

## LESSON 8

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*Format*

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### LESSON PREVIEW

Fractions, mixed numbers, and radical expressions are studied. The "linked expression" is defined and its format is discussed in detail, including the special case of the nested linked expression.

*Note:* The font size in some of the examples is larger than standard, for clarity. Some of the larger symbols may appear to be bold. Do not consider the size difference or the darker image to be a variant typeform.

## ***INTRODUCTION TO FRACTIONS***

Two types of fractions are presented in this lesson: simple fractions (including mixed numbers) and complex fractions.

A fraction is composed of three parts: a numerator, a denominator, and a fraction line.

$$\begin{array}{ccc} \text{numerator} \swarrow & & \\ & \frac{3}{4} & \leftarrow \text{fraction line} \\ \nwarrow \text{denominator} & & \end{array}$$

Fractions are printed in a variety of ways. The numerator may be printed above the denominator, or they may be printed on the same level. The fraction line may be horizontal or diagonal. Here are three examples of the fraction "three fourths" printed in different styles.

$$\frac{3}{4} \qquad 3/4 \qquad 3/4$$

The numerator and/or denominator may also consist of or contain words or abbreviations. Here are four examples.

$$\text{m/s} \qquad (\textit{Spoken: meters per second})$$

$$\text{ft./sec.} \qquad (\textit{Spoken: feet per second})$$

$$\frac{\text{rise}}{\text{run}} \qquad (\textit{Spoken: rise over run})$$

$$3.5\%/year \qquad (\textit{Spoken: 3.5 percent per year})$$

In a technical transcription, fractions are transcribed in Nemeth.

### ***Simple Fractions***

**[NC 13.1]**

#### **8.1 Definition**

For the purposes of the Nemeth Code, a simple fraction is one whose numerator and denominator contain no fractions, except possibly at the superscript or subscript level. These are simple fractions:

$$\frac{1}{2} \quad \frac{a^2}{b^2} \quad \frac{72 \text{ mi.}}{4 \text{ hr.}} \quad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}} \quad 5/12$$

This is not a simple fraction because the numerator contains a fraction:  $\frac{1/3}{5}$



Example 8-2

Rate formula:  $\text{rate} = \frac{\text{distance}}{\text{time}}$  or  $r = \frac{d}{t}$ .

Rate formula: rate = distance / time or r = d / t.

*Reminder: Words in Nemeth are transcribed without contractions.*

Example 8-3

Slope formula:  $m = \frac{y_2 - y_1}{x_2 - x_1}$  or  $m = \frac{\Delta y}{\Delta x}$ .

Slope formula: m = (y2 - y1) / (x2 - x1) or m = Δy / Δx.

*Reminder: A numeric subscript does not need a subscript indicator when it is a right subscript to a letter.*

Example 8-4

Multiplying fractions is easy!  $\frac{3}{4} \cdot \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$

Multiplying fractions is easy! 3/4 · 1/2 = (3 · 1) / (4 · 2) = 3/8

*Spacing: There is no space before or after the operation signs (multiplication dots); there is a space before and after the comparison signs (equals signs).*

Example 8-5

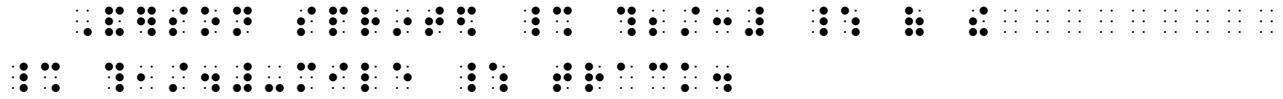
Use the reciprocal of the coefficient to solve for x in  $\frac{3}{8}x = 72$ .

Use the reciprocal of the coefficient to solve for x in 3/8 x = 72.

*Spacing: The coefficient (fraction) is unspaced from the variable (x).*

Example 8-6

Anderson sprinted  $\frac{2}{3}$  of the  $\frac{1}{4}$ -mile track.



*Spacing: The fraction in this hyphenated expression is unspaced from the hyphen.*

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*Instructions:* Transcribe the first two lines as a paragraph, with one space between expressions. A blank line must precede the itemized portion. When you proofread, check that you closed each fraction, that you returned to the baseline after each superscript, that displayed expressions are placed in the proper cell, and that you terminated Nemeth where appropriate.

**PRACTICE 8A**

**Horizontal Simple Fraction Line**

Here are some examples of simple fractions.  $\frac{1}{2}$   $\frac{15}{16}$   $\frac{x}{y}$   $\frac{a+b}{c+d}$   $\frac{\Delta y}{\Delta x}$   $\frac{(x+y)}{(x-y)}$

$$\frac{9}{12} \left( \frac{3}{2}a + \frac{1}{2}b \right) \frac{3x}{17y} x - \frac{1}{4}(x - 2x)$$

1.  $V = \frac{1}{3}\pi r^2 h$

2.  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

3.  $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$

4. Write an equation to show that  $\frac{3}{4}$  of  $\frac{1}{2}$  is  $\frac{3}{8}$ .

5.  $x^2 \frac{dy}{dx} = \frac{4x^2 - x - 2}{(x+1)(y+1)}$

6. Solve this differential equation:

$$x \frac{dy}{dx} + 2y = e^{x^2}$$

7. The number  $\pi$  is the ratio of the circumference of a circle to its diameter. That is,

$$\pi = \frac{\text{circumference}}{\text{diameter}}.$$

8.  $\frac{12}{33} = \frac{m}{11}$

9.  $\frac{4}{32} = \frac{10.5}{x}$

10.  $\frac{1}{4} + \frac{3}{4} - \frac{1}{2} = \frac{1}{2}$

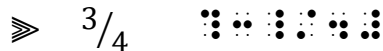
## 8.4 The Diagonal Simple Fraction Line

The type of fraction line used in the print copy (horizontal or diagonal) is replicated in the braille transcription. In a simple fraction, the diagonal fraction line is transcribed as the symbol shown in the box below. Note that the diagonal simple fraction line consists of two braille cells.



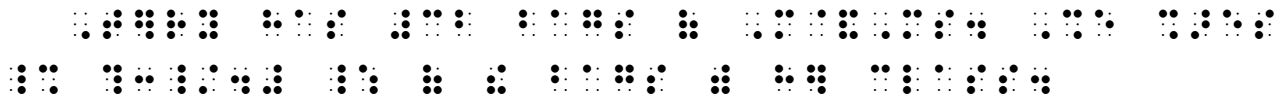
When a diagonal fraction line is printed, it may not be clear where the fraction begins and where it ends. The transcriber must not attempt to analyze the math. Instead, application of the following rules will prevent misinterpretation of the expression.

- 8.4.1 **Use of Simple Fraction Indicators with the Diagonal Simple Fraction Line.** When the numerator and denominator are printed at different levels of writing on either side of the diagonal line, the construction is a fraction and so simple fraction indicators are used. Do not confuse this type style with superscripts and subscripts. In this sample, the numeral 3 is the numerator and the numeral 4 is the denominator.

➤  $3/4$  

### Example 8-7

Terry has 32 bags of M&Ms. She shares  $3/4$  of the bags with her class.



- 8.4.2 **Type Size on the Baseline of Writing.** If the numerator and denominator are printed at the same level of writing on either side of the diagonal line, the transcriber must notice the type size.

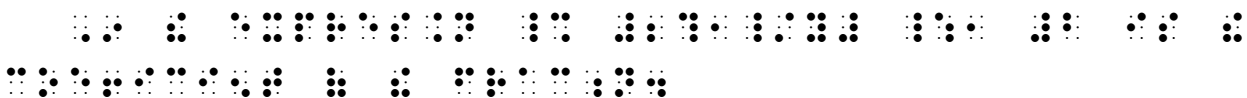
- a. **Fraction in Smaller Type.** If the type is in a different size from that normally used for similar expressions throughout the text, identify this as a fraction by using simple fraction indicators. In the sample below, note that the fraction is printed on the baseline of writing—it is not a subscript.

➤  $2 \text{ } 1/y$  

*The numeral 1 is smaller than the numeral 2. The space between the coefficient and the fraction is not shown in braille.*

### Example 8-8

In the expression  $2 \text{ } 1/y$ , 2 is the coefficient of the fraction.













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*Instructions:* Determine whether the slash is used mathematically, that is, does it require a switch to Nemeth? If it does, then determine if fraction indicators are required.

**PRACTICE 8B**

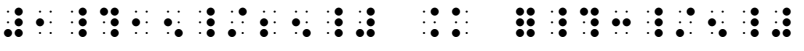
**Diagonal Simple Fraction Line**

- A) How many  $\frac{2}{3}$ 's are there in  $\frac{5}{6}$ ?
  - B) Energy is absorbed at the rate of 880 J/s for each square meter of the surface.
  - C)  $y(0) = \pi/4$
  - D)  $1 \text{ ft/sec} \approx 0.6818 \text{ mph}$
  - E) In  $y \frac{1}{5}$ ,  $y$  is the coefficient of the fraction  $\frac{1}{5}$ .
  - F) True/False: The rise/run ratio is 5 in graph A.
  - G)  $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
  - H) A 5-year CD went from earning interest at the rate of 12.06%/year in 1984 to earning less than 0.87%/year in 2015.
-



8.5.2 **Mixed Numbers and Omissions.** If any part of a mixed number contains a sign of omission, the mixed number fraction indicators are used.

$\gg \frac{7}{4} = 1\frac{?}{4}$ 


$\gg 1^{15/25} = ?^{3/5}$ 


8.5.3 **Nonuse of Mixed Number Fraction Indicators.** If the fractional part of the expression contains a letter, it no longer fits the Nemeth Code's definition of "mixed number." Appropriate fraction indicators are used (or are not used) according to the rules.

$\gg 3\frac{1}{y}$ 


$\gg x\frac{3}{8}$ 


### PRACTICE 8C

#### Mixed Numbers

1. Find the premium for a  $1\frac{1}{2}$ -yr. policy at the yearly rate of 24¢ per \$100.
2.  $2\frac{1}{2}$  ft + 8 in = ? inches
3.  $(\frac{1}{2} \times 3\frac{1}{2}) + (3\frac{1}{2} \times 2)$
4.  $13\frac{1}{2} + 2\frac{2}{3} = 16\frac{1}{6}$
5.  $7/4 = 1\frac{?}{4}$
6.  $\frac{9}{4} = 2\frac{x}{4}$

#### Fraction Review

Compute each **unit rate** (price/pound).

- a. \$1.50 for  $\frac{2}{3}$  pound of potatoes
- b. \$4.20 for  $\frac{1}{2}$  pound of Edam cheese
- c. \$6.00 for  $\frac{3}{4}$  pound of deli smoked turkey
- d. \$12.50 for  $1\frac{1}{2}$  pounds of sliced ham

## Complex Fractions

[NC 13.5]

### 8.6 Definition of Complex Fraction

A complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers.

$$\begin{array}{l} \text{numerator} \\ \text{denominator} \end{array} \quad \frac{4 \frac{3}{4}}{5} \quad \leftarrow \text{complex fraction line}$$

Here are more samples of complex fractions.

$$\frac{\frac{4}{3}}{12} \quad \frac{\frac{a}{b} - \frac{c}{d}}{\frac{a}{b} + \frac{c}{d}} \quad \frac{1}{3/8} \quad \frac{1/3}{2/4}$$

*Reminder:* A fraction is not a complex fraction if the only fractions it contains are at the superscript or subscript level. Such a fraction is a simple fraction.

This is a simple fraction, not a complex fraction:  $\frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

#### 8.6.1 Use of Complex Fraction Indicators and Complex Fraction Lines. The opening and closing complex fraction indicators are used to enclose a complex fraction.

Complex Fraction Indicators

⠠⠠ Opening

⠡⠡ Closing

The main complex fraction line is represented by its appropriate braille symbol—either horizontal or diagonal.

⠠⠠ Horizontal Complex Fraction Line —

⠠⠠⠠ Diagonal Complex Fraction Line /



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*Instructions:* Begin each complex fraction on a new braille line, not side by side as printed.  
Read left to right.

**PRACTICE 8D**

**Complex Fractions**

$$\frac{\frac{1}{8} + \frac{3}{4}}{7}$$

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{7}{9}}$$

$$\frac{1/3+1/4}{4/5-1/2}$$

$$\frac{\frac{\pi}{8}}{2\pi}$$

$$\frac{\frac{a}{b}}{c}$$

$$\frac{33\frac{1}{3}}{100}$$

$$\frac{3/5}{6}$$

$$\frac{3}{5} \bigg/ \frac{7}{9}$$

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## *More Fraction Rules*

### 8.7 Fractions and the Baseline Indicator

When a fraction is on the baseline level, ensure that the components of the fraction (the fraction indicators and the fraction line) are notated on the baseline of writing. When there is a level in effect at the end of a numerator or a denominator, correct placement of the baseline indicator assures accurate reading.

$\gg \frac{a^2}{b}$ 
⠠⠠⠠⠠⠠⠠⠠⠠

*The baseline indicator precedes the fraction line following the superscript in the numerator.*

$\gg \frac{a}{b^2}$ 
⠠⠠⠠⠠⠠⠠⠠⠠

*The baseline indicator precedes the closing fraction indicator following the superscript in the denominator.*

$\gg \frac{\frac{x}{x-1} - 1^2}{\frac{x}{x+1} + 1^2}$ 
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

*These fractions are all on the baseline.*

$\gg \frac{1}{x^2} \times y$ 
⠠⠠⠠⠠⠠⠠⠠⠠

*This fraction is in the superscript position.*

$\gg \frac{y_2 - y_1}{x_2 - x_1}$ 
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

*Recall that a return to the baseline after each numeric subscript is assumed when a baseline indicator is not used.*

### 8.8 Further Observations Regarding Spacing

Spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction. No space is left between a fraction and a letter, a numeral, a sign of grouping, a braille indicator, or another fraction when these items are part of the same expression.

Example 8-24

Explain why  $\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$ .

*The plus sign is unspaced from the fractions before and after it; there is a space before and after the equals sign. There is no space between the components of a mixed number.*

Example 8-25

Multiply the fractions.  $\left(\frac{5}{12}\right)\left(\frac{4}{12}\right) = \text{---}$

*No space is left between factors even though one may appear in print.*

Example 8-26

Differentiating the first two terms,  $\frac{1}{2}x^{1/2} + \frac{5}{6}x^{-3/2}$ .

*In the braille transcription, no spaces occur in this long math expression.*

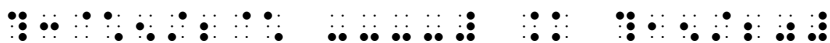
### 8.9 Fractions and the Ellipsis and Long Dash

- a. **Spacing Next to a Fraction Indicator.** No space is left between an opening or closing fraction indicator and an ellipsis or long dash in the numerator or denominator of a fraction.

$$\gg \frac{\dots \times 5}{2 \times 10} = \frac{15}{20}$$

*The space following this ellipsis is required. Review Section 1.12 in Lesson 1.*

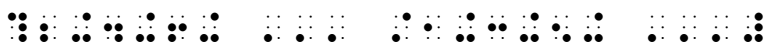
$$\gg \frac{3 \times 5}{2 \times \underline{\quad}} = \frac{15}{20}$$



The space preceding this long dash is required. Review Section 1.12 in Lesson 1.

- b. **Spacing Next To a Fraction Line.** A space is left between a fraction line and an ellipsis or long dash.

$$\gg \frac{2+4+6+\dots}{1+3+5+\dots}$$



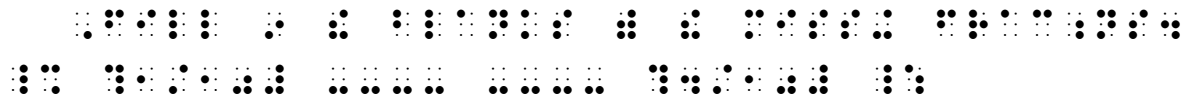
- c. **Spacing Between Fractions.** A space is left between a fraction and an ellipsis or long dash preceding or following the fraction.

$$\gg \frac{1}{10} \dots \frac{10}{10}$$



Example 8-27

Fill in the blanks with the missing fractions.  $\frac{1}{10}$  — —  $\frac{4}{10}$



**8.10 Fractions in an Enclosed List**

Fractions and mixed numbers may be part of an enclosed list. (Review the rules for "enclosed list" in Lesson 4.)

$$\gg \left\{ 0, \frac{1}{2}, 1, 1\frac{1}{2} \right\}$$



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*Instructions:* Determine the formatting before beginning your transcription. Where does each paragraph begin? Which expressions are embedded and which are displayed? What is the proper cell placement for the displayed expressions?

### PRACTICE 8E

These are simple fractions:

$$\frac{1}{2} \quad \frac{a^2}{b^2} \quad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$$

This is not a simple fraction:  $\frac{1/3}{2/3}$

Review the rules in Section 6.12.5 regarding an ellipsis on the baseline of writing when it follows a superscript.

$$x^{\frac{1}{2}} \dots x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \dots \frac{x^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1}$$

Plot the points  $\left(-\frac{1}{2}, 4\right)$ ,  $\left(3, 4\frac{1}{4}\right)$ , and  $\left(-9, \frac{3}{4}\right)$ . Then express  $\frac{a^{3/4}}{b^{5/4}}$  in radical form.

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# ***RADICAL EXPRESSIONS***

## [NC Rule 16]

### 8.11 Terminology

Here are the parts of a radical expression.

$\sqrt{144}$        $\sqrt{\phantom{x}}$  is the *radical sign*.

144 is the *radicand*.

The horizontal bar above the radicand is the *vinculum*. The vinculum shows the extent to which the radical sign applies.

There may be a figure placed to the left and slightly above the radical sign, called the *index* of the radical. For example, this radical sign has an index of three.  $\sqrt[3]{\phantom{x}}$  When there is no index, the radical sign may be referred to as the "square root" sign.

### 8.12 The Termination Indicator

When a radical expression has a vinculum, the radical sign is placed before the radicand and the termination indicator is placed after the radicand.

|  |
|--|
| $\sqrt{\phantom{x}}$ Radical Sign $\sqrt{\phantom{x}}$ |
| $\sqrt{\phantom{x}}$ Termination Indicator             |

➤  $\sqrt{x}$        $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$

➤  $\sqrt{64}$        $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$

*Reminders: An English-letter indicator is not needed for an English letter (in regular type) which occurs in an unspaced sequence of mathematical symbols. A numeric indicator is not used when a numeral is not preceded by a space.*

- a. **No Vinculum.** When a vinculum is not shown in print, or when the radical sign occurs without a radicand, a termination indicator is not used.

➤  $\sqrt{(x - 1)}$        $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$

➤  $\sqrt{\phantom{x}}$        $\sqrt{\phantom{x}}$

#### Example 8-28

The  $\sqrt{\phantom{x}}$  is called a "radical sign."

$\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$   $\sqrt{\phantom{x}}$

### 8.13 Spacing

The spacing before and after a radical expression is subject to the spacing rules for the signs preceding or following the radical expression.

$$\Rightarrow \sqrt{9} - \sqrt{4} = 1$$

No space is left between a radical expression and a letter, a numeral, a fraction, a sign of grouping, a braille indicator, or another radical expression when these items are unspaced in print and belong to the same expression.

$$\Rightarrow \sqrt{5}y$$

$$\Rightarrow \sqrt{x^2}$$

$$\Rightarrow 2a\sqrt{4ab}$$

$$\Rightarrow \sqrt{4}\sqrt{87} = 2\sqrt{87}$$

$$\Rightarrow \sqrt{y} dx + (1 + x) dy = 0, y(0) = 1$$

$$\sqrt{y} dx + (1 + x) dy = 0, y(0) = 1$$

*Reminder: In print, derivative notation dx, dy, etc. is often preceded and followed by a space within an expression, for clarity. In braille, the terms are not spaced unless a space is required with the item preceding or following them. (Review Section 4.15.1 in Lesson 4.)*

Example 8-29

Simplify.  $\frac{2 - \sqrt{\frac{1}{4}}}{3 - \sqrt{\frac{1}{2}}}$

$$\frac{2 - \sqrt{\frac{1}{4}}}{3 - \sqrt{\frac{1}{2}}}$$

### 8.14 Index of Radical

A small number or letter that may appear next to the radical sign is the *index* of the radical. This print sample shows an index "3".

$$\sqrt[3]{27}$$

In braille, the index-of-radical indicator and the index precede the radical sign.



### 8.15 Nested Radical Expressions [NC 16.3]

When radical expressions are nested one within the other, the appropriate number of order-of-radical indicators shows the depth of each inner radical expression.

| Order-of-Radical Indicators |                       |
|-----------------------------|-----------------------|
| ⋮                           | First Inner Radical   |
| ⋮⋮                          | Second Inner Radical  |
| ⋮⋮⋮                         | Third Inner Radical   |
| ⋮⋮⋮⋮                        | Termination Indicator |

The appropriate order-of-radical indicator is placed before its radical sign and before its termination indicator.

$\Rightarrow \sqrt{x + \sqrt{x + y + z}}$ 
⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮

When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

$\Rightarrow \sqrt{x + \sqrt{y + \sqrt{z}}}$ 
⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮

**8.15.1 Nested Radical Expression with an Index.** If an inner radical expression has an index, the appropriate order-of-radical indicator is placed before the index-of-radical indicator as well as before its termination indicator. When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

$\Rightarrow \sqrt{\sqrt[3]{16}} = \sqrt[3]{\sqrt{16}}$ 
⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮
⋮⋮
⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮

$\Rightarrow \sqrt{\sqrt[3]{\sqrt[4]{10}}}$ 
⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮⋮

### 8.16 Radical Expressions and the Baseline Indicator

When a radical expression is on the baseline level, assure that the components of the construction (the radical symbol or the indicators) are notated on the baseline of writing. Place the baseline indicator before the component when it follows a superscript or a subscript.



$$\gg (r^2\sqrt{r})^2 \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot$$

$$\gg \sqrt{x^2 + y^2} \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot$$

The baseline indicator is not used after a numeric subscript that does not require a subscript indicator.

$$\gg \sqrt{x_1 + y_2} \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot$$

### 8.17 Radical Expressions and the Ellipsis and Long Dash

When an ellipsis or a long dash occurs within a radical, no space should be left between the ellipsis or long dash and the termination indicator. However, a space must be left between the radical sign and an ellipsis or long dash.

$$\gg \sqrt{a + b + c + \dots} \\ \cdot$$

$$\gg \sqrt{x + \sqrt{x + \sqrt{\dots}}} \\ \cdot$$

A space is required between a radical expression and an ellipsis or long dash preceding or following the expression.

$$\gg \sqrt{4} \dots \sqrt{64} \quad \cdot$$

### 8.18 Radical Expressions and Abbreviations

When an abbreviation occurs within a radical, no space is left between the abbreviation and the termination or order-of-radical indicator following it.

$$\gg \sqrt{9 \text{ ft}} \quad \cdot$$

However, a space is required between a radical sign and an abbreviation.

$$\gg \sqrt{\text{ft.}} \quad \cdot$$

A space is also required between a radical expression and an abbreviation preceding or following the expression.

$$\gg 2\sqrt{12} \text{ sq. in.} \quad \cdot$$

### 8.19 Enclosed Lists with Radical Expressions

Radical expressions may be part of an enclosed list.

$$\gg (\sqrt{9}, 3, \sqrt{4}, 2\sqrt{6})$$



## PRACTICE 8G

### Nested Radical Expressions

$$(1) \sqrt{-\frac{1}{2} - i\frac{\sqrt{3}}{2}}$$

$$(2) \sqrt{\sqrt{13} + \sqrt{15} + \sqrt{117}}$$

$$(3) \sqrt{1 - \sqrt{a-b}} \times \sqrt{1 + \sqrt{a-b}}$$

$$(4) \sqrt{b^3\sqrt{b\sqrt{b}}}$$

$$(5) \sqrt[a]{\sqrt[b]{\sqrt[c]{\sqrt{abc}}}}$$

$$(6) \sqrt{a^2}\sqrt{b^4}\sqrt{c} = ab^2\sqrt{c}$$

$$(7) (s^2\sqrt[3]{s^4})^2$$

$$(8) \sqrt[3]{x^2\sqrt{64x^6}}$$

$$(9) \sqrt{\sqrt[3]{\sqrt[4]{\sqrt[5]{\sqrt{b^{48}}}}}}$$

$$(10) \sqrt{x_1 + \sqrt{x_2}}$$

$$(11) q^{\sqrt{r}} + s$$

$$(12) \sqrt{c + d + e + \dots}$$

## LINKED EXPRESSIONS

[NC 26.5]

### 8.20 Definition of Linked Expression

A linked expression contains at least one sign of comparison. The part preceding the first sign of comparison is called the *anchor*. Each remaining part, beginning with a sign of comparison and ending before the next sign of comparison, is called a *link*. In its simplest form,  $x = y$  is a linked expression where  $x$  is the anchor and  $= y$  is the link.

$$12.5\% > \frac{1}{10} \quad \text{The anchor is } 12.5\% \text{ and the link is } > \frac{1}{10}$$

$$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$$

*The anchor is 6 × 245, followed by three links each beginning with an equals sign.*

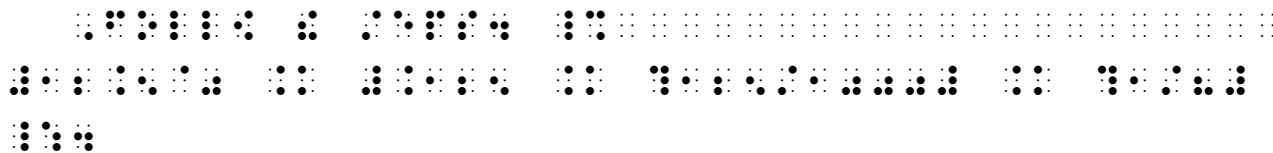
## 8.21 Division of Linked Expressions

A linked expression is considered to be one unit. Recall that a mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. Observe the following points when an anchor and its link or links will not fit on one line.

- (1) If the entire linked expression will fit on one braille line, do not divide it. See [Example 8-31](#). Revisit Examples 1-20, 1-21, and 1-22 in Lesson 1.
- (2) If an anchor and its link do not fit on one braille line, use as much of the current line as possible before dividing. Then divide before the comparison sign of a link. See [Example 8-32](#) and [Example 8-33](#).
- (3) It is not necessary to divide at every link. See [Example 8-34](#). (*Exception:* If the linked expression is printed in the nested layout discussed in [Section 8.22](#), other rules apply.)
- (4) If the anchor or a link will not fit on one braille line, further rules apply. Those rules will be covered in Lesson 15.

### Example 8-31

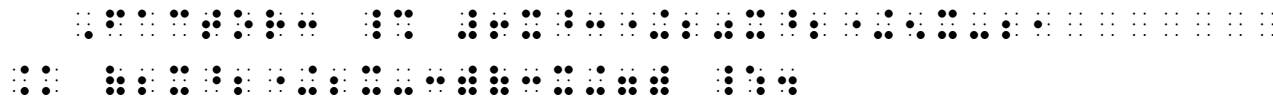
Follow the steps.  $12.5\% = .125 = \frac{125}{1000} = \frac{1}{8}$ .



*Although the anchor and the first link will fit on line 1, since the entire linked expression fits on one line, the anchor begins on line 2.*

### Example 8-32

Factor:  $6x^3 + 20x^2 + 5x - 21 = (2x^2 + 2x - 3)(3x + 7)$ .



*The anchor and its link will not fit on one line. The anchor fits on line 1. The link begins on line 2 in cell 1, the runover position for narrative.*





## 8.22 Special Case—Nested Linked Expressions [NC 26.5.3.b]

An expression with two or more links may be subject to special Nemeth format rules if it appears in print in a particular arrangement as described in this section.

### *Print Layout*

- The expression is displayed.
  - There is an exception regarding itemized problems – see [Section 8.22.b](#).
- The first line contains only the anchor or only the anchor and the first link.
- Each following link begins on a new line, and the comparison signs beginning each link are vertically aligned.
  - An exception applies to the last line – see [Section 8.22.a](#).

The following linked expression meets the three requirements.

To factor  $ab + c^2 + ac + bc$ , the terms can be grouped in pairs with a common factor.

$$\begin{aligned}ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c).\end{aligned}$$

- a. It is common for the last line of the expression to contain more than one link. As long as the other conditions are met, this layout meets the requirements for this nested format.

We can reduce  $12\frac{1}{2}\%$  to lowest terms in the following way:

$$\begin{aligned}12\frac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \frac{125}{1000} = \frac{1}{8}\end{aligned}$$

- b. The next sample shows an itemized problem with no narrative. Although the expression is not displayed, this arrangement follows all of the other print layout specifications: the comparison signs are vertically aligned, and – other than on the first and last line – no sign of comparison is preceded by any expression on its left. Rules regarding this layout are outlined in [Section 8.22.4](#).

$$\begin{array}{l} 1. \quad 12\frac{1}{2}\% = 12.5\% \\ \quad \quad \quad = .125 \\ \quad \quad \quad = \frac{125}{1000} = \frac{1}{8} \end{array}$$

When the print layout meets the definition of a nested linked expression, one of the following Nemeth formats is applied.

**8.22.1 Margin Requirements for a Nested Linked Expression.** The margins which are applied to a nested linked expression follow a reliable pattern, which can be generalized as follows.

- The anchor begins two cells to the right of the runover margin of the material to which it is displayed.
- Each link that starts on a new line begins two cells to the right of the anchor cell.
- Runovers to anchor or links begin four cells to the right of the anchor cell.

*Note:* Rules regarding how to divide a link that will not fit on the line will be discussed in Lesson 15. In this lesson, in order to illustrate runovers within a nested linked expression, a runover line will begin with a sign of operation.

**8.22.2 Nested Linked Expression Displayed to Narrative.** When a nested linked expression occurs in unitemized explanatory portions of the text (3-1), the anchor begins in cell 3 and each link begins in cell 5. In braille, each link begins on a new line, even when the print copy shows more than one link on the last line.

*Reminder:* A line is not skipped above or below displayed mathematical material unless a blank line is required under other rules or guidelines.







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## PRACTICE 8I

### Recognizing a Nested Linked Expression

To test the equation  $R_t = \frac{R}{n}$ , use four 1000- $\Omega$  resistors wired in series to predict a total resistance of 250  $\Omega$ .

$$R_t = \frac{R}{n} = \frac{1000 \Omega}{4}$$

$$\frac{1000 \Omega}{4} = 250 \Omega$$

Then, by using the resistance theory equation

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n},$$

plug the 150- $\Omega$  and 1000- $\Omega$  resistors into the equation as  $R_1$  and  $R_2$ .

$$\frac{1}{R_t} = \frac{1}{150 \Omega} + \frac{1}{1000 \Omega}$$

$$= 0.007 + 0.001$$

$$= 0.008$$

$$R_t = \frac{1}{0.008} = 125 \Omega$$

---

8.22.3 **Nested Linked Expression Displayed to Itemized Material.** Apply the general pattern when a nested linked expression is displayed to itemized material: begin the anchor two cells to the right of the current runover margin; begin each link two cells to the right of the anchor cell; begin runovers four cells to the right of the anchor cell.

- a. **Itemized Text with No Subdivisions.** When a nested linked expression occurs in itemized text containing no subdivisions (1-3), the anchor begins in cell 5 and each link begins in cell 7. Runovers begin in cell 9.
- b. **Itemized Text with Subdivisions.** When a nested linked expression occurs in itemized text containing subdivisions (1-5; 3-5), the anchor begins in cell 7 and each link begins in cell 9. Runovers begin in cell 11.









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## PRACTICE 8J

### Multiplying Mixed Numbers

A.  $2\frac{1}{2} \cdot 1\frac{1}{4} = \left(2 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{4}\right)$   
 $= 2 + \frac{2}{4} + \frac{1}{2} + \frac{1}{8}$   
 $= 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8}$   
 $= 2 + 1 + \frac{1}{8} = 3\frac{1}{8}$

B. What will the remainder be in this problem?

$$4\frac{1}{3} \times 3\frac{2}{5} \times 9\frac{11}{15} \times 2\frac{3}{4}$$
$$= \frac{13}{3} \times \frac{17}{5} \times \frac{146}{15} \times \frac{11}{4}$$
$$= \frac{13 \times 17 \times 146 \times 11}{3 \times 5 \times 15 \times 4}$$
$$= \frac{354,926}{900}$$

= 394 with a remainder of \_\_\_\_.

---

*For further practice, see Addendum 1—Reading Practice.*

|                                       |
|---------------------------------------|
| Submit Exercise 8 to your instructor. |
|---------------------------------------|

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# ANSWERS TO PRACTICE MATERIAL

## PRACTICE 8A

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*Line 20: Because Nemeth continues after the displayed mathematical material, the opening switch is placed at the end of the text preceding the math. See Section 7.1.1.b in Lesson 7.*



## PRACTICE 8C

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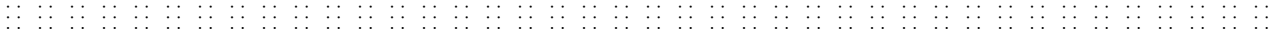





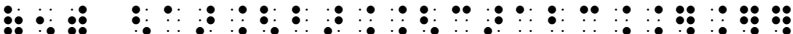





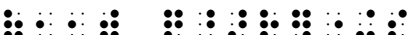

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# PRACTICE 8G

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## PRACTICE 8H

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$$\frac{2x^2 + 3x - 2}{x^2 - 4} = \frac{(x+2)(x-1)}{(x-2)(x+2)} = \frac{x-1}{x-2}$$

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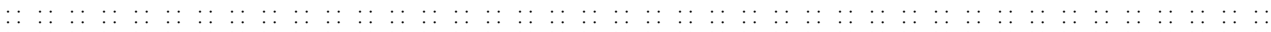












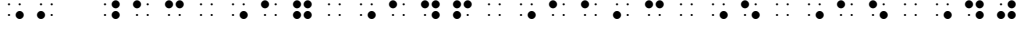

12

*Lines 10-11: Notice that the print copy divided this equality after the equals sign, but the braille transcription follows Nemeth rules and divides before the comparison sign.*





# PRACTICE 8J

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