

LESSON 5

- [SIGNS OF OPERATION, cont.](#)
- [SIGNS OF COMPARISON, cont.](#)

Format

- [Nemeth Instructions](#)
- [Simple Tables](#)

[Answers to Practice Material](#)

LESSON PREVIEW

Many more operation signs and signs of comparison are explored, including negated forms. Table format is introduced, with a table consisting of mathematical symbols and their names. Mathematical use of the colon meaning "such that" is shown. The concept of symbols compounded vertically and symbols compounded horizontally is introduced with certain signs of comparison. Considerations for format of instructions are investigated.

This lesson introduces many more symbols that you will come across in all areas of mathematics. Look carefully at the print symbol as many look similar to each other. Understand the context – some symbols are used as signs of operation as well as signs of comparison. The function of the symbol must be determined in order to apply proper spacing rules.

SIGNS OF OPERATION, cont.

5.1 Review of Signs of Operation

In Nemeth, no space is left before or after a sign of operation unless it is preceded or followed by a sign of comparison, an ellipsis, a dash, an unrelated word, or an abbreviation with a related value. Signs of operation are mathematical symbols and are punctuated accordingly. The following signs of operation have already been introduced.

⠠⠠⠠⠠	Plus	+
⠠⠠⠠⠠	Minus	−
⠠⠠⠠⠠⠠⠠	Multiplication Cross	×
⠠⠠⠠⠠	Multiplication Dot	•
⠠⠠⠠⠠⠠⠠	Division	÷

5.2 Signs of Operation Using Plus and Minus

The following signs are a combination of the plus and minus signs, written either side by side or one atop another. These are considered to be one symbol. The components must not be divided between braille lines.

⠠⠠⠠⠠⠠⠠	"Plus or Minus"	±
⠠⠠⠠⠠⠠⠠	"Minus or Plus"	∓
⠠⠠⠠⠠⠠⠠	Plus followed by Minus	+ −
⠠⠠⠠⠠⠠⠠	Minus followed by Plus	− +
⠠⠠⠠⠠⠠⠠	Minus followed by Minus	− −

Note that, in the first two symbols, the upper sign is transcribed first, followed immediately by the lower sign.

➤ 38 ± 7 ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

➤ 38 ∓ 7 ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

In the side-by-side symbols, the multipurpose indicator (dot 5) prevents these symbols from being read as "plus or minus", "minus or plus", or a dash.

⠄	Multipurpose Indicator
---	------------------------

- 10 + -5 ⠄⠄⠒⠄⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄
- 10 - +5 ⠄⠄⠒⠄⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄
- 10 - -5 ⠄⠄⠒⠄⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄

Note that the multipurpose indicator is used between other side-by-side operation symbols not shown in the table. It indicates that the signs are printed horizontally, not vertically.

➤ -10 + +5 ⠄⠄⠒⠄⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄

Example 5-1

±5 means +5 and -5.

⠄⠄⠒⠄⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄

Review the rules in Section P8 of the Preliminary Lesson regarding the use/nonuse of the numeric indicator with positive and negative numbers.

Example 5-2

Can $3 \pm 1 = +4$ and $+2$?

⠄⠄⠒⠄⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄

Example 5-3

Compare: $20 + -3$; $20 - -3$; $-20 - -3$; $-20 + +3$.

⠄⠄⠒⠄⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄
⠄⠄⠒⠄⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄⠒⠒⠒⠄

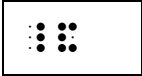
5.3 Signs of Operation That Look Like Literary Symbols

Some mathematical symbols use characters also seen in literary writing. When the following signs are used in mathematical context, the Nemeth symbols shown in the box below are used. As mathematical operation signs, the spacing rules for operation signs are followed.

	Ampersand	&
	Asterisk	*
	Crosshatch	#
	Dagger, Single	†
	Dagger, Double	‡
	Paragraph Mark	¶
	Section Mark	§

- a. When these symbols are used in literary context and have no mathematical meaning, the symbols and rules of UEB apply.
- b. Some of these symbols have other mathematical applications, which will be discussed in later lessons.

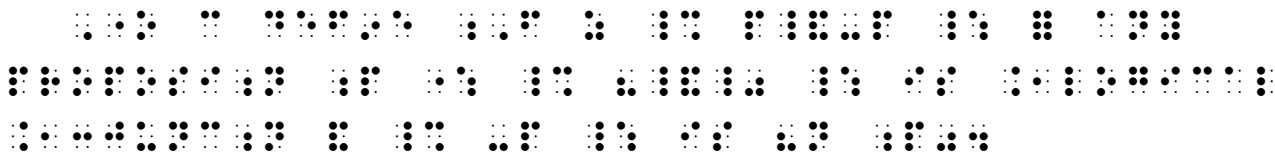
5.3.1 **Ampersand**



The ampersand functioning as a symbol of operation is commonly encountered in the study of logic, where it means "and". The symbol may be referred to as "logical conjunction".

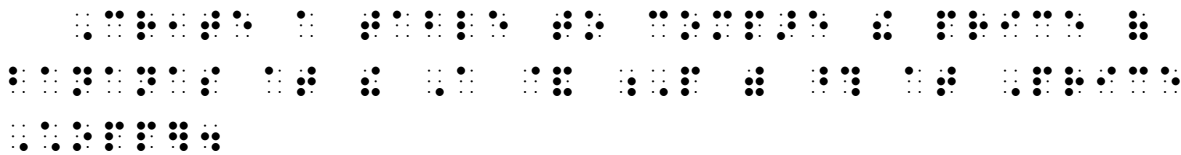
Example 5-4

One can define F as $p \& \neg p$ for any proposition p where "&" is *logical conjunction* and $\neg p$ is "not p ".



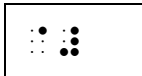
Example 5-5

Create a table to compare the price of bananas at the A & P with those at Price Chopper.



This ampersand is used as a literary device, using the UEB symbol and following the spacing rules of UEB.

5.3.2 Asterisk

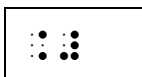


- The asterisk as an operation symbol is commonly encountered in material about calculators where the symbol represents multiplication. When a numeral follows an asterisk, the numeric indicator is used even though the numeral is not preceded by a space.

$$\begin{aligned} \gg 3 * 9 = 27 & \quad \dots \\ \gg .3 * .9 = 0.27 & \quad \dots \end{aligned}$$

- The asterisk used as a reference marker will be discussed in Lesson 13.

5.3.3 Crosshatch



- When the crosshatch represents a symbol of operation, the Nemeth symbol is used.

Example 5-6

What operations can the # symbol signify in $x\#y = y\#x$?

$$\dots$$

Example 5-7

$A\#B$ is the connected sum of the manifolds A and B .

$$\dots$$

- When a numeral or a decimal point and a numeral follows a crosshatch, the numeric indicator is transcribed even though the numeral is not preceded by a space.

$$\gg 1 \# .2 = .2 \# 1 \quad \dots$$

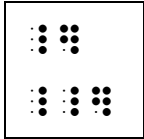
- When the crosshatch is used in literary context such as "Problem #1" or as a hashtag in a social media reference or URL, the UEB symbol is used. When the crosshatch denotes pounds (weight), the Nemeth symbol is used.

Example 5-8

Notation Shortcut #4: "23 pounds" can be written "23#".



5.3.4 **Dagger and Double Dagger**



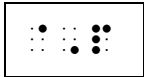
a. The dagger and double dagger may be used as operation symbols in binary operations.

➤ $A \dagger B = B \dagger A$

➤ $A \ddagger B = B \ddagger A$

b. The dagger used as a reference marker will be discussed in Lesson 13.

5.3.5 **Paragraph Mark**

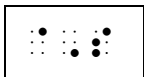


In mathematical context, the Nemeth symbol is used. A numeric indicator is required before a numeral following a paragraph mark.

➤ $A \P B$

➤ $3 \P 4 = 4 \P 3$

5.3.6 **Section Mark**



In mathematical context, the Nemeth symbol is used. A numeric indicator is required before a numeral following a section mark.

➤ $A \S B$

➤ $3 \S 4 = 4 \S 3$

Instructions: Before transcribing the practice, review the spacing rules for operation signs in [Section 5.1](#) and in Section P8.1 in the Preliminary Lesson. Transcribe this entire list in Nemeth.

PRACTICE 5A

$$4 \pm 1, 400 \pm 10, 6 \mp 1, 600 \mp 10$$

$$\mu \pm 1.645 \sigma$$

$$50 - +5 = 45$$

$$50 + -5 = ?$$

$$-3 - -3 = 0$$

$$A \& B = B \& A$$

$$a * (b * c) = (a * b) * c$$

$$(1 + 2) * (3 + 4) = 3 * 7$$

$$\#A = \#B$$

$$.5\#9 = .9\#5$$

$$[(p \dagger p) \dagger (q \dagger q)]$$

$$s \upharpoonright t = u \upharpoonright v$$

$$1 \upharpoonright 3 = 4 \upharpoonright 3$$

$$m \S y = y \S m = y$$

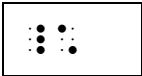
$$5 \S 6 = 6 \S 5 = 6$$

5.4 Signs of Operation Unique to Mathematics

⠠⠠⠠⠠	Backslash	\
⠠⠠	Dot	·
⠠⠠⠠	Hollow Dot	◦
⠠⠠⠠	Intersection ("cap")	∩
⠠⠠⠠	Logical Product	∧
⠠⠠⠠	Logical Sum	∨
⠠⠠⠠	Minus with Dot Over	÷
⠠⠠⠠	Slash	/
⠠⠠⠠	Tilde, Simple	~
⠠⠠⠠⠠	Tilde, Extended	⋈
⠠⠠⠠	Union ("cup")	∪
⠠⠠	Vertical Bar	
⠠⠠⠠	Vertical Bar, Negated	∤ or ∉

Examples of each symbol are shown in Sections 5.4.1-5.4.12. Note that, as with the other operation signs you have learned, these signs are unspaced from related mathematical terms regardless of the spacing shown in print.

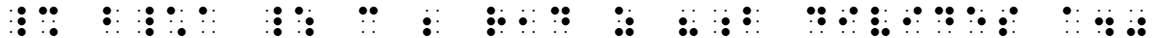
5.4.1 Backslash



The backslash slants upper left to lower right. In the context of operation signs, the backslash means "divides" or "is a factor of".

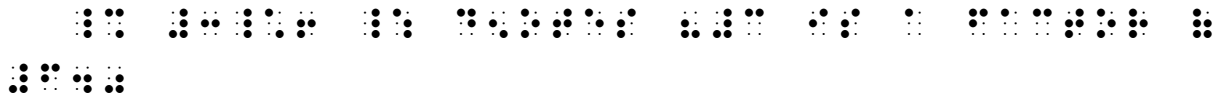
Example 5-9

$b \backslash a$ can be read as " b divides a ."

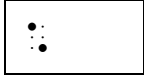


Example 5-10

$3 \backslash 6$ denotes "3 is a factor of 6."



5.4.2 Dot



In addition to operating as a multiplication sign, the dot may also be used to denote "and" in the study of logic. In either case, the symbol is transcribed without a space.

Example 5-11

In logic, $p \cdot q$ is read "p and q".



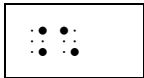
Example 5-12

Torque is expressed as $N \cdot m$.



An English-letter indicator is required for the single-letter abbreviations N (Newton) and m (meter). See Section 4.21 in Lesson 4. The operation sign is not spaced because N and m have no related value. See Section 4.20.1.b in Lesson 4.

5.4.3 Hollow Dot



The hollow dot may be used as a sign of operation. It is also seen in function notation. The raised hollow dot used to represent degrees will be discussed in Lesson 6.

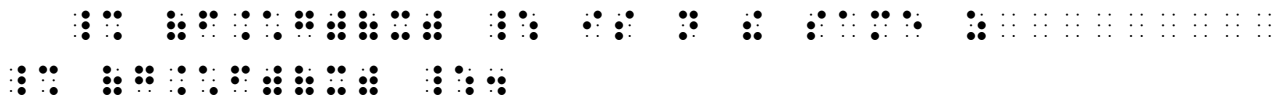
Example 5-13

$$a \circ (b \circ c) = (a \circ b) \circ c$$



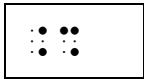
Example 5-14

$(f \circ g)(x)$ is not the same as $(g \circ f)(x)$.



Be sure to transcribe the hollow dot symbol – this is not the letter "o".

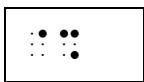
5.4.4 Intersection



This operation symbol is also called a "cap".

$$\gg A \cap B = B \cap A \quad \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

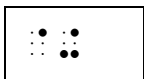
5.4.5 Logical Product



In the study of logic, this operation sign means "and" or "meet".

$$\gg p \wedge q \wedge r \quad \dots \dots \dots \dots \dots \dots \dots \dots$$

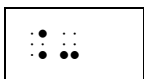
5.4.6 Logical Sum



In the study of logic, this operation sign means "or" or "join".

$$\gg p \vee q \vee r \quad \dots \dots \dots \dots \dots \dots \dots \dots$$

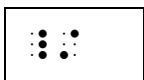
5.4.7 Minus with Dot Over



This operation sign means "proper difference".

$$\gg x \dot{-} y = 0 \quad \dots \dots \dots \dots \dots \dots \dots \dots$$

5.4.8 Slash



The term "slash" refers to the forward slash, which slants from lower left to upper right. In Nemeth, no space is left between the slash and any numeral, word, part of a word, or abbreviation to which it applies.

- a. With Numerals When a slash represents a fraction line in a fraction where the numerator and denominator are printed on the baseline of writing, a switch to Nemeth is required. The numeric indicator is not needed for a numeral immediately following a slash.

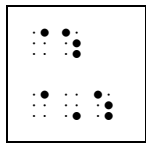
Example 5-19

The chapter test will be given on Friday, 10/23. Bring your calculator and/or iPad to class. Practice problems can be found at <https://www.Math.edu/Chapter12/Practice.html>.

- 1 ⠠⠄ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠
- 2 ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠
- 3 ⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠
- 4 ⠠⠠⠠
- 5 ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

All slashes are in literary context.

5.4.9 **Tilde and Extended Tilde**



- a. The simple tilde has one peak. In logic, the tilde may be used as a sign of operation meaning "not".

➤ $\sim p$ ⠠⠠⠠⠠

➤ $\sim(\sim p)$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠

- b. When two symbols for the tilde follow one another, a multipurpose indicator (dot 5) is inserted between them to indicate that they are written horizontally.

➤ $\sim \sim p \vee q$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- c. The extended tilde has more than one peak.

➤ $\sim \sim s \vee t$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- d. When the tilde symbol is used to replace the word "approximately", "about", or "around" it is transcribed as an unspaced symbol.

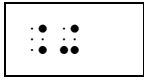
Example 5-20

The test will begin in ~15 seconds.

⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠ ⠠⠠⠠⠠ ⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

- e. The tilde is also used as a sign of comparison. Consider the context to determine its meaning. (See [Section 5.6.12.](#)) When the tilde's meaning cannot be determined from context, follow print spacing.

5.4.10 Union



This operation symbol is also called a "cup".

$$\gg A \cup B = B \cup A$$

5.4.11 Vertical Bar



- a. When the vertical bar means "is a factor" or "divides", it is functioning as an operation sign.

Example 5-21

In $b|a$, b is a factor of a .

$$12|6 \quad 6|12 \quad 6|24 \quad 24|6 \quad 24|12 \quad 6|24 \quad 24|6 \quad 24|12 \quad 12|24 \quad 6|12$$

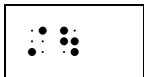
Example 5-22

$6|12$ can be read as "6 divides 12."

$$6|12 \quad 12|6 \quad 6|24 \quad 24|6 \quad 24|12 \quad 12|24 \quad 6|24 \quad 24|6 \quad 24|12 \quad 12|24 \quad 6|12$$

- b. The vertical bar is also used as a sign of grouping and as a sign of comparison. Consider the context to determine its meaning. (See Lesson 2. See also [Section 5.6.14.](#)) When the vertical bar's meaning cannot be determined from context, follow print spacing.

5.4.12 Vertical Bar, Negated



This symbol means "does not divide".

$$\gg 5 \nmid n$$

- Line 2: The column headings are not mathematical. Contractions are used.*
- Line 3: Column separation lines are inserted according to Braille Formats guidelines.*
- Line 4: The opening Nemeth Code indicator is placed in cell 1.*
- Line 5: The first row is transcribed in Nemeth. Guide dots are inserted according to Braille Formats guidelines.*
- Line 6: Words in Nemeth are uncontracted.*
- Line 7: The ellipsis indicates that there will be further entries in your transcription.*
- Line 8: Terminate Nemeth on the line after the completion of the table, in cell 1.*

5.5.1 **Omissions in a Simple Table.** In a table transcribed in Nemeth, when a dash, underscore, ellipsis, or other omission symbol is printed in an otherwise blank entry field, the appropriate Nemeth symbol is transcribed. (See Lesson 1.) Guide dots are inserted, as needed. When the entry field is blank, follow Braille Formats regarding the insertion of a series of guide dots across the width of a column.

Instructions: Include the box lines for this table.

PRACTICE 5B

<u>Name</u>	<u>Symbol</u>
Dot	•
Vertical Bar	
Logical Product	∧
Simple Tilde	~
Logical Sum	∨
Extended Tilde	≈
Backslash	\
Slash	/
Hollow Dot	◦
Intersection	∩
Union	∪
Minus with Dot Over	÷

SIGNS OF COMPARISON, cont.

Five comparison signs were presented in the Preliminary Lesson.

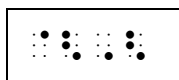
⠠⠨⠠	Equals	=
⠠⠨⠠⠨	Greater Than	>
⠠⠨⠠⠨	Less Than	<
⠠⠨⠠⠨	Proportion	::
⠠⠨⠠	Ratio	:

5.6 More Comparison Signs

⠠⠨⠠⠨⠠⠨	Equivalence	≡
⠠⠨⠠⠨⠠⠨	Greater Than with Curved Sides	⋙
⠠⠨⠠	Identity	≡
⠠⠨⠠⠨	Inclusion	⊂
⠠⠨⠠⠨	Less Than with Curved Sides	⋘
⠠⠨⠠	Membership	∈
		(also ε or Ε)
⠠⠨⠠	Parallel to	∥
⠠⠨⠠	Perpendicular to	⊥
⠠⠨⠠	Relation	R
⠠⠨⠠⠨	Reverse Inclusion	⊃
⠠⠨⠠	Reverse Membership	∋
⠠⠨⠠	Tilde, Simple	~
⠠⠨⠠⠨	Tilde, Extended	≈
⠠⠨⠠	Variation	∝
⠠⠨⠠	Vertical Bar	

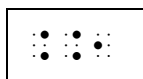
Examples of each symbol are shown in Sections 5.6.1-5.6.14. Note that, as with the other comparison signs you have learned, these signs are preceded and followed by a space regardless of the spacing shown in print.

5.6.1 Equivalence ("is equivalent to")



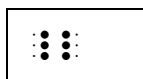
➤ $x \cong y$ ⠠⠨⠠⠶ ⠠⠨⠠⠽

5.6.2 Greater Than with Curved Sides ("is greater than")



➤ $7 > 4 > 3$ ⠠⠠⠲ ⠠⠠⠳ ⠠⠠⠼

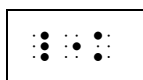
5.6.3 Identity (Triple Bar)



This symbol is used in several different contexts. Most commonly it means "is identical with" or "is congruent to". The transcriber uses the same symbol regardless of its meaning. Do not confuse the triple bar with the Greek letter Xi or the triple bond in Chemistry.

➤ $A + B \equiv B + A$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

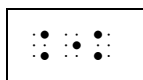
5.6.4 Inclusion



This symbol is generally used for sets and their elements, meaning "is contained in" or "is a subset of".

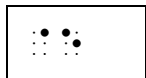
➤ $A \subset D$ ⠠⠠⠠⠠⠠⠠⠠⠠

5.6.5 Less Than with Curved Sides ("is less than")



➤ $5 < 9 < 11$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

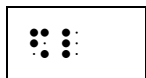
5.6.6 **Membership**



This symbol is generally used for sets and their elements, meaning "is an element of" or "belongs to". It must not be mistaken for the Greek uncapitalized epsilon even though it may be referred to as such.

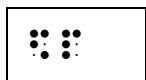
➤ $5 \in B$ ⠠⠨ ⠠⠃

5.6.7 **Parallel To ("is parallel to")**



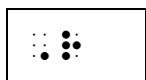
➤ $AB \parallel CD$ ⠠⠁⠃ ⠠⠇⠒⠠⠃⠇

5.6.8 **Perpendicular To ("is perpendicular to")**



➤ $PQ \perp RS$ ⠠⠑⠒ ⠠⠇⠒⠠⠗⠗

5.6.9 **Relation**



When a letter R is used between two expressions to show relation ("is related to"), the letter is treated as a sign of comparison with a space before and after it regardless of print spacing. Read the surrounding narrative to realize the meaning of the letter "R" in order to transcribe it correctly. Note that other letters or signs may also be used to show relation.

Example 5-23

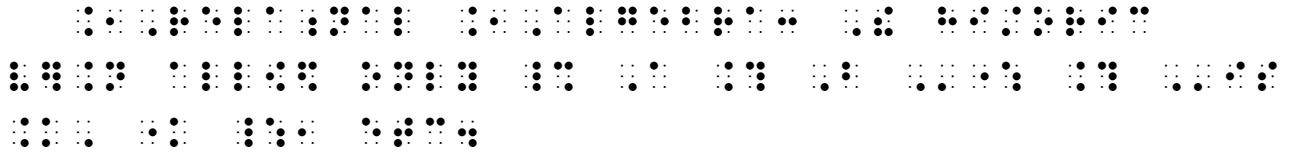
The statement $(x, y) \in G$ is read "x is R-related to y", and is denoted by xRy.

⠠⠭⠠⠇⠠⠗ ⠠⠒⠠⠇⠠⠗ ⠠⠇

The letter R is functioning as a comparison sign here, so it is preceded and followed by a space in the transcription.

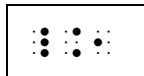
Example 5-24

Relational Algebra: The historic version allowed only $A\theta B$ where θ is $=, <, etc.$



Greek letter theta is the relation symbol in this example. As a sign of comparison, it is preceded and followed by a space in the transcription.

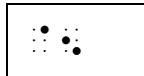
5.6.10 Reverse Inclusion



This symbol may mean "contains" or, in logic, "implies".

⤵ $D \supset A$ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠

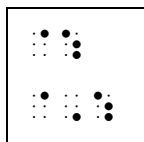
5.6.11 Reverse Membership



This symbol means "contains the element".

⤵ $B \ni 5$ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠

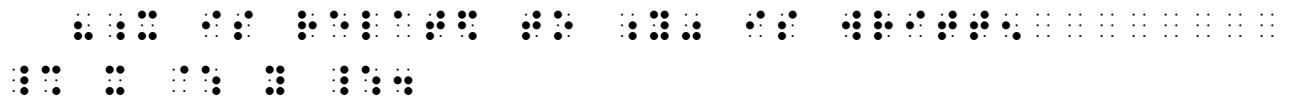
5.6.12 Tilde and Extended Tilde



- a. The simple tilde has one peak. When used as a comparison sign, the tilde means "is related to" or "is similar to".

Example 5-25

"x is related to y" is written $x \sim y$.



- b. The extended tilde has more than one peak.

⤵ $x \rightsquigarrow y$ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠

- c. The tilde functioning as a sign of comparison is often encountered in the study of set theory and relations. The tilde is also used as a sign of operation. Consider the context to determine its meaning. (See [Section 5.4.9](#).) When the tilde's meaning cannot be determined from context, follow print spacing.

5.6.13 Variation ("varies as")



➤ $x \propto y$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

5.6.14 Vertical Bar



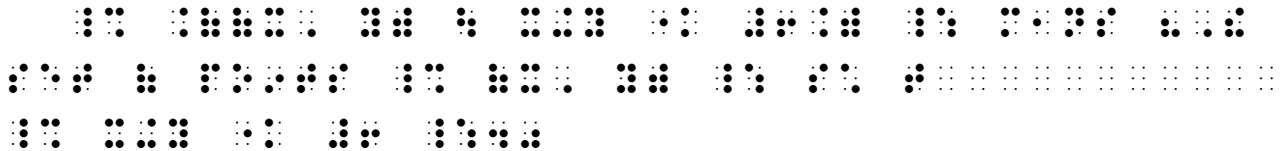
- a. When used as a sign of comparison, the vertical bar means "such that" or "given". It usually occurs in an expression within braces. The print copy may or may not show the vertical bar as a spaced symbol. In braille, however, comparison signs must be preceded and followed by a space.

➤ $\{x \in E | \Phi(x)\}$



Example 5-26

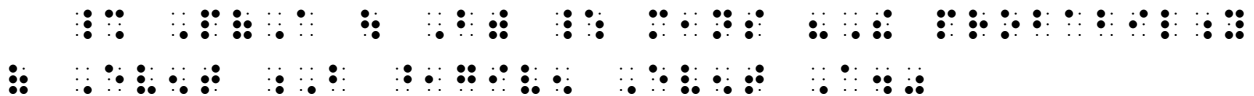
$\{(x, y) | x + y < 6\}$ means "The set of points (x, y) such that $x + y < 6$."



- b. The vertical bar functioning as a sign of comparison is often encountered in the study of conditional probability and logic. A vertical bar may also appear in other situations as a sign of comparison. Consider the context to determine its meaning. Recall that this symbol is also used as a sign of grouping and as a sign of operation. Apply proper spacing according to its function. See Lesson 2. See also [Section 5.4.11](#). When the vertical bar's meaning cannot be determined from context, follow print spacing.

Example 5-27

$P(A|B)$ means "The probability of Event B given Event A ."



Inside the switches, letters P and A do not need an English-letter indicator because they are not preceded and followed by a space; letter B does not need an English-letter indicator because it immediately follows a comparison symbol. Review Section 4.18 in Lesson 4 regarding an English letter touching only one grouping symbol.

5.7 Special Case: A Colon Meaning "Such That"

In set notation or mapping notation, a colon may be used to mean "such that". Although functioning as a sign of comparison, the Nemeth spacing rules governing comparison signs are not followed because a colon is a mark of punctuation.

- Spacing in print may vary. Regardless of the spacing shown in print, in braille the colon is not preceded by a space. A punctuation indicator is required before the colon. Follow print for spacing after the colon.
- Follow Nemeth rules for a "single letter" for the letter on either side of the colon regarding use/nonuse of the English-letter indicator. (See Lessons 3 and 4.)

➤ $m:W$ ⠠⠽⠺

In the print copy, this colon is unspaced. Letter m needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letter W needs an English-letter indicator because it is preceded by punctuation and is followed by a space.

➤ $\{x : x > 0\}$ ⠠⠆⠺⠆⠑⠛⠏⠠⠽⠺

In the print copy, this colon is preceded and followed by a space. The first x needs an English-letter indicator because it is touching only one grouping sign and is followed by punctuation. The second x does not need an English-letter indicator because it is followed by a comparison sign.

➤ $f:(x, y)$ ⠠⠿⠺⠸⠸

In the print copy, this colon is unspaced. Letter f needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letters x and y do not need an English-letter indicator because they are in an enclosed list.

➤ $p:r = q:s$ ⠠⠽⠺⠆⠑⠆⠑⠛⠠⠽⠺

In the print copy, these colons are unspaced. Letter p needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letters r and q do not need an English-letter indicator because they each are next to a comparison sign. Letter s needs an English-letter indicator because it is preceded by punctuation and is followed by a space.

Instructions: Review the spacing rules for comparison signs before transcribing the practice. Assume all tildes and vertical bars are comparison signs in these expressions.

PRACTICE 5C

$$A \ni x$$

$$\{x \in A \mid x \sim a\}$$

$$j \cong k$$

$$r \propto s$$

$$f(x) \equiv D(x) \cdot q(x)$$

$$-12 < -4 < 0$$

$$.9 > .5$$

$$(A \cup E) \subset (F \cup B)$$

$$Q \supset Z$$

$$PQR \sim P'Q'R'$$

$$AB \parallel MN$$

$$CD \perp OP$$

$$EF \parallel GH \perp QR$$

$$\{m \mid 3(m - 6) = -9\}$$

$$\{x : x \text{ has the property } T\}$$

5.8 Signs of Comparison Compounded Vertically

When two or more simple signs of comparison are arranged one under the other, the combination becomes a single comparison sign compounded vertically. The symbol for the uppermost sign is written first, immediately followed by and unspaced from the symbol for the lower sign.

Comparison signs compounded vertically not shown in this section are transcribed in accordance with this principle.

5.8.1 Greater Than or Equal To

⠠⠠⠠⠠⠠⠠	Bar Over Greater Than	$\bar{>}$ or $\bar{>}$
⠠⠠⠠⠠⠠⠠⠠⠠	Equals Sign Over Greater Than	$\bar{=}$ or $\bar{=}$
⠠⠠⠠⠠⠠⠠	Bar Under Greater Than	\geq or \geq
⠠⠠⠠⠠⠠⠠⠠⠠	Equals Sign Under Greater Than	\cong or \cong

The "equal to" sign may be printed as an equals sign or as a single line – either a horizontal bar or an oblique line. Note that both the horizontal bar and the oblique line are represented by the same braille symbol (dots 156).

$$\begin{aligned} \Rightarrow a \bar{>} b & \quad \cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot\cdot \\ \Rightarrow a \bar{=} b & \quad \cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot\cdot \\ \Rightarrow x \geq y & \quad \cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot\cdot \\ \Rightarrow |x| \cong 0 & \quad \cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot\cdot\cdot\cdot \end{aligned}$$

5.8.2 Inclusion ("is a subset of")

⠠⠠⠠⠠⠠⠠	Bar Over Inclusion	$\bar{\subset}$
⠠⠠⠠⠠⠠⠠⠠⠠	Equals Sign Over Inclusion	$\bar{=} \subset$
⠠⠠⠠⠠⠠⠠	Bar Under Inclusion	\subseteq
⠠⠠⠠⠠⠠⠠⠠⠠	Equals Sign Under Inclusion	\subseteq

$$\begin{aligned} \Rightarrow C \bar{\subset} B' & \quad \cdot\cdot\cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot \\ \Rightarrow C \bar{=} B' & \quad \cdot\cdot\cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot \\ \Rightarrow (D \cap E) \subseteq (E \times E) & \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \quad \cdot \end{aligned}$$

» $(D \cap E) \subseteq (E \times E)$

5.8.3 Intersection

	Bar Under Intersection	\cap
	Equals Sign Under Intersection	\sqcap

- a. The intersection sign is a sign of comparison when modified by a bar or equals sign below it. It is also called a "cap".

» $X \cap Y$

» $X \sqcap Y$

- b. An unmodified intersection sign is a sign of operation. See [Section 5.4.4](#).

5.8.4 Less Than or Equal To

	Bar Over Less Than	\gtrless or \lessgtr
	Equals Sign Over Less Than	\gtrless or \lessgtr
	Bar Under Less Than	\lessgtr or \lesseqgtr
	Equals Sign Under Less Than	\lesseqgtr or \lesseqgtr

» $v - 1 \gtrless 5$

» $v - 1 \lessgtr 5$

» $6 \leq x \leq 9$

» $6 \lesseqgtr x \lesseqgtr 9$

5.8.5 Logical Product

⠠⠠⠠⠠	Bar Over Logical Product	$\overline{\wedge}$
⠠⠠⠠⠠⠠	Bar Over and Bar Under Logical Product	$\overline{\underline{\wedge}}$
⠠⠠⠠⠠⠠⠠	Bar Over and Equals Sign Under Logical Product	$\overline{\underline{=}}\wedge$
⠠⠠⠠⠠	Bar Under Logical Product	$\underline{\wedge}$
⠠⠠⠠⠠⠠	Equals Sign Over Logical Product	$\overline{=}\wedge$
⠠⠠⠠⠠⠠⠠	Equals Sign Over and Bar Under Logical Product	$\overline{=}\underline{\wedge}$
⠠⠠⠠⠠⠠⠠⠠	Equals Sign Over and Equals Sign Under Logical Product	$\overline{=}\underline{=}\wedge$
⠠⠠⠠⠠⠠	Equals Sign Under Logical Product	$\underline{=}\wedge$

- a. The logical product sign is a sign of comparison meaning "meet" when modified by a bar or equals sign above or below it.

$$\gg ABD \overline{\wedge} A'B'D'$$

$$\text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠}$$

$$\gg \{A\} \underline{\wedge} K \quad \text{⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠}$$

$$\gg p \underline{\wedge} q \quad \text{⠠⠠⠠⠠⠠⠠⠠}$$

- b. An unmodified logical product sign is a sign of operation. See [Section 5.4.5](#).

5.8.6 Logical Sum

⠠⠨⠠⠠⠠⠨	Bar Over Logical Sum	$\bar{\vee}$
⠠⠨⠠⠠⠠⠠⠨	Bar Over and Bar Under Logical Sum	$\bar{\underline{\vee}}$
⠠⠨⠠⠠⠠⠠⠠⠨	Bar Over and Equals Sign Under Logical Sum	$\bar{\equiv}$
⠠⠠⠠⠠⠨	Bar Under Logical Sum	$\underline{\vee}$
⠠⠠⠠⠠⠠⠨	Equals Sign Over Logical Sum	$\overline{\vee}$
⠠⠠⠠⠠⠠⠠⠨	Equals Sign Over and Bar Under Logical Sum	$\overline{\underline{\vee}}$
⠠⠠⠠⠠⠠⠠⠠⠨	Equals Sign Over and Equals Sign Under Logical Sum	$\overline{\equiv}$
⠠⠠⠠⠠⠨	Equals Sign Under Logical Sum	$\underline{\equiv}$

a. The logical sum sign is a sign of comparison meaning "join" when modified by a bar or equals sign above or below it.

$\gg ABC \bar{\vee} A'B'C'$
 $\gg P(E \underline{\vee} F)$

b. An unmodified logical sum sign is a sign of operation. See [Section 5.4.6](#).

5.8.7 Reverse Inclusion

⠠⠠⠠⠠⠠⠠⠨	Bar Over Reverse Inclusion	\supseteq
⠠⠠⠠⠠⠠⠠⠠⠨	Equals Sign Over Reverse Inclusion	$\overline{\supseteq}$
⠠⠠⠠⠠⠠⠨	Bar Under Reverse Inclusion	\supseteqbar
⠠⠠⠠⠠⠠⠠⠠⠨	Equals Sign Under Reverse Inclusion	$\underline{\supseteq}$

$\gg B \supseteq A$
 $\gg D \underline{\supseteq} C$

5.8.8 Tilde

⠠⠨⠠⠨⠠⠨	Bar Over Single Tilde	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨⠠⠨	Equals Sign Over Single Tilde	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨⠠⠨	Double Tilde	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨	Bar Over Double Tilde	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨	Equals Sign Over Double Tilde	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨	Bar Under Single Tilde	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨⠠⠨	Equals Sign Under Single Tilde	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨	Bar Under Double Tilde	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨	Equals Sign Under Double Tilde	⠠⠨⠠⠨

➤ $3.14159 \approx 3.1416$

⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨

➤ $ABC \cong DEF$ ⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨

5.8.9 Union

⠠⠨⠠⠨⠠⠨	Bar Under Union	⠠⠨⠠⠨
⠠⠨⠠⠨⠠⠨⠠⠨	Equals Sign Under Union	⠠⠨⠠⠨

a. The union sign is a sign of comparison when modified by a bar or equals sign above or below it. It may also be referred to as a "cup".

➤ $A \underline{\cup} B$ ⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨⠠⠨

b. An unmodified union sign is a sign of operation. See [Section 5.4.10](#).

Instructions: Transcribe "Signs of Comparison Compounded Vertically" as a cell-5 heading. Change the two-column format to a nested list by starting each phrase in cell 1, with each math expression starting on a new line in cell 3. *Code Switching Guidelines:* To maintain clarity within the nested list layout, place the opening Nemeth Code indicator at the end of the UEB phrase, and place the Nemeth Code terminator on the same line with the related math.

PRACTICE 5D

Signs of Comparison Compounded Vertically

Greater Than or Equal To	$ab \overline{\geq} de$ $ y \geq 0$
Less Than or Equal To	$q - 7 \overline{\leq} 5z$ $-6 \leq x \leq -1$
Inclusion and Reverse Inclusion	$C' \overline{\subset} F'$ and $D \supset C$ $(B \cap E) \overline{\subseteq} (E \times E)$
Intersection and Union ("Cup")	$X \overline{\cap} Y$ $X \overline{\cup} Y$ $A \overline{\cup} B$
Logical Product and Logical Sum	$QRS \overline{\wedge} Q'R'S'$ $y \overline{\wedge} z$ and $M(E \overline{\vee} H)$ $ABC \overline{\vee} A'B'C'$
Tilde	$3.14159 \approx 3.1416$ $ABC \cong DEF$

5.9 Signs of Comparison Compounded Horizontally

When two or more signs of comparison are arranged side by side, the combination becomes a single comparison sign compounded horizontally. A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that they are printed horizontally, not vertically. Comparison signs compounded horizontally not shown in this section are transcribed in accordance with this principle.

5.9.1 Greater Than ...

⠠⠠⠠⠠⠠⠠⠠⠠	Greater Than Followed by Less Than	><
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Greater Than Followed by Equals Followed by Less Than	>=<

⦿ $n > < 1$ ⠠⠠⠠⠠⠠⠠⠠⠠

⦿ $n > = < 1$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

5.9.2 Less Than ...

⠠⠠⠠⠠⠠⠠	Less Than Followed by Greater Than	<>
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	Less Than Followed by Equals Followed by Greater Than	<=>

⦿ $n < > 1$ ⠠⠠⠠⠠⠠⠠⠠⠠

⦿ $n < = > 1$ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

5.10 Negated Signs of Comparison

In print, a sign of comparison may be negated by a vertical or a slanted line drawn through it. The print negation symbol may be slanted in either direction. In braille, ⠠⠠ represents any of the print negation lines. ⠠⠠ is placed immediately before the sign of comparison being negated.

Some samples are shown in the box below. Negated signs of comparison not illustrated here are transcribed according to the same principle.

⠠⠠⠠⠠⠠⠠	Negated Equals Sign	≠ or ≠
⠠⠠⠠⠠⠠⠠	Negated Parallel To	∦
⠠⠠⠠⠠⠠⠠	Negated Perpendicular To	⊄
⠠⠠⠠⠠⠠⠠	Negated "Bar Under Greater Than"	≧̄
⠠⠠⠠⠠⠠⠠	Negated Membership	∉ or ∋

$$\begin{aligned}
\Rightarrow 4 \times 13 &\neq 14 && \dots\dots\dots\dots\dots\dots\dots && \dots\dots\dots\dots && \dots\dots\dots\dots \\
\Rightarrow 4 &\not\geq 7 && \dots\dots && \dots\dots\dots\dots && \dots\dots \\
\Rightarrow 9 &\notin D && \dots\dots && \dots\dots\dots\dots && \dots\dots \\
\Rightarrow CD &\nparallel EF && \not\perp && GH && \\
&&& \dots\dots\dots\dots && \dots\dots\dots\dots && \dots\dots\dots\dots && \dots\dots\dots\dots && \dots\dots\dots\dots && \dots\dots\dots\dots && \dots\dots\dots\dots && \dots\dots\dots\dots
\end{aligned}$$

Instructions: Use the principles learned in [Section 5.10](#) to construct the negation symbols that are not shown in the sample box.

PRACTICE 5E

- (1) $x \notin A$ means "x is not an element of A".
- (2) By typing <=, the symbol \leq will appear. By typing /< , the symbol \nless will appear.
- (3) $A \not\subseteq B$ means that at least one element of A is not an element of B.
- (4) $WXY \sim VXW$
- (5) Since $L \nparallel M$ and $M \nparallel N$, does it follow that $L \nparallel N$?
- (6) The domain is all $x \neq -4, 0, 5$.

Format: Nemeth Instructions

5.11 Margins for Instructions Preceding Itemized Material (5-3)

The Nemeth Code makes a distinction regarding instructions preceding a set of itemized problems. Following a blank line, the instructions begin in cell 5 with runovers in cell 3. The related itemized material begins on the next line.

Exceptions: (a) Instructions may begin on line 1 if no running head is used. (b) When instructions immediately follow a cell-5 or a cell-7 heading, the blank line before the instructions is unnecessary. (c) If the itemized material itself requires a leading blank line, such as for spatial material (to be studied later in this course), a blank line is inserted.

The print document may use a distinctive typeform for instructions. Typeform used solely as a visual enhancement is disregarded in the braille transcription, according to UEB and *Braille Formats* guidelines.

In [Example 5-31](#), the dashed line indicates a print page turn.

Example 5-31

Problem Set 7F Tell whether the following ratios are equivalent.

1. $3 : 2 = 75 : 50$

2. $6 : 4 = 15 : 30$

Which math sentence is true? Which is false?

3. $328 \div 4 = 41 \times 2$

4. $672 - 415 < 312 \div 3$

54

Multiply.

5. $11,251.54 \times 1436$

6. $1000 \times 476,792$

5.12.3 **Margins for Narrative Preceding Itemized Material (3-1).** Only explicit textual matter is formatted as (5-3) "instructions". If the narrative is purely explanatory, regular paragraphing is applied and a blank line precedes the itemized material.

Example 5-37

Multiplication is a short way of adding quantities of the same size. For example, $6 + 6$ becomes two 6's or 2×6 and $7 + 7 + 7$ becomes three 7's or 3×7 .

1. What would $10 + 10 + 10$ become?
2. How is 5×5 obtained?
3. Express "4 sixes" in two ways—as addition; as multiplication.

1. $10 + 10 + 10 = 30$
2. $5 \times 5 = 25$
3. $4 \times 6 = 24$ or $6 + 6 + 6 + 6 = 24$
4. $10 + 10 + 10 = 30$
5. $5 \times 5 = 25$
6. $10 + 10 + 10 + 10 + 10 = 50$
7. $5 \times 5 = 25$
8. $4 \times 6 = 24$
9. $10 + 10 + 10 = 30$
10. $4 \times 6 = 24$

Instructions: Treat "Signs of Comparison" and "Adding and Subtracting Integers" as cell-5 headings.

PRACTICE 5F

Signs of Comparison

These examples illustrate the basic spacing rules for comparison signs learned in this unit.

- (1) $5 < 9 < 11$
- (2) $11.7 > 1.17$
- (3) $550 : 11 :: ? : 12$

Adding and Subtracting Integers

Find the sum or difference as indicated by the signs.

- 1) $-6 + -5 = \underline{\quad}$
 - 2) $5 + -19 = \underline{\quad}$
 - 3) $-7 - -13 = \underline{\quad}$
 - 4) $29 - -24 = \underline{\quad}$
-

For further practice, see Addendum 1—Reading Practice.

Submit Exercise 5 to your instructor.

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ANSWERS TO PRACTICE MATERIAL

PRACTICE 5A

- 1
- 2
- 3

PRACTICE 5C

- 1 ⠠⠠
- 2 ⠠⠠ ⠠⠠ ⠠⠠
- 3 ⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠ ⠠⠠ ⠠⠠ ⠠⠠⠠ ⠠⠠⠠⠠
- 4 ⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠
- 5 ⠠⠠ ⠠⠠⠠ ⠠⠠
- 6 ⠠⠠⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
- 7 ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠
- 8 ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠
- 9 ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
- 10 ⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠
- 11 ⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠
- 12 ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠
- 13 ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠
- 14 ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠
- 15 ⠠⠠⠠⠠ ⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠
- 16 ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠

Notes regarding the last item: The words are part of the math and so are transcribed uncontracted without switching out of Nemeth. Each single letter requires an English-letter indicator because each is preceded and followed by a space and/or punctuation (or begins a new line). The presence of a single grouping symbol is ignored when determining whether an English-letter indicator is needed.

