single, Normal	|
⠳⠳	Double, Normal	||
⠘⠳	Single, Enlarged	
⠘⠳⠘⠳	Double, Enlarged	
Barred Brace		
⠘⠸⠼	Left, Normal	⦃
⠘⠸⠼	Right, Normal	⦄
⠘⠸⠘⠼	Left, Enlarged	
⠘⠸⠘⠼	Right, Enlarged	
Bracket		
⠘⠼	Left, Normal	[
⠘⠼	Right, Normal	]
⠘⠘⠼	Left, Enlarged	
⠘⠘⠼	Right, Enlarged	
Angle Bracket		
⠘⠸⠼	Left, Normal	<
⠘⠸⠼	Right, Normal	>
⠘⠸⠘⠼	Left, Enlarged	
⠘⠸⠘⠼	Right, Enlarged	

Barred Bracket		
⠈⠸⠷	Left, Normal	⟦
⠈⠸⠾	Right, Normal	⟧
⠈⠸⠠⠷	Left, Enlarged	
⠈⠸⠠⠾	Right, Enlarged	
Half Bracket		
⠈⠘⠷	Upper Left, Normal	⌈
⠈⠘⠾	Upper Right, Normal	⌉
⠈⠘⠠⠷	Upper Left, Enlarged	
⠈⠘⠠⠾	Upper Right, Enlarged	
⠈⠰⠷	Lower Left, Normal	⌊
⠈⠰⠾	Lower Right, Normal	⌋
⠈⠰⠠⠷	Lower Left, Enlarged	
⠈⠰⠠⠾	Lower Right, Enlarged	

Example 17-17





⟨1 2 3 4 5 6 7 8 9 10⟩

⠸⠩

⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶

⠨⠨⠠⠷⠂ ⠨⠨⠠⠾

⠨⠨⠠⠷⠆ ⠨⠨⠠⠾

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⠨⠨⠠⠷⠢ ⠨⠨⠠⠾

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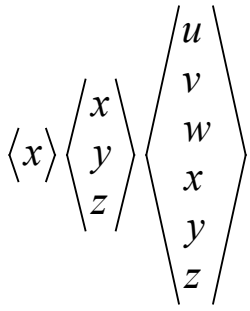
⠨⠨⠠⠷⠂⠴⠨⠨⠠⠾

⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶

⠸⠱

Instructions: Follow "top alignment" rules for these side-by-side arrays. Place the leftmost character in cell 1.

PRACTICE 17E



DETERMINANTS AND MATRICES

[NC 25.8]

17.12 Definition and Recognition

Determinants and matrices (singular: matrix) are arrangements of items in rows and columns which are enclosed between left and right grouping symbols. Items can be numbers, symbols, or mathematical expressions.

Here is a 2 × 2 ("two by two") determinant enclosed between enlarged vertical bars.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Here is a 2 × 3 ("two by three") matrix enclosed between enlarged brackets.

$$\begin{bmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{bmatrix}$$

A matrix can also be composed of only one column or only one row. Row matrices will be studied at the end of this section.

Determinants and matrices may also be referred to as "arrays."

17.13 Transcription Rules for Determinants and Matrices

Observe the following rules regarding the transcription of the two samples shown in Section 17.12.

17.13.1 **Blank Lines.** Determinants and matrices are spatial arrangements. Thus, a blank line is to be left above and below each array. If the arrangement begins at the top of a braille page, it may begin on line 1 provided no running head is in use; if the arrangement ends at the bottom of a braille page, it may end on line 25. In either case, the rightmost symbol of the arrangement must not lie within three cells of the page number. (See Lesson 9 for details regarding layout of a spatial arrangement at the top or bottom of a page.)

17.13.2 **Grouping Symbols.** For an array consisting of two or more rows, enlarged grouping symbols are used on each braille line. Grouping symbols are vertically aligned.

17.13.3 **Alignment and Spacing of Items.** Each entry is moved as far left as possible in its column. Consequently, each left grouping symbol will be in direct contact with the first entry of each row in the array.

One column of blank cells is left between the columns of the arrangement. That is, one blank cell separates the widest entry in a column from the beginning of the next column. Even if entries contain a space, only one space is left between columns. See Example 17-20.

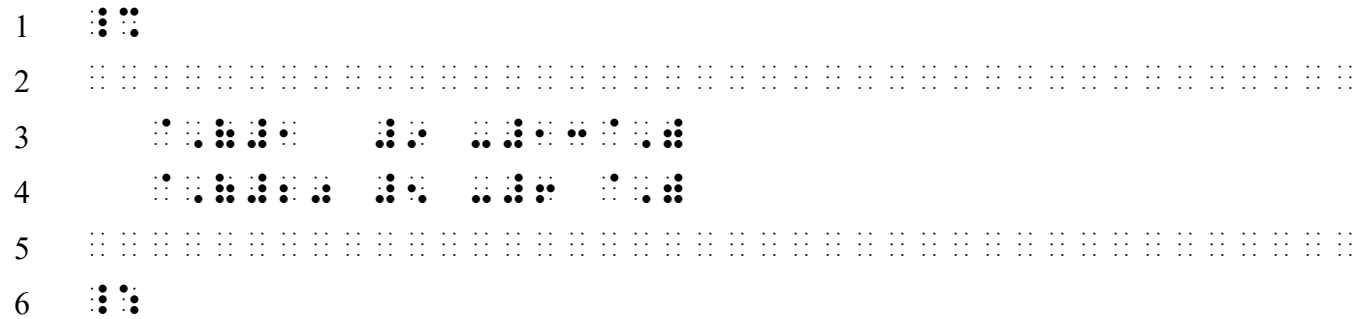
At least one right grouping symbol must be in direct contact with an entry in the array. The widest entry in the rightmost column determines the placement of the right enlarged grouping symbols.

17.13.4 **Numeric and Letter Indicators.** The numeric indicator is used with numeric entries in an array, even when such entries are in direct contact with a grouping symbol. The English-letter indicator is not used with any English letter in an array.

Example 17-18

$$\begin{bmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{bmatrix}$$

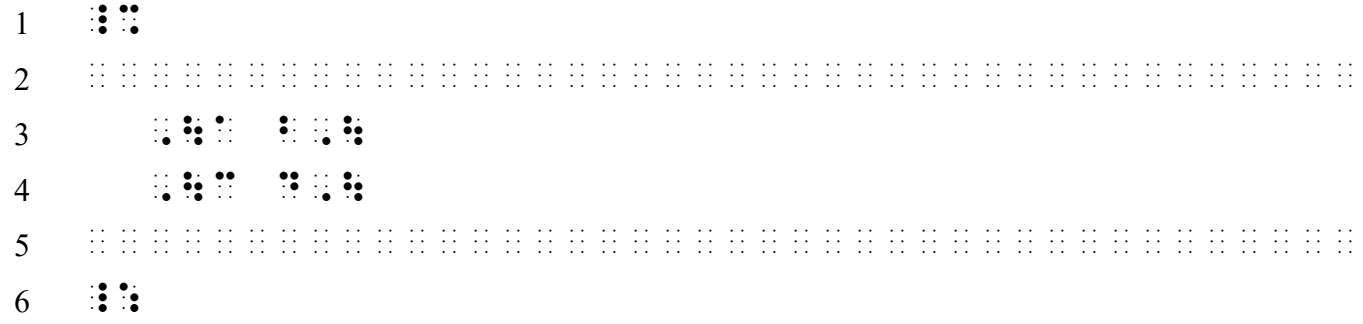
Print observation: Enlarged brackets enclose this two-row array. The numerical entries are centered in their columns.



Braille observation: The numeric indicator is used for each number. Items begin in the same cell in each column (entries are left aligned within each column). One blank cell separates the widest entry in a column from the beginning of the next column. The widest entry in the last column determines the placement of the right enlarged grouping symbols.

Example 17-19

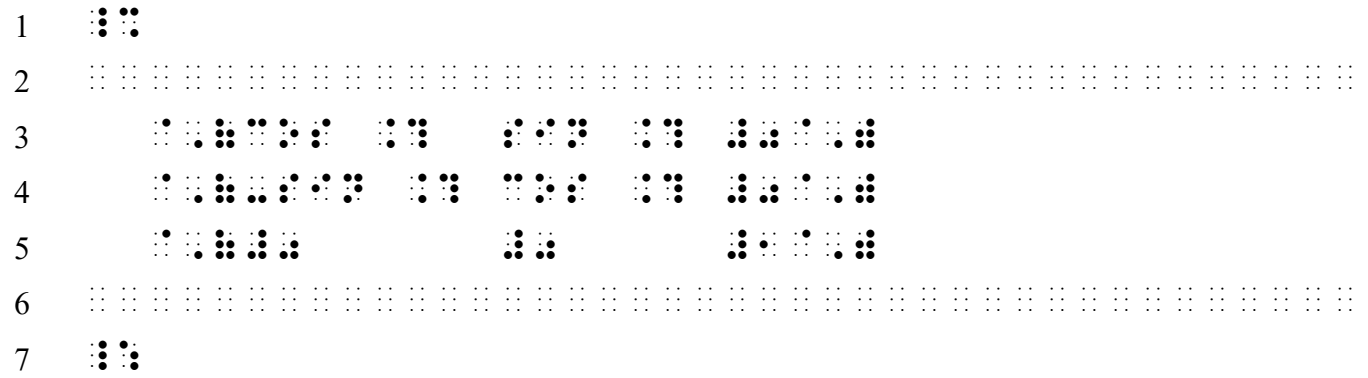
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$



Enlarged vertical bars enclose this two-row array. Each English letter is transcribed without a letter indicator.

Example 17-20

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



One space separates each column, even when entries contain a space.
Columns are left aligned, regardless of print layout.

17.13.5 **Placement of Identifiers, Symbols, and Punctuation.** Material outside of an array (such as identifiers, punctuation, operation or comparison signs) are placed on the top line of the arrangement even though the material may be centered in print.

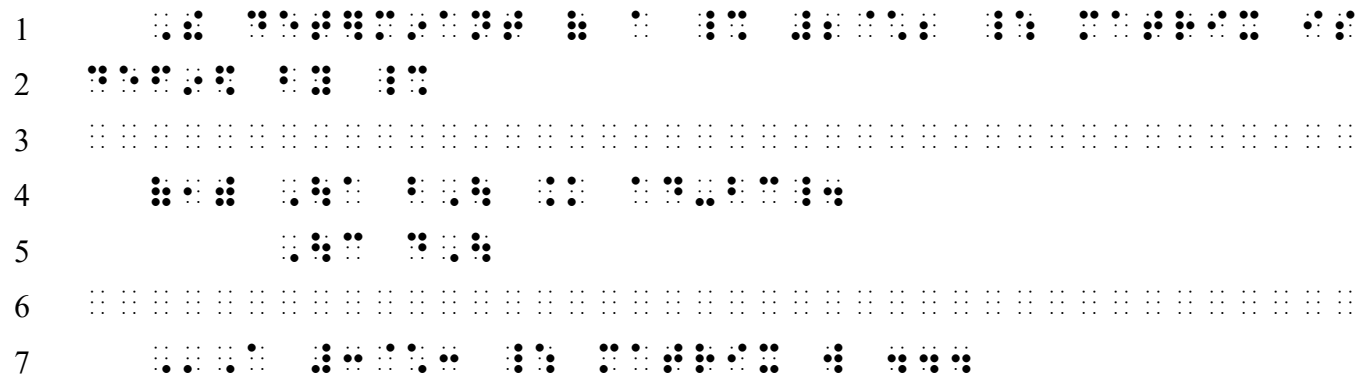
Example 17-21

The determinant of a 2 × 2 matrix is defined by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc. \qquad (1)$$

A 3 × 3 matrix will ...

Print observation: An equals sign and expression are centered to the array. A period ends the statement. The expression is identified to the right with a numeral in parentheses.

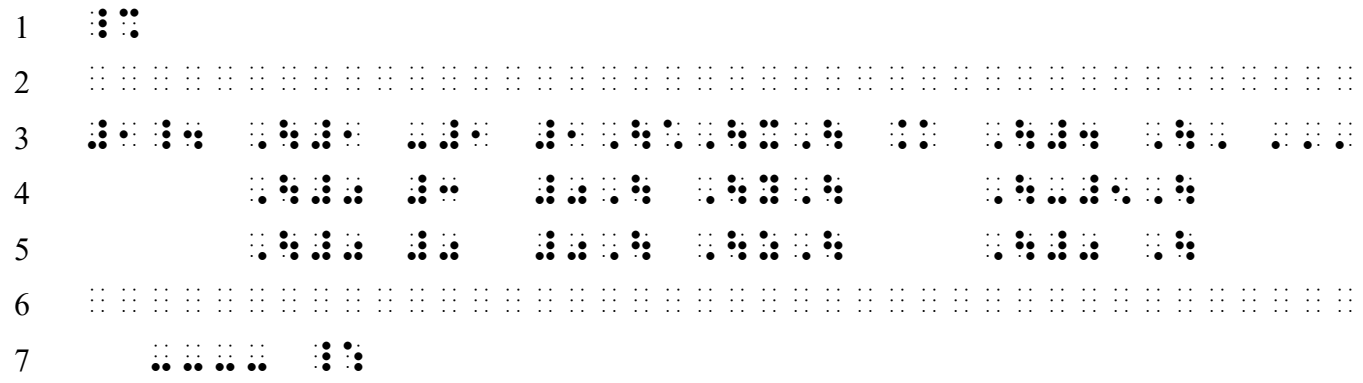


Braille observation: The identifier is moved to the left of the displayed expression according to the rules of the Nemeth Code. (Lesson 7) The identifier, the comparison symbol and the link, and the period are placed on the top line of the arrangement. A punctuation indicator precedes the period.

Example 17-22

1. $\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 4 \\ -5 \\ 0 \end{vmatrix}, \cdots$

Print observation: Three 3-row arrays are printed side-by-side. Aligned with row 2 are symbols before, between, and after the arrays: an identifier, a multiplication dot, an equals sign, a comma, and an ellipsis. The numbers are right adjusted within their columns. Assume that more math follows.



Braille observation:
Line 3: The material outside of the array is placed on the top line of the arrangement—the identifier, the operation symbol, the comparison symbol, and the punctuation. The multiplication dot is unspaced from the items being multiplied, following spacing rules for operation signs.
Line 7: A long dash is inserted to show that further math would be placed in the runover position (cell 3).

PRACTICE 17F

1. $\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

2. $\begin{pmatrix} 1 & -\frac{3}{4} & \frac{5}{3} \\ 2 & 5 & 12 \end{pmatrix}$

3. $\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \end{bmatrix}$

4. $\begin{vmatrix} ab & cd \\ ac & ce \end{vmatrix}$

5. Explain why points $(a_1,\ b_1), (a_2,\ b_2),$ and $(a_3,\ b_3)$ are collinear if and only if

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$$

PRACTICE 17G

A. The unit vectors of a three dimensional Cartesian coordinate system are shown below.

$$\hat{\mathbf{i}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ \hat{\mathbf{j}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \text{and } \hat{\mathbf{k}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

17.14 Multiplying Arrays

Recall that a multiplication problem can be printed without a multiplication symbol. When each factor is enclosed in grouping signs, it is understood that the side-by-side factors are to be multiplied. For example, $3 \cdot 2$ and $(3)(2)$ both mean "three times two". Similarly, when arrays are being multiplied, the multiplication symbol is often not printed. This array

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

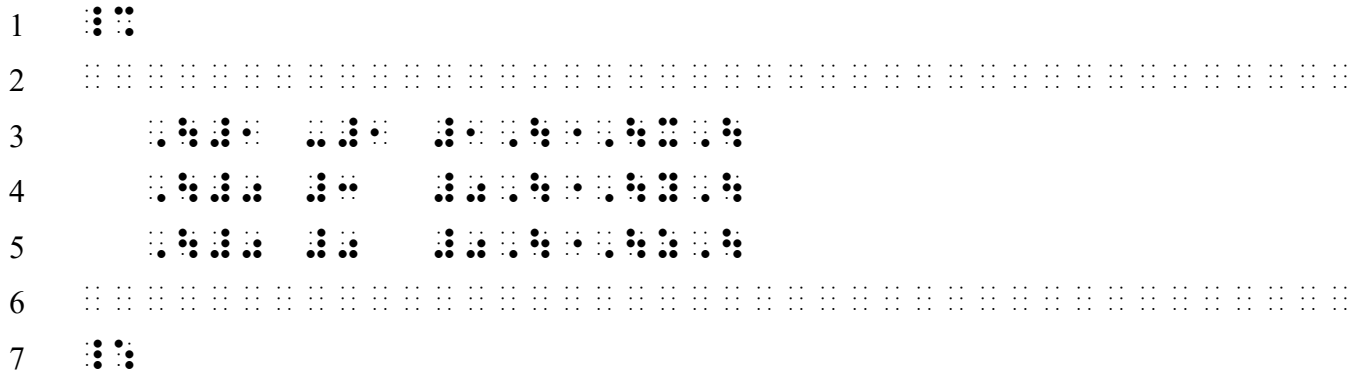
can also be printed without the multiplication dot, like this:

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

The spacing between the arrays shown (in print) is there only to distinguish between the two factors. The space is not inserted in braille. When vertical bars are used, a multipurpose indicator is transcribed between the two symbols, otherwise it will be misread as a double vertical bar.

Example 17-23

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$



Lines 3-5: The multipurpose indicator is transcribed between each side-by-side enlarged vertical bar, on every line of the arrangement.

17.15 Ellipses, Single Dots, and Blank Entries

The following symbols represent ellipses, a single dot, or a blank entry in a matrix or a determinant.

⠀⠀	Blank Entry	
⠈⠀⠀	Ellipsis, Diagonal, lower left to upper right	⋰
⠈⠀⠈	Ellipsis, Diagonal, upper left to lower right	⋱
⠀⠀⠀	Ellipsis, Horizontal	…
⠒⠀⠀	Ellipsis, Vertical	⋮
⠄	Single Dot	·

Each symbol is positioned as far left as possible in its column. A transcriber's note must explain the use of the short dash to represent the blank space. Sample transcriber's note:

A short dash represents a blank entry.

Example 17-24

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & \cdots & . \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

Print observation: A horizontal ellipsis is in each entry in column 3. A single dot is printed in row 3, columns 1, 2, and 4.

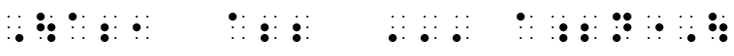
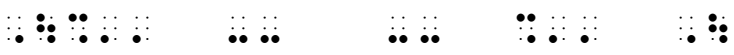
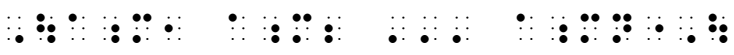
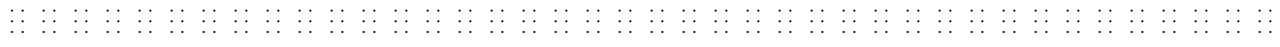
1 ⠒⠒

2 ⠀⠀⠀

Example 17-25

$$\det(\mathrm{A}) = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{vmatrix}$$

Print observation: The leftmost and rightmost columns show a vertical ellipsis between the second and the last entry in the column. In columns 2 and 3, the same area is blank. The first, second, and last row show a horizontal ellipsis printed in the third column.

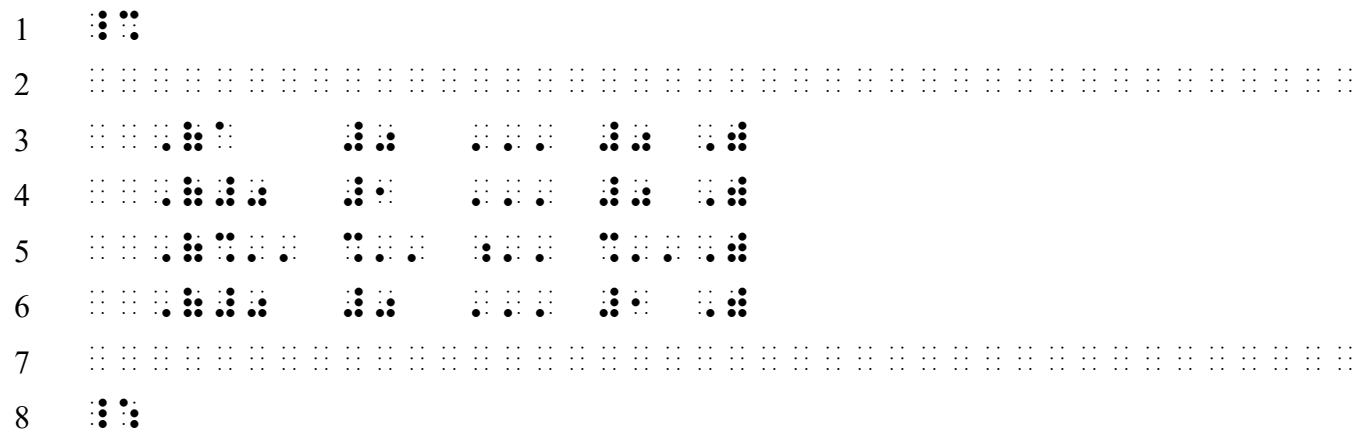
1
2
3
4
5 
6 
7 
8 
9

Braille observation: A transcriber's note explains the use of the short dash: "A short dash represents a blank entry." In rows 1, 2, and 4, a baseline indicator is needed to assure each right grouping symbol is on the same level as the related left grouping symbol.

Example 17-26

$$\begin{pmatrix} a & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

Print observation: Horizontal, vertical, and diagonal ellipses occur in this matrix.

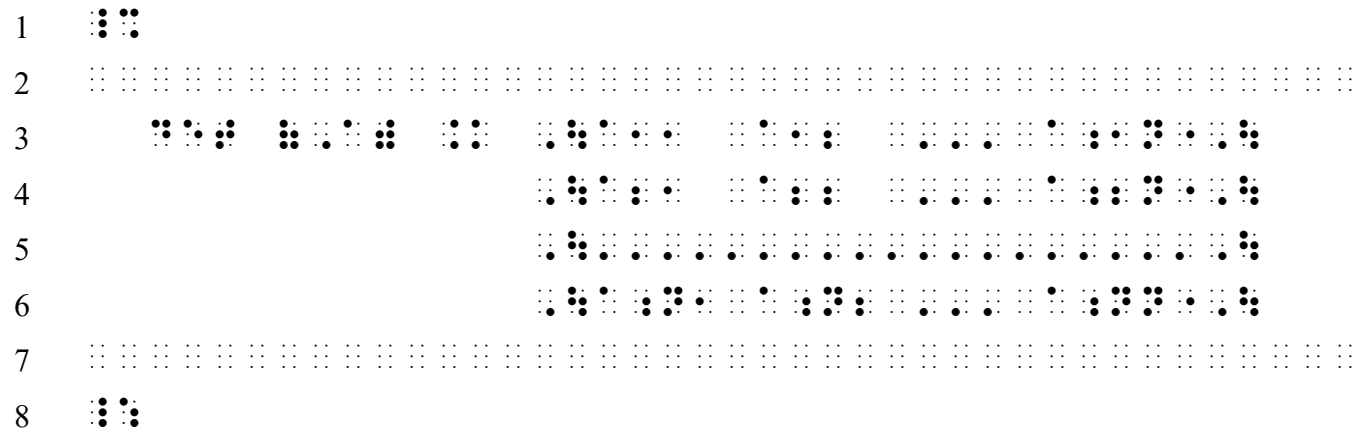


17.15.1 **A Row of Dots Printed Without Spaces Between Columns.** When dots are strung completely across the omitted row and the dots occupy space between the columns, a row of unspaced dot 3's is transcribed across the full width of the array, beginning in the first cell of the first column and extending to the end of the longest entry in the last column.

Example 17-27

$$\det(\mathrm{A}) = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ . & . & . & . \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix}$$

Print observation: The third row is printed as a series of dots across the width of the array.



Braille observation: On rows 1, 2, and 4 a baseline indicator is needed to assure each right grouping symbol is on the same level as the related left grouping symbol.

PRACTICE 17H

In the next equation, matrix Y is expressed as the product of matrices B and X.

$$\begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ y_r \end{vmatrix} = \begin{vmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ b_{31} & b_{32} & \cdots & b_{3n} \\ . & . & . & . \\ b_{r1} & b_{r2} & \cdots & b_{rn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{vmatrix}$$

Instructions: Place a transcriber's note before item 2 explaining the treatment of the blank entries.

PRACTICE 17I

1. $A = \begin{bmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{K1} & \cdots & a_{KM} \end{bmatrix}$

2. $B = \begin{bmatrix} b_{11} & b_{12} & \cdots \\ \vdots & \ddots & \\ b_{K1} & & b_{KK} \end{bmatrix}$

17.16 Augmented Matrix

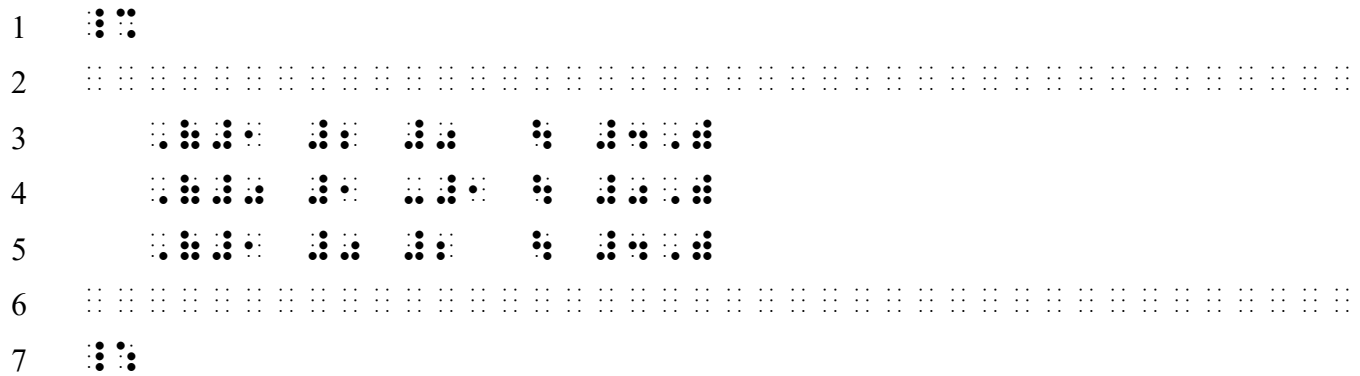
The vertical line in an augmented matrix (the "augmentation line") is represented in braille with the vertical bar symbol, preceded and followed by at least one space, in each row of the matrix. Print may use a solid, dashed, or gray line. In all cases, a vertical bar is transcribed.

⠳	Vertical Bar ⠸

If space is an issue, the vertical line may be represented as a tactile drawing. See *Guidelines and Standards for Tactile Graphics* for drawing techniques.

Example 17-28

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 2 & 4 \end{array}\right)$$



17.17 Runovers in Arrays

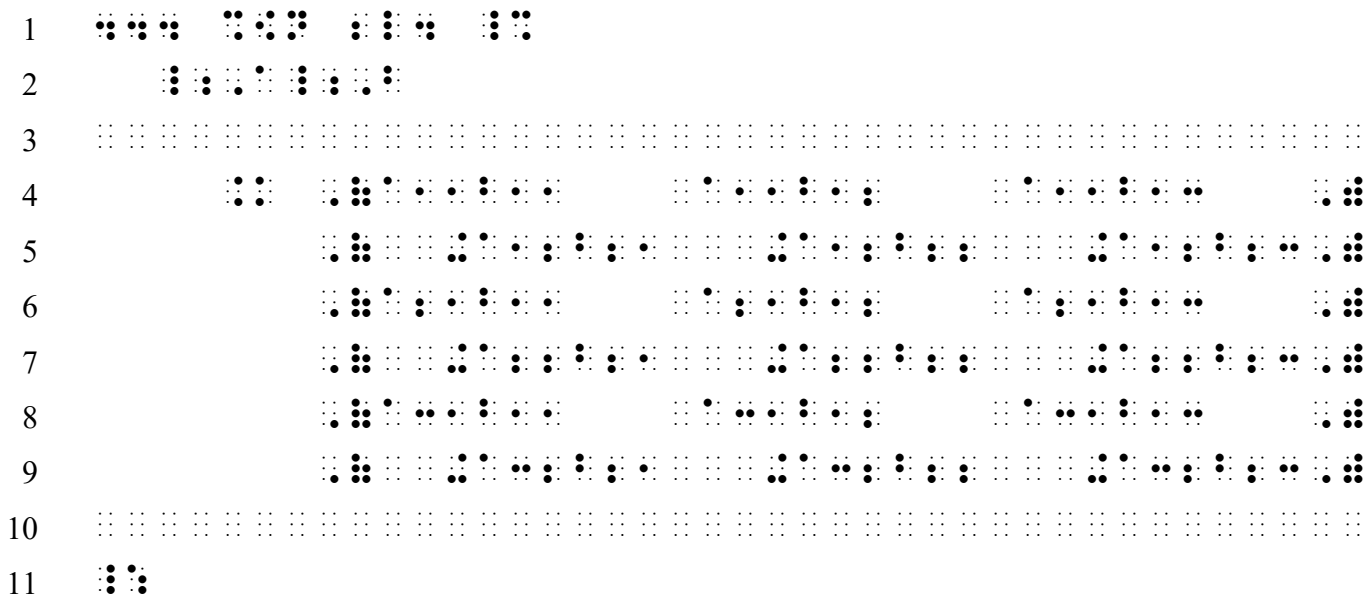
When row entries are too wide to fit on one braille line within the current margins, the arrangement may begin in cell 1 if this will allow each row to be contained on one line. If that strategy fails, apply one of the following techniques.

17.17.1 **Runovers With Indentation.** This method is the preferred runover technique. Entries may be divided (see Lesson 15). Runovers are indented two cells from the first cell of the entry. The guidelines for division of mathematical expressions can be disregarded if there is no suitable division site. There is no need to explain the runover format to the reader.

Example 17-29

... shown below.

$$\mathbf{AB} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{pmatrix}$$



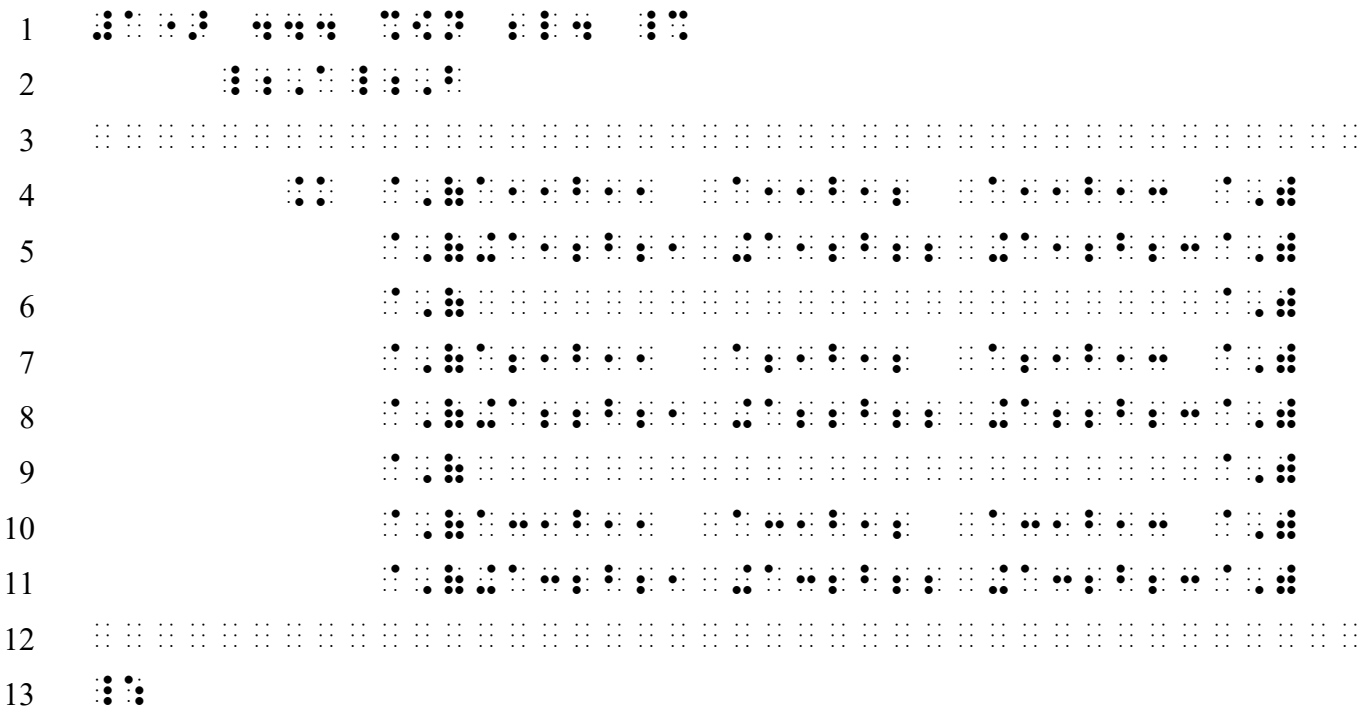
One blank cell separates the widest entry in a column, including any runovers, from the beginning of the next column.

17.17.2 **Runovers Without Indentation.** If the technique shown in Section 17.17.1 is not feasible, entries may be run over to new lines without indentation. Preference rules for runovers of mathematical expressions need not be observed if space would be saved. In order to distinguish each row, a blank line is inserted between them. Enlarged grouping symbols are transcribed on those blank lines within the arrangement.

Example 17-30

1) ... shown below.

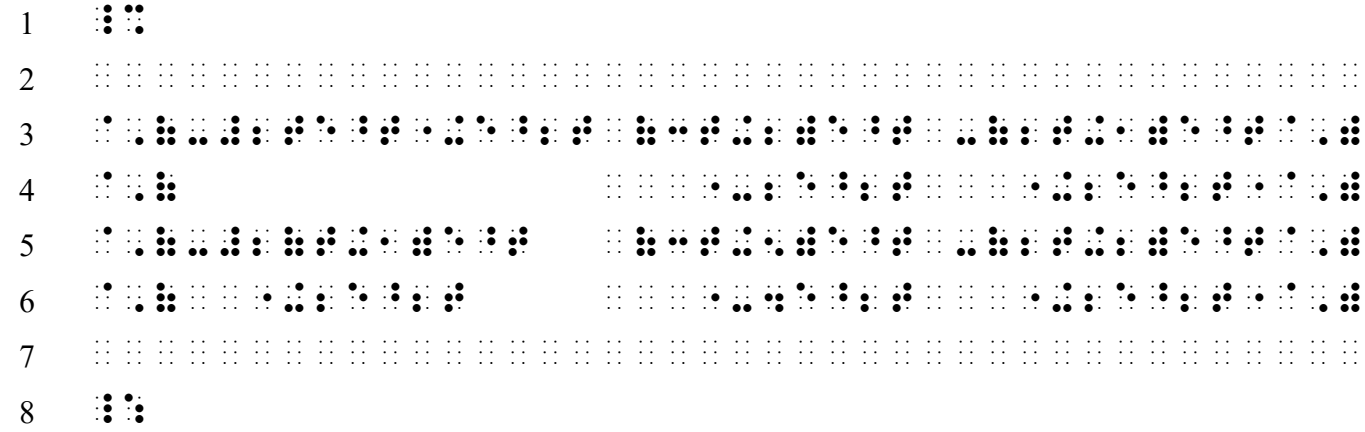
$$\mathbf{AB} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$$



17.17.3 **Runovers and Level Indicators.** As noted in examples throughout this section, a baseline indicator is needed when a row entry in the rightmost column ends at a level other than the baseline, and it touches the right grouping symbol (subscripts which don't require a subscript indicator excepted). When a row entry requires a runover, this rule applies only to the last symbol of the entry. A baseline indicator is not used before a right enlarged grouping symbol if the expression which touches the right grouping symbol is continued on the next line.

Example 17-31

$$\begin{bmatrix} -2te^{t} + e^{2t} & (3t+2)\,e^{t} - 2e^{2t} & -(2t+1)\,e^{t} + 2e^{2t} \\ -2\,(t+1)\,e^{t} + 2\,e^{2t} & (3t+5)\,e^{t} - 4e^{2t} & -(2t+2)\,e^{t} + 2e^{2t} \end{bmatrix}$$



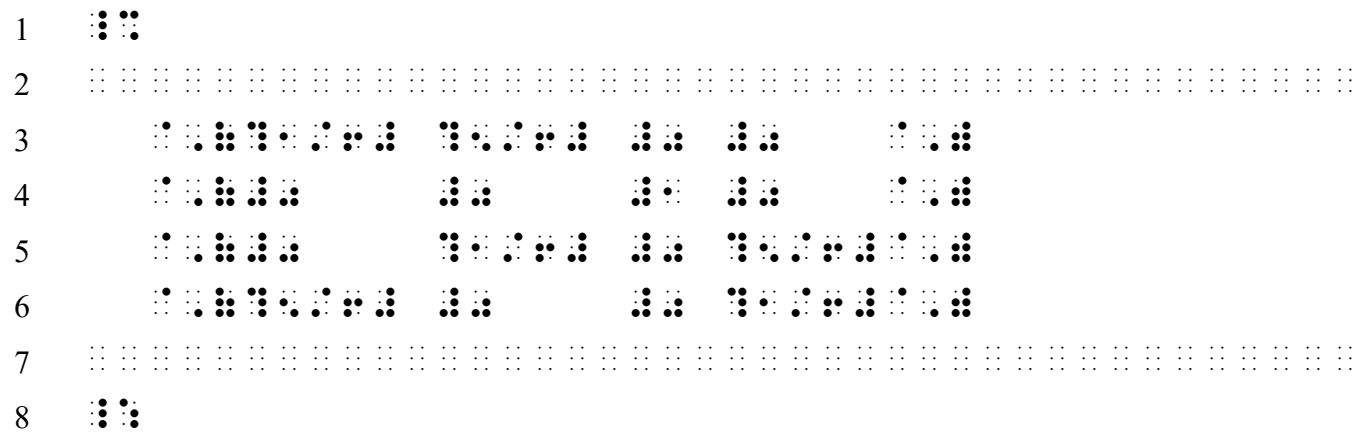
Lines 3-6: Note that this example uses full margins in addition to applying runover techniques.
Lines 3 and 5: A baseline indicator is not needed between the superscript "t" and the right grouping symbol because the expression continues on the next line.

Lines 4 and 6: The runover line of each row entry ends with the letter t in the superscript position. A baseline indicator is required before transcribing the right grouping symbol which follows, unspaced.

17.17.4 **Fractions in Arrays.** Fractions in an array may be transcribed linearly but may be arranged spatially if linear fractions take up too much space.

Example 17-32

$$\begin{bmatrix} \frac{1}{6} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} \end{bmatrix}$$

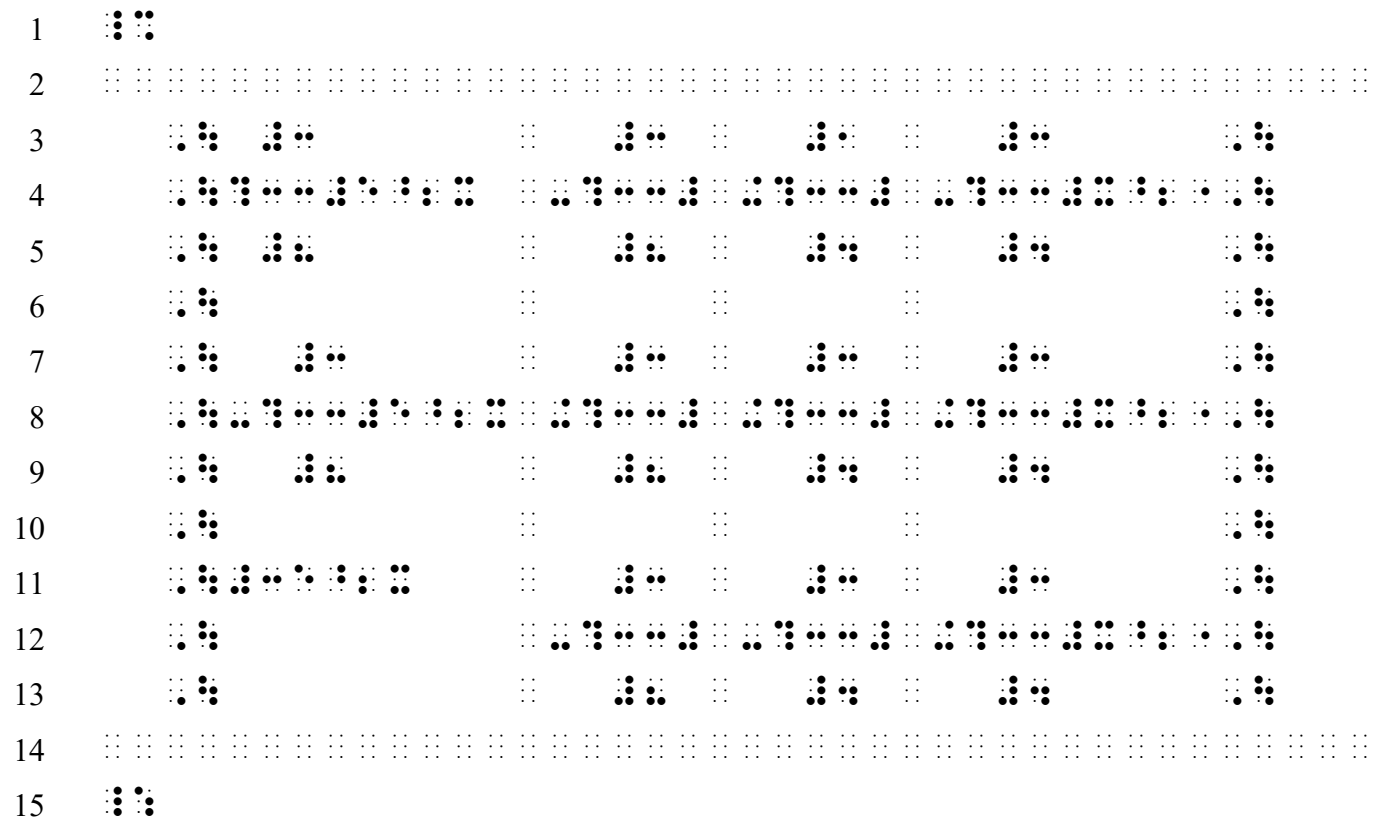


Observe the following details when transcribing the fractions spatially.

- A line is skipped between the rows containing the spatial fraction. Enlarged grouping symbols are transcribed on those blank lines within the arrangement.
- Operation signs and variables are placed on the same line as the spatial fraction line.
- Each entry is moved as far up as possible in its row. This includes entries that are not fractions.
- If the last item in a row is a superscript, or a subscript that uses a subscript indicator, a return to the baseline must occur before the right grouping symbol is transcribed. If the row extends fully to the grouping symbol, a baseline indicator is required to return to the baseline. If the row does not extend to the right grouping symbol, the space returns the reader to the baseline.

Example 17-33

$$\begin{vmatrix} \frac{3}{8}e^{2x} & -\frac{3}{8} & +\frac{1}{4} & -\frac{3}{4}x^2 \\ -\frac{3}{8}e^{2x} & +\frac{3}{8} & +\frac{3}{4} & +\frac{3}{4}x^2 \\ 3e^{2x} & -\frac{3}{8} & -\frac{3}{4} & +\frac{3}{4}x^2 \end{vmatrix}$$



The single-line entry in row 3, column 1, is transcribed on the top line of the row even though it is centered in its row in print.

17.17.5 **Keying.** When the array will not fit on the page even after trying the methods described in this section, a key may be devised. The technique of keying will be discussed in the next lesson.

Instructions: Use spatial fractions.

PRACTICE 17J

The derivative of T can be expressed as follows.

$$dT = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \cdot & \cdot & \cdots & \cdot \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \cdots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}$$

Instructions: Use runovers with indentation. Review spacing rules and "keep together" rules for abbreviated function names in Lesson 14.

PRACTICE 17K

In this Jacobian determinant,

$$\begin{vmatrix} 0 & 5 & 0 \\ 8x_1 & -2x_3\cos(x_2x_3) & -2x_2\cos(x_2x_3) \\ 0 & x_3 & x_2 \end{vmatrix} = -8x_1 \begin{vmatrix} 5 & 0 \\ x_3 & x_2 \end{vmatrix} = -40x_1x_2$$

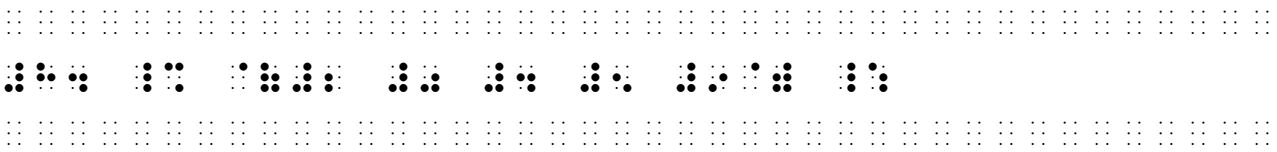
the orientation of the resulting object is reversed.

17.18 Row Matrix

A row matrix has only one row. Because it is a matrix, it is transcribed as a spatial arrangement —that is, a line is left above and below the arrangement. Regular grouping symbols are used, not enlarged symbols, despite the larger appearance of the grouping symbols in print. Here is an example of a 1×5 row matrix.

Example 17-34

8. $\begin{bmatrix} 2 & 0 & 4 & 5 & 9 \end{bmatrix}$



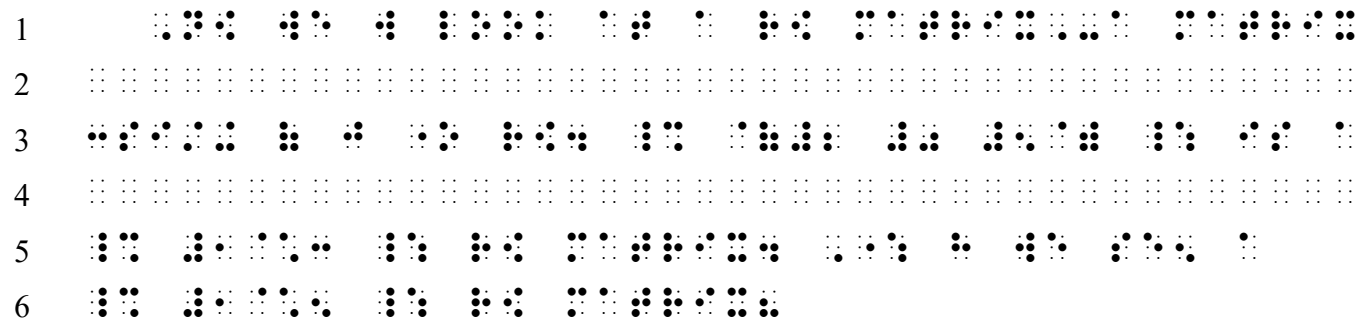
This is not an enclosed list because there are no commas between items. Assume that context will make it clear that this is a row matrix. A blank line precedes and follows the matrix. The brackets are tall in print, but are transcribed as regular brackets.

17.19 Embedded Arrays

If the array is embedded within narrative text, the required blank line is inserted on the line before and the line after the line where the arrangement lies. Text is placed on the same line as the array if it fits.

Example 17-35

Now we will look at a row matrix—a matrix consisting of just one row. $\begin{bmatrix} 2 & 0 & 5 \end{bmatrix}$ is a 1×3 row matrix. Where have we seen a 1×5 row matrix?



A blank line precedes and follows the line which contains the embedded row matrix. The brackets are tall in print, but are transcribed as regular brackets.

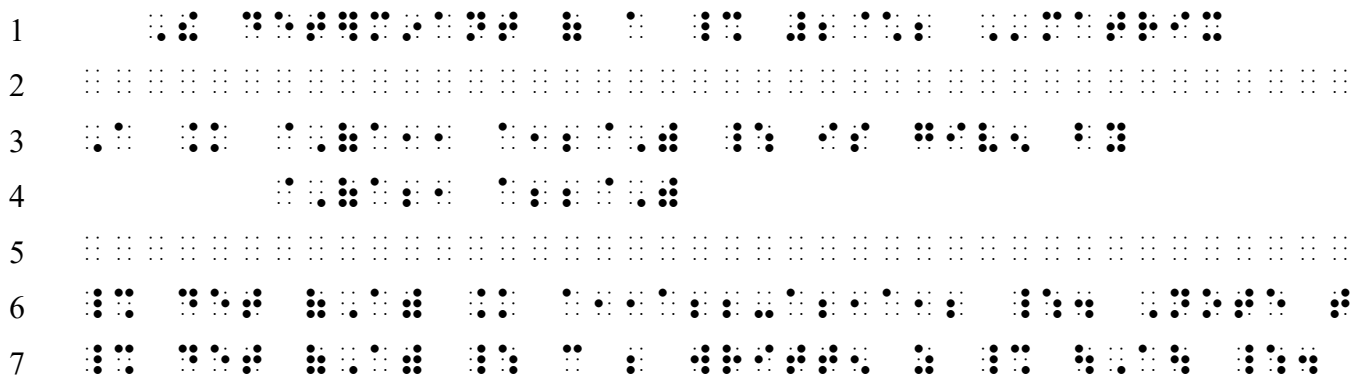
a. **Top Alignment.** If the embedded array consists of more than one line, the surrounding text is placed on the top line of the arrangement only – preceding and/or following the top row of the array, as it fits in context of the narrative. The opening Nemeth Code indicator and the Nemeth Code terminator are placed only on the top line of the embedded arrangement. The switch indicators apply to the whole arrangement.

Example 17-36

The determinant of a 2×2 matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by

$\det(A) = a_{11}a_{22} - a_{21}a_{12}$. Note that $\det(A)$ can be written as $|A|$.

Print observation: The two rows of the embedded matrix are vertically centered to the equals sign in print.



Braille observation: The first row of the embedded matrix is transcribed on the same line as "A =". The second row is aligned beneath the first row; no text is transcribed on that line. The Nemeth Code terminator is on the main line of text. Note that "det" is an abbreviated function name and therefore is followed by a space in the transcription.

PRACTICE 17L

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$, find AB. The answer is

$$AB = \begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}.$$

PRACTICE 17M

1) Here are three examples of matrix operations.

a) $\begin{bmatrix} 5 & 4 \end{bmatrix} + \begin{bmatrix} 20 & 30 \end{bmatrix} = \begin{bmatrix} 25 & 34 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 1 & 5 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 7 \\ 1 & 9 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \end{bmatrix}$

17.20 Use of Tactile Graphics for Enlarged Grouping Signs

Enlarged grouping symbols may be drawn in place of the braille equivalents, especially when space saving is a factor. Refer to BANA's *Guidelines and Standards for Tactile Graphics* for drawing techniques. See Section 12.6.3 in Lesson 12 for horizontal grouping signs.

For further practice, see Addendum 1—Reading Practice.

Submit Exercise 17 to your instructor.

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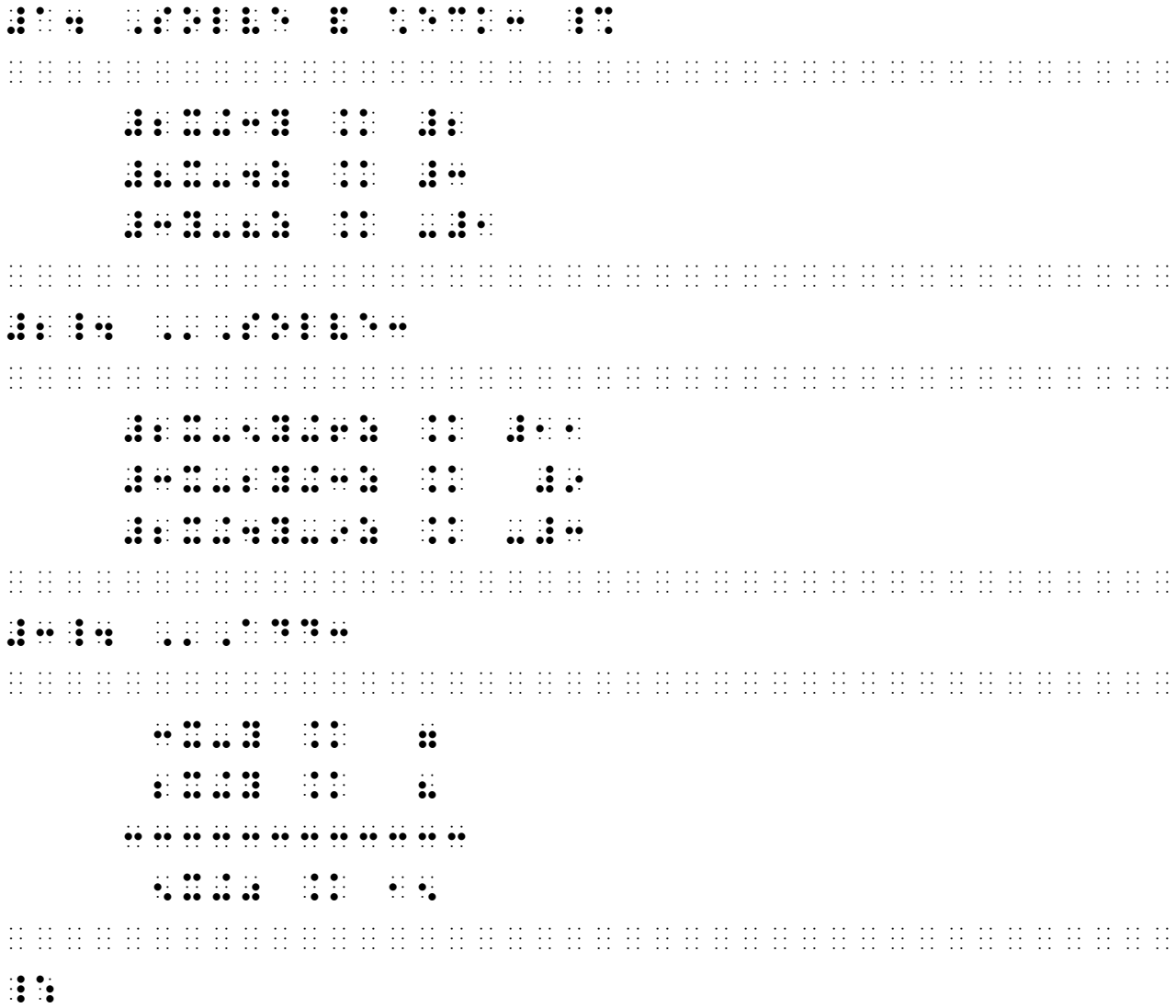
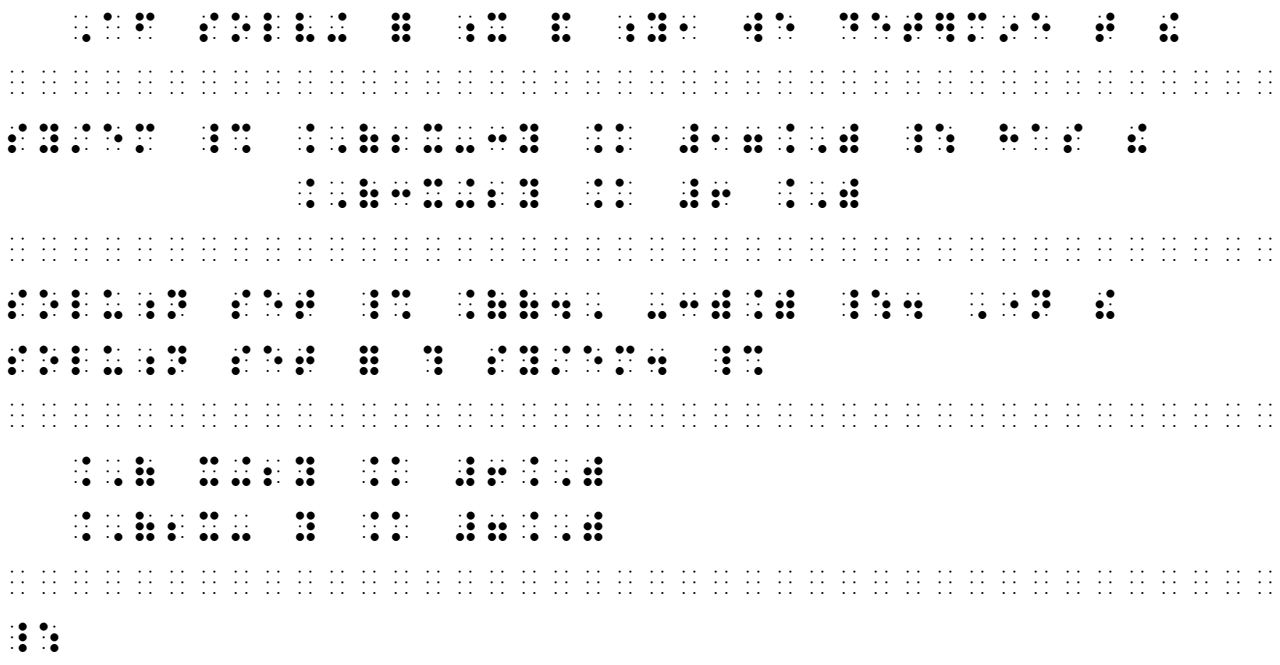
ANSWERS TO PRACTICE MATERIAL

PRACTICE 17A

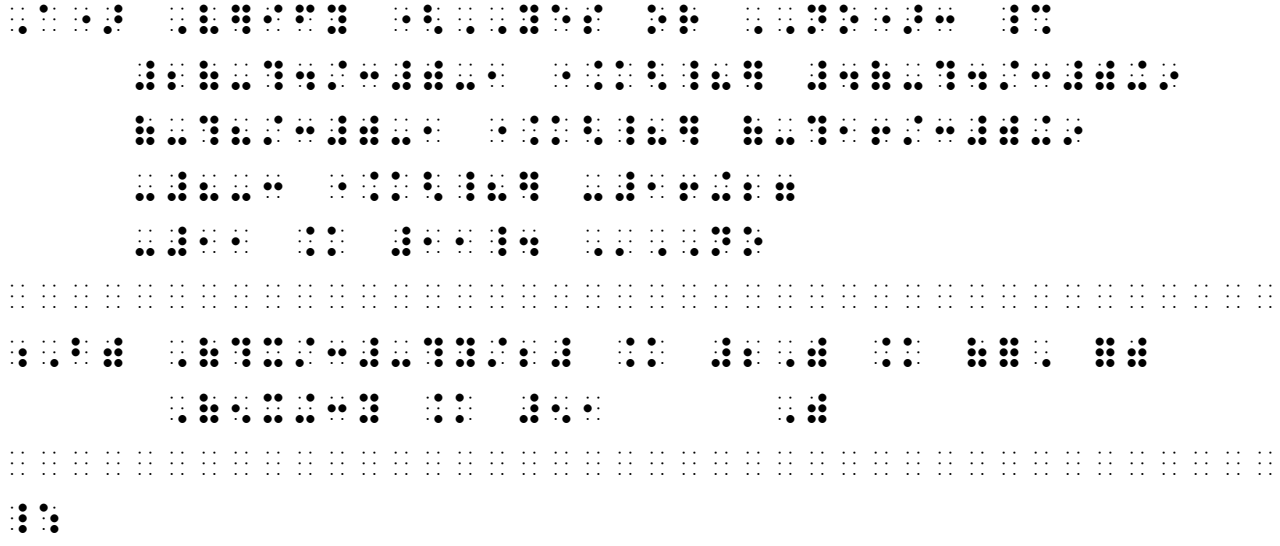


Lines 3-5, 9-11, and line 17: The left margin for material displayed to itemized text is cell 5, even if the text has no runovers.

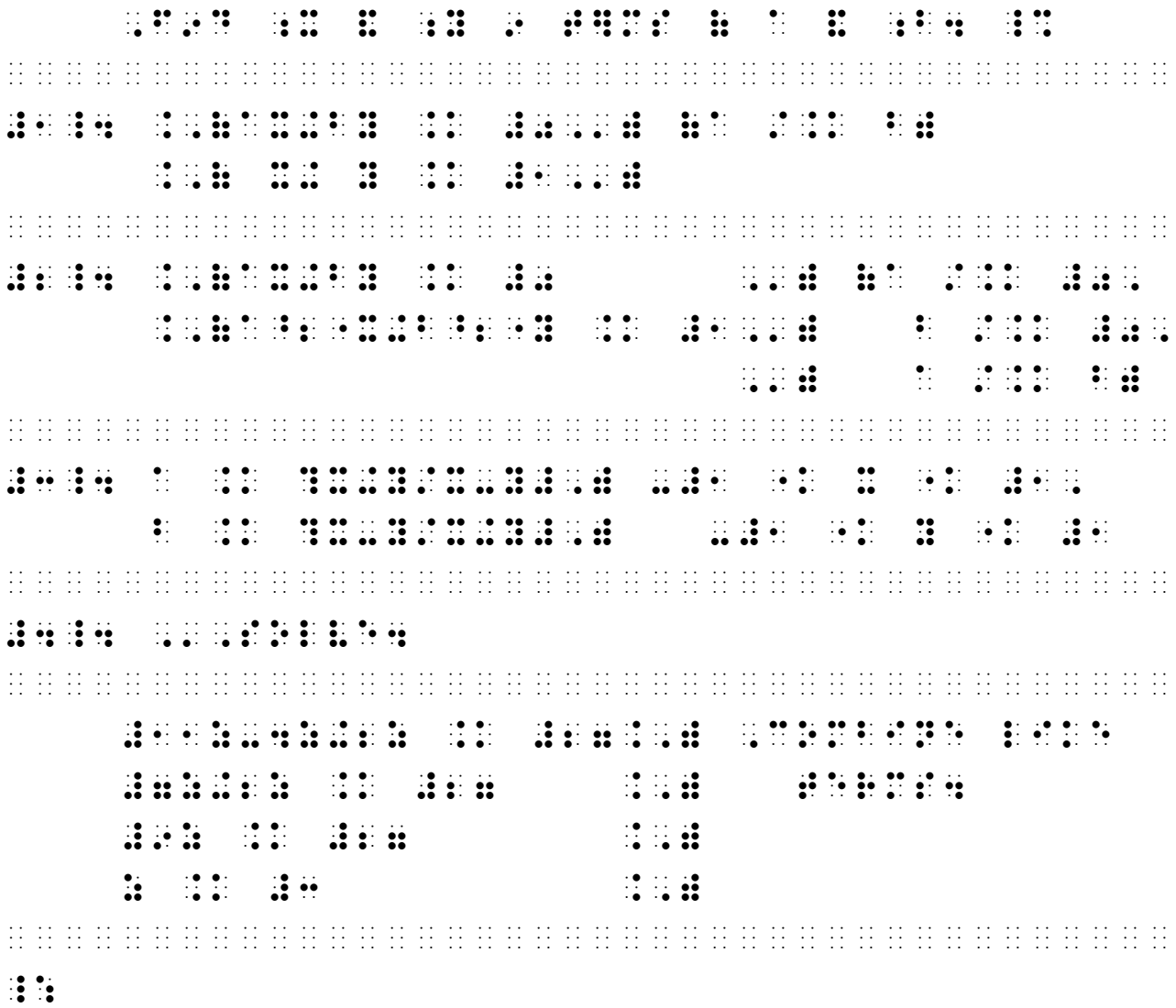
PRACTICE 17B



PRACTICE 17C

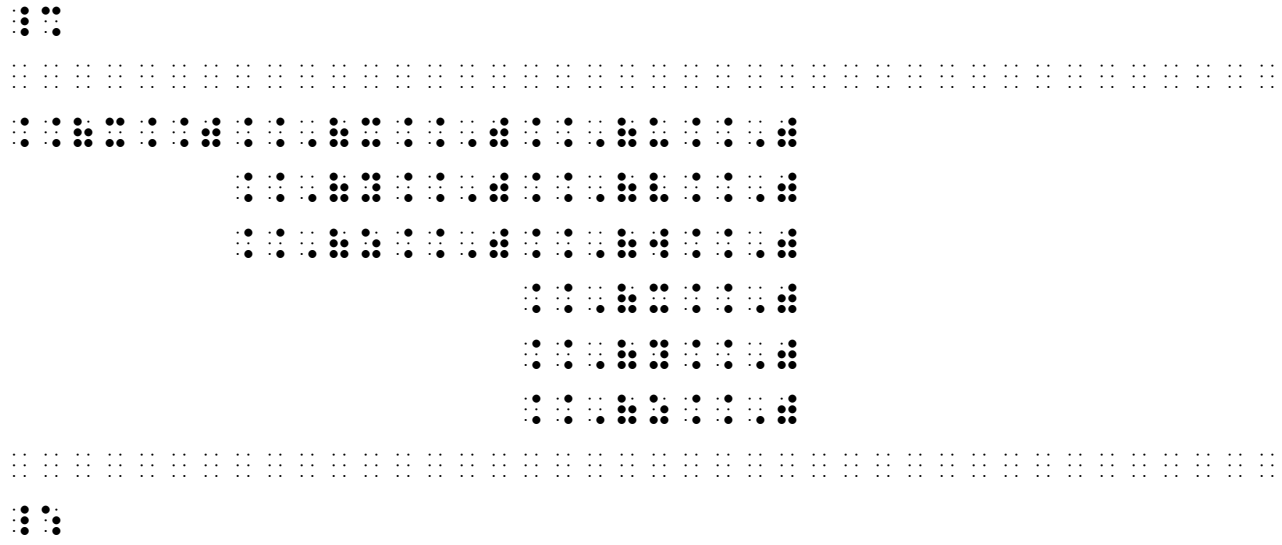


PRACTICE 17D

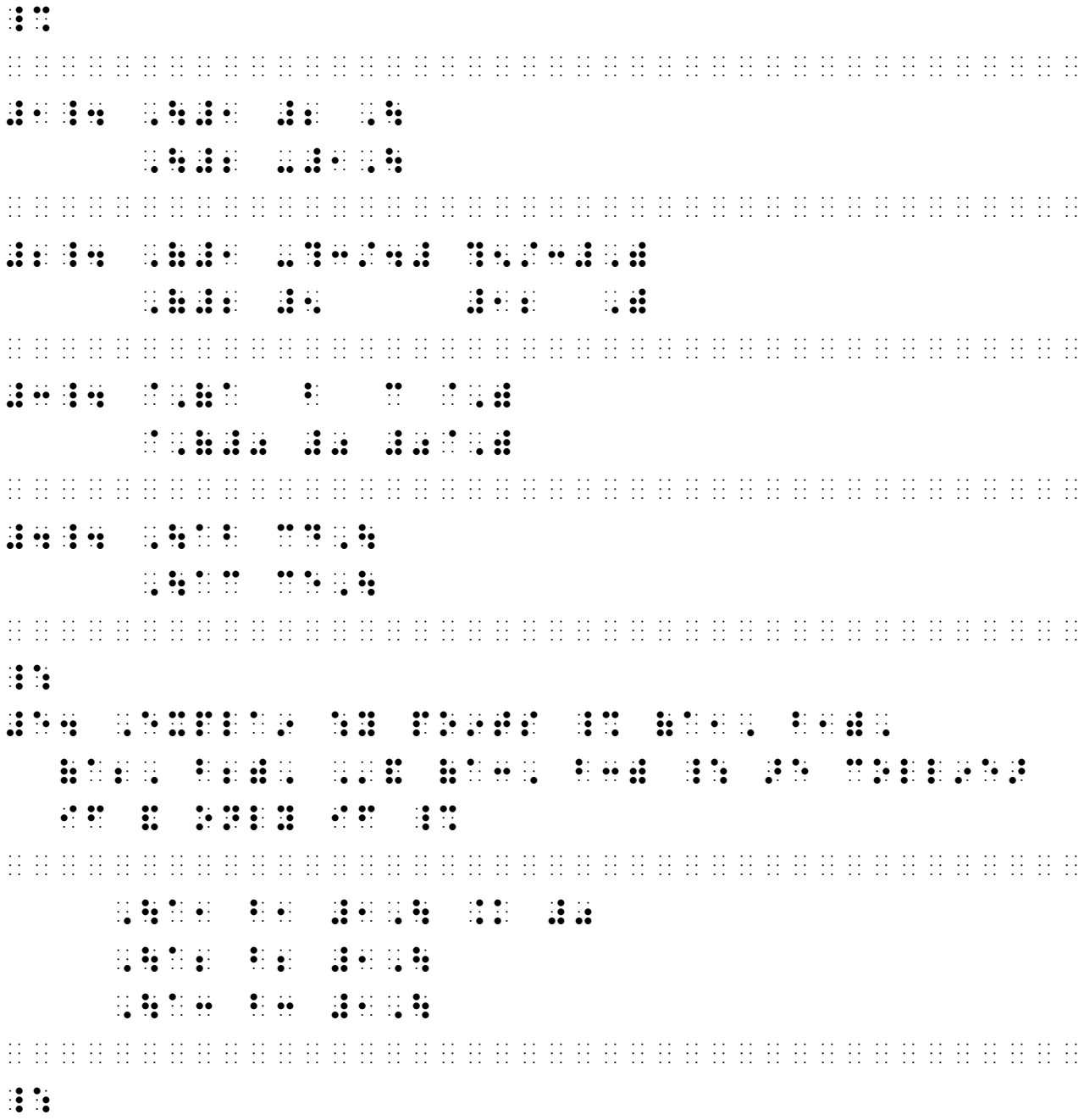


Line 4: A space is inserted before x and y because terms are aligned, as printed.
Lines 6-7: Although the equals signs are aligned in print, terms are not aligned. In braille, each line is left adjusted.
Line 8: An additional transcriber-inserted grouping symbol is needed because the remark takes three lines.
Lines 15-18: Although this is not a system of equations (there is only one variable), alignment guidelines in Section 17.2(c) are followed. Terms are aligned on the left of the equals signs; to the right, place values are not aligned.

PRACTICE 17E

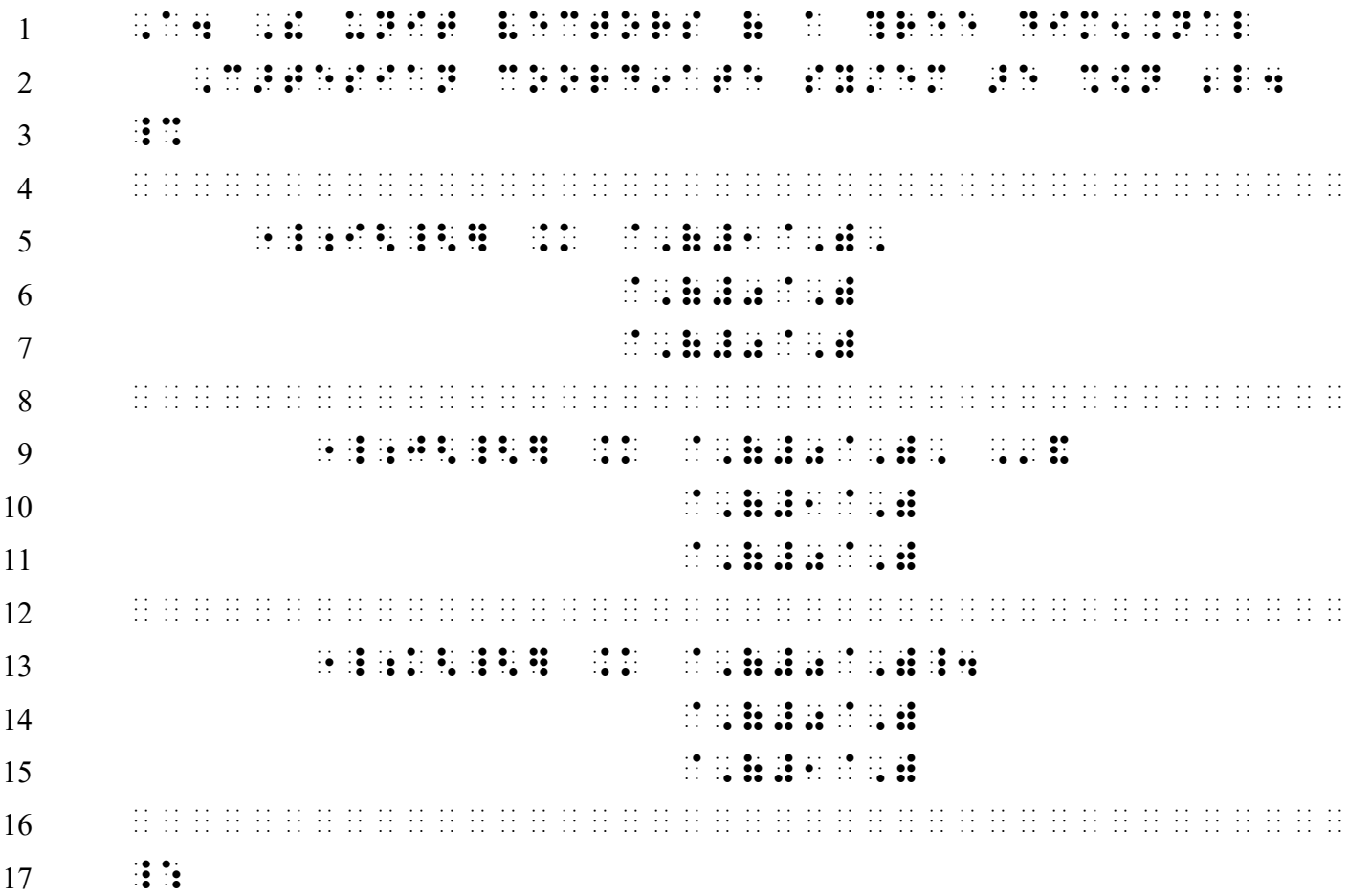


PRACTICE 17F



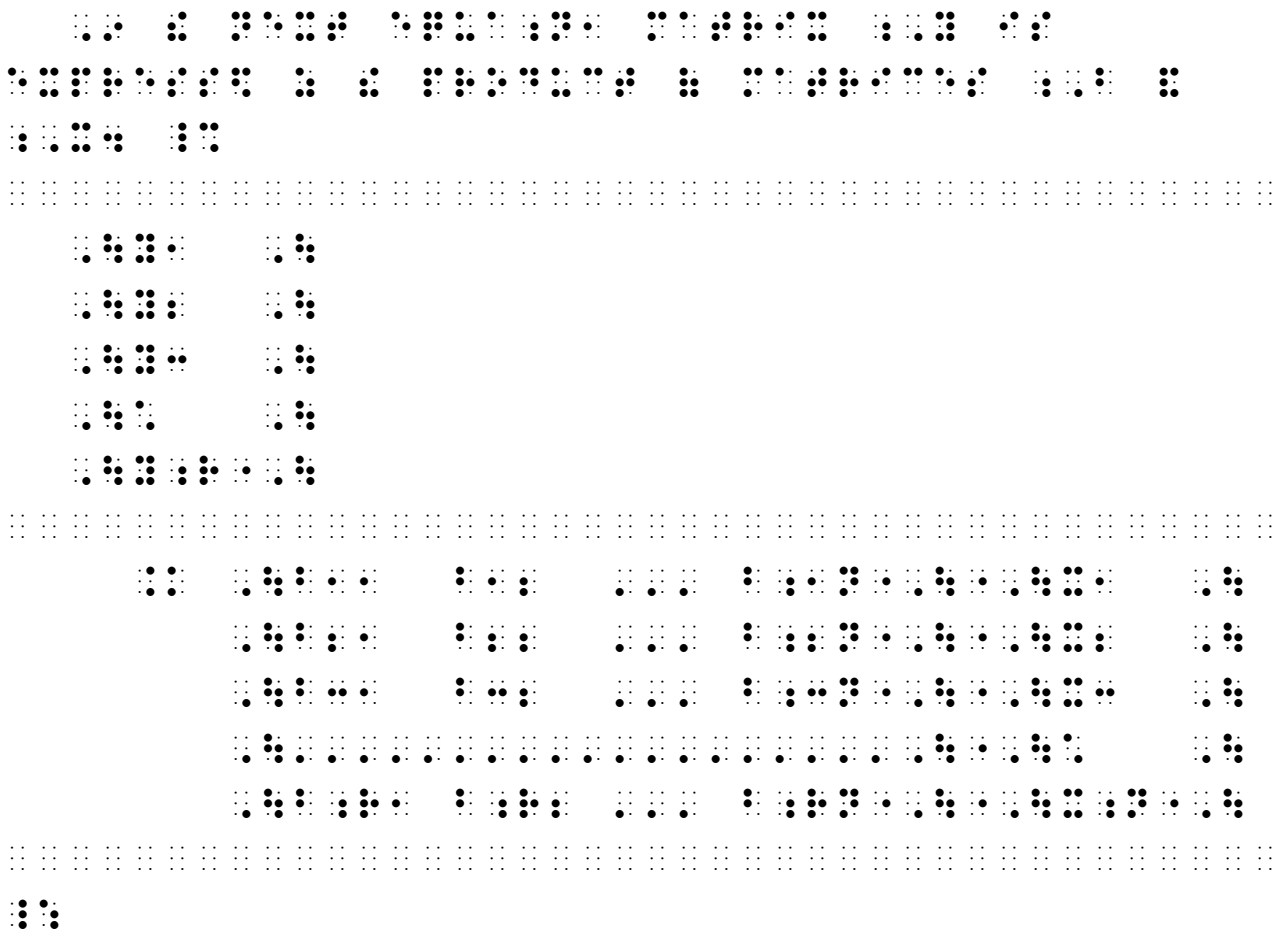
Line 1: The opening Nemeth Code indicator is placed outside of the itemized spatial arrangement. (See Section 17.8.)
Line 2: A blank line precedes spatial material.
Lines 15 and 24: The Nemeth Code terminator is placed in cell 1 following spatial material, regardless of the surrounding margins.

PRACTICE 17G

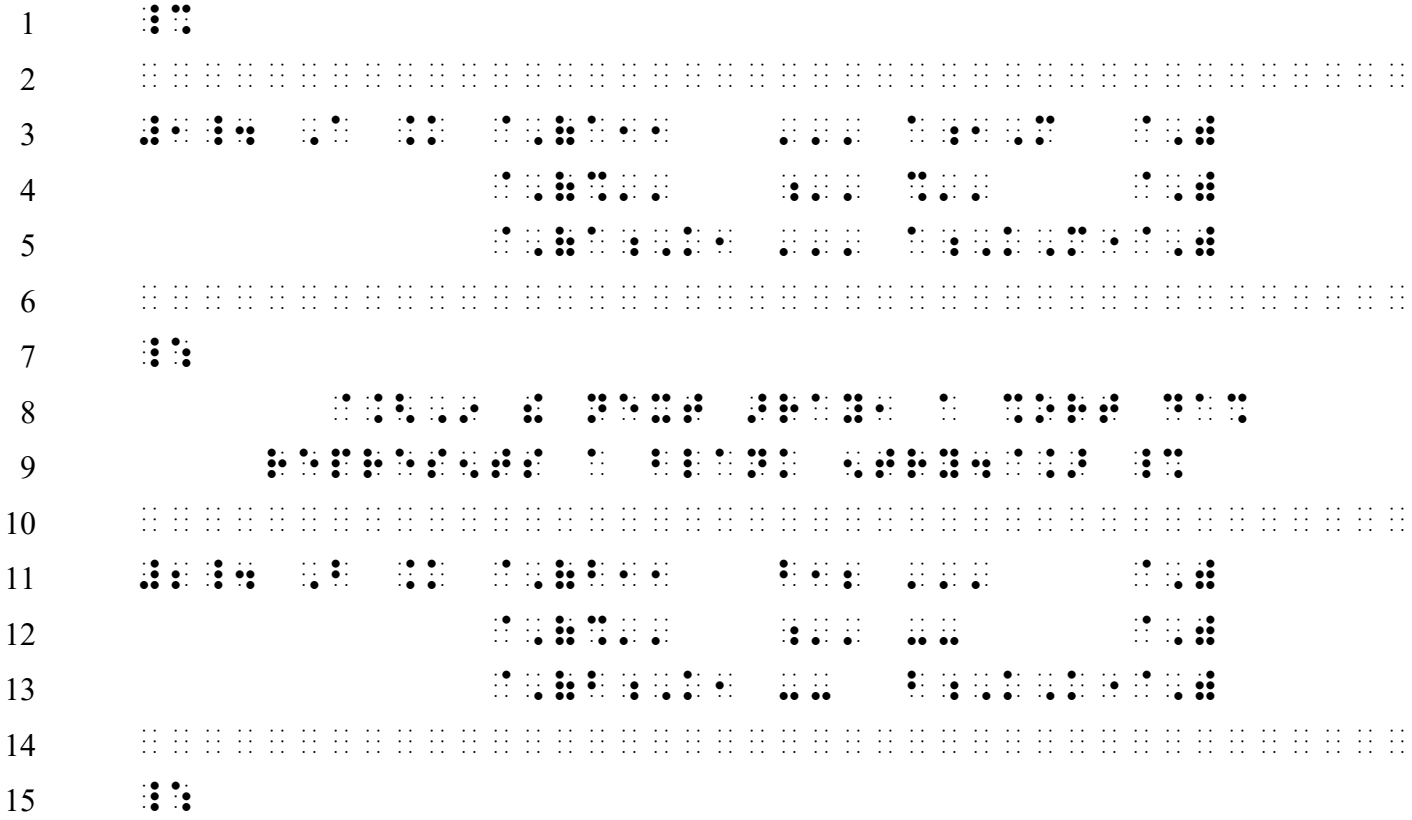


Line 3: The opening Nemeth Code indicator does not fit at the end of the text on line 2. It is placed in cell 1 when spatial material follows, regardless of the surrounding margins. (See Section 9.29.i in Lesson 9.)
Line 17: The Nemeth Code terminator is placed in cell 1 following spatial material.

PRACTICE 17H

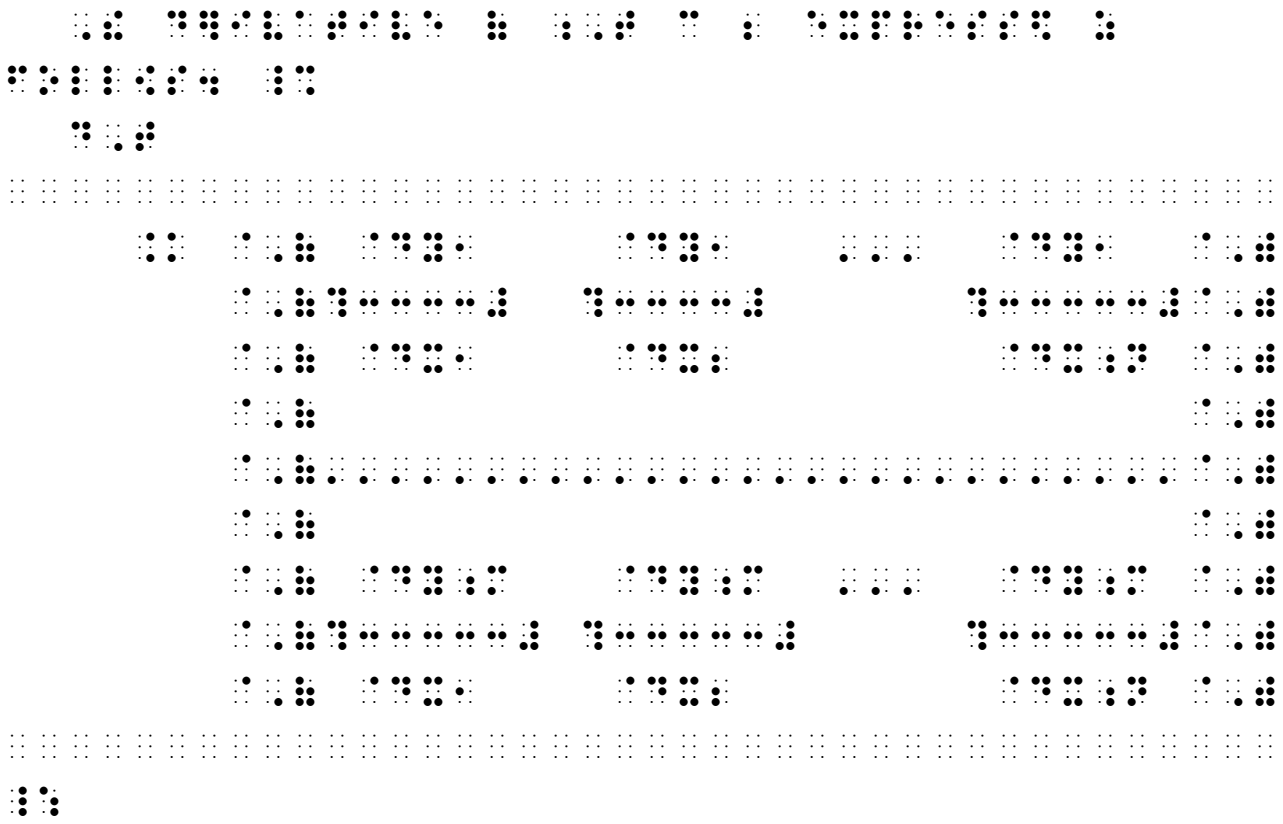


PRACTICE 17I



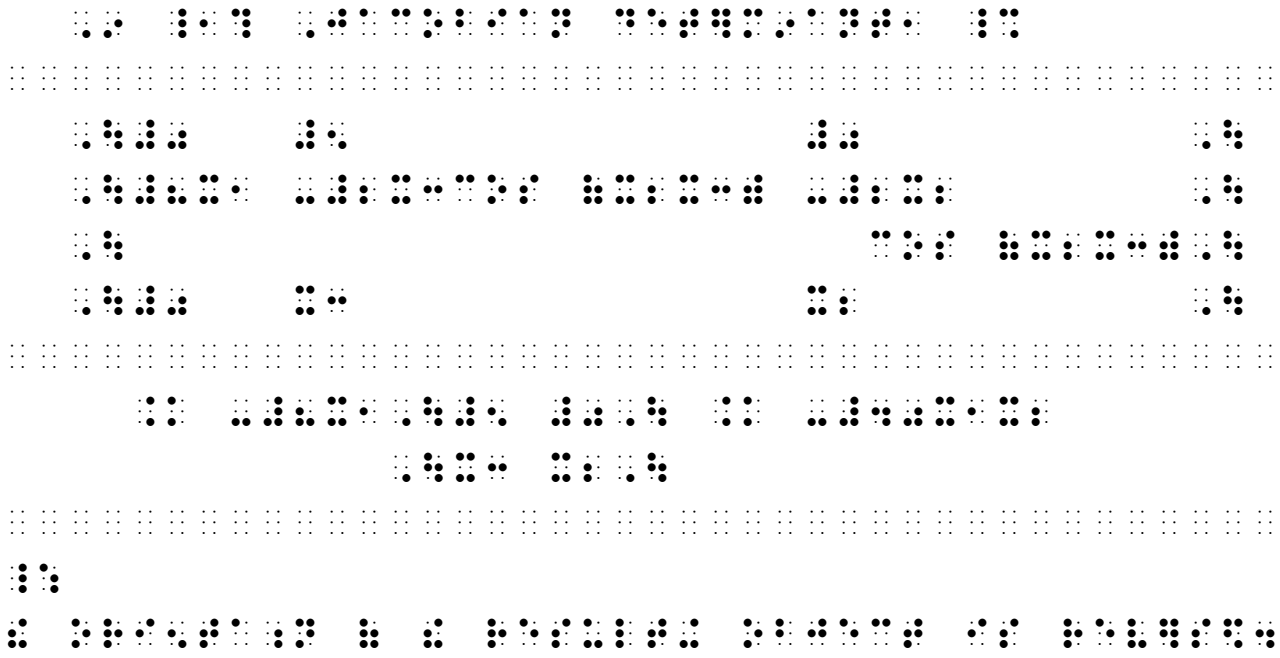
Notice the placement of the blank lines. Blank lines 2 and 6 precede and follow the spatial material on lines 3-5; blank lines 10 and 14 precede and follow the spatial material on lines 11-13. The code switch indicators are placed on either side of those blank lines.

PRACTICE 17J

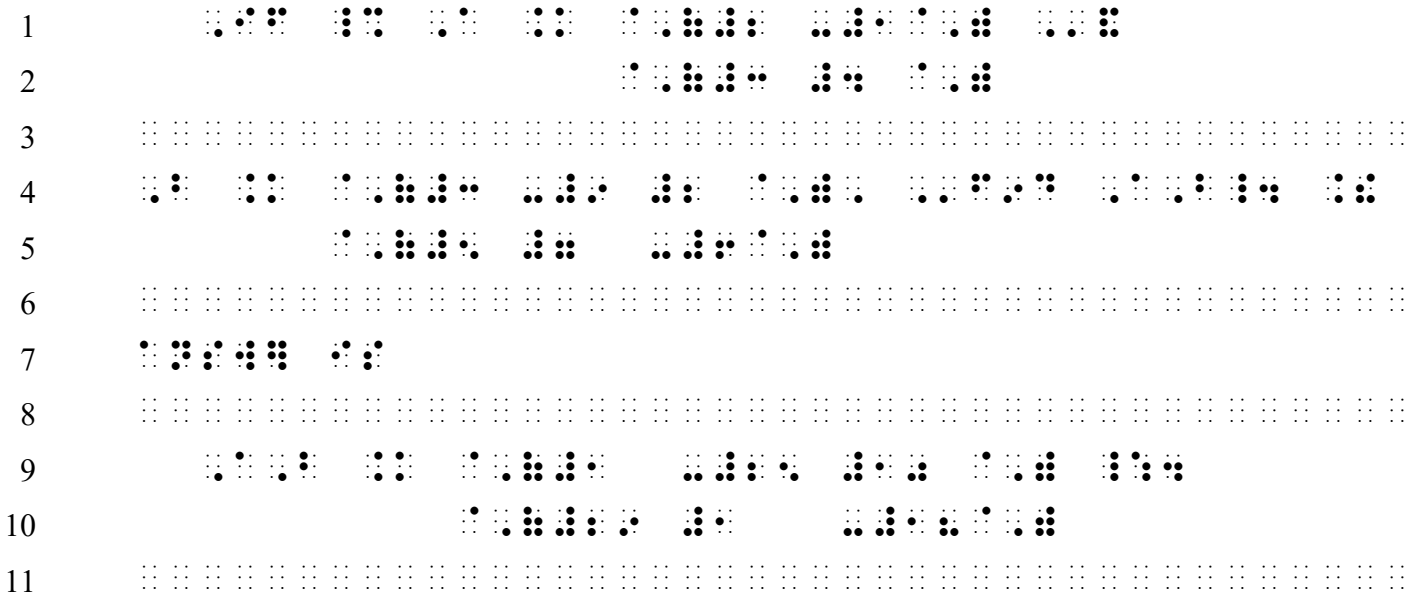
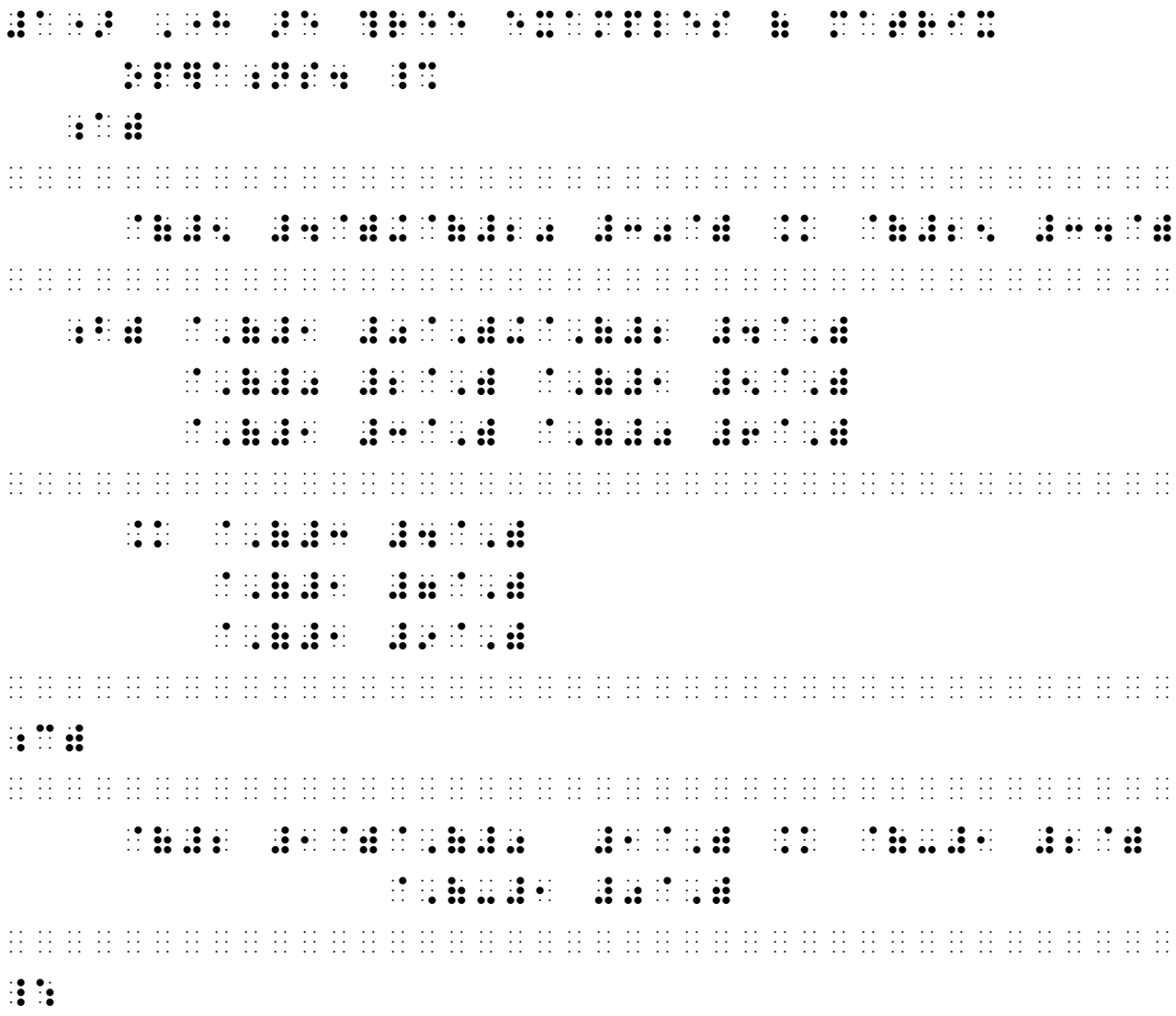


Notice the placement of the blank lines. The anchor (line 3) is not spatial. There is no blank line preceding the anchor. Blank lines 4 and 14 precede and follow the spatial material (the link) on lines 5-13.

PRACTICE 17K



PRACTICE 17L



PRACTICE 17M