AN INTRODUCTION TO BRAILLE MATHEMATICS USING UEB WITH NEMETH A Course for Transcribers

LESSON 9

ARROWS

- <u>Construction of Braille Arrows</u>
- <u>Vertical, Slanted, and Curved Arrow Shafts</u>
- Boldface and Compounded Arrows
- INTRODUCTION TO SPATIAL ARRANGEMENTS
 - <u>Spatial Arrangements with Addition and Subtraction</u>
 - Introduction to Cancellation
 - Arrangement on the Page
 - <u>Placement of Code Switch Indicators</u>

Answers to Practice Material

LESSON PREVIEW

The construction of braille arrows is demonstrated, showing several styles of arrowheads and arrow shafts. Braille format for spatially arranged addition and subtraction problems is studied.

Note: Due to the amount of vertical space used with spatial arrangements, this lesson consumes more pages than most.

ARROWS

[NC Rule 22]

9.1 Arrows Used in Mathematics

Here are some samples of arrows which bear mathematical meaning.

$A \rightarrow B$	$\stackrel{\longleftrightarrow}{PQ}$	$2C0 \rightleftharpoons CO_2 + C$

 $f: a \mapsto b \qquad P \uparrow Q \qquad A \to B \leftrightarrow C \leftarrow D$

Note that some chemical arrows are constructed differently from math arrows. In a chemistry transcription, the arrows of *Chemical Notation Using the Nemeth Braille Code* should be used.

9.2 Construction of Braille Arrows

The components of an arrow are transcribed in the following order.

- (1) The shape indicator.
- (2) The arrow direction, if it must be indicated (vertical or slanted). (Sections <u>9.9.1</u> and <u>9.9.2</u>.)
- (3) The typeface of the arrow, if it must be indicated. (Section 9.10.)
- (4) The left arrowhead or dot, if any. (Sections <u>9.5</u>, <u>9.7</u>, and <u>9.8</u>.)
- (5) The arrow shaft, if required. (Sections 9.4 and 9.9.)
- (6) The right arrowhead or dot, if any. (Sections <u>9.4</u>, <u>9.7</u>, and <u>9.8</u>.)

An explicit terminator is not necessary.

Shape Indicator

9.3 Spacing and Punctuation with Arrows

When arrows function as a sign of comparison, the entire construction is spaced according to the rules governing comparison signs. When not functioning as a sign of comparison, an arrow is spaced according to its context. Spacing issues will be illustrated throughout this lesson.

Punctuation associated with an arrow is mathematical.

9.4 Horizontal Arrow Shafts

The length of a shaft is indicated by the number of times the arrow shaft symbol is used. Two braille symbols represent the ordinary shaft length. By comparison, one braille symbol represents a short shaft, and three or more symbols indicate a longer shaft.

	Horizontal Arrow Shafts									
•••	Ordinary length, single									
••• ••	Short, single	—								
••••••••	Long, single									
** **	Ordinary length, double	—								
••	Short, double	=								
** ** **	Long, double									
•••••••••	Dashed									
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Dotted	•••								
··· ·· ·· ··	Wavy	\sim								

9.5 Barbed and Solid Arrowheads

An arrowhead may be barbed, solid, blunted, straight, or curved, and may occur at the left, at the right, or at both ends of an arrow shaft. An arrowhead may also appear with only its upper or lower barb. First, we will look at barbed and solid arrowheads appearing at the end of a horizontal arrow shaft.

The short line in the barbed images represents the point where the arrow shaft meets the arrowhead.

Barbed and Solid Arrowheads									
••	Barbed left full	₹							
••	Solid left	•							
• · · •	Barbed right full	\geq							
• · · •	Solid right	►							
	Barbed left lower	7							
	Barbed right lower	7							
	Barbed left upper	2							
	Barbed right upper	7							

Notice that solid arrowheads are transcribed using the same symbol as full barbed arrowheads.

Many types of arrows can be constructed using the given shaft and arrowhead symbols. Some samples are shown on the next page.

Note: Code switch indicators are omitted in the isolated examples in this section.

Consider the shaft in the barbed arrows shown below to be the standard length for the book. A two-cell "ordinary length" shaft is transcribed. Notice the similarity between the print and braille symbols.

\rightarrow	•••••••••••	\leftarrow	••••••••••••••••••••••••••••••••••••••	\leftrightarrow	
\Rightarrow	•••••••••••	⇐	••••••••••	\Leftrightarrow	00 .0 0. 0. 0. 00 00 .0 .0 .0 00 00 0.
<u> </u>	••••••••••••••••••••••••••••••••••••••	<u>~</u>	•• •• •• •• •• •• •• •• ••	~`	••••••••••••••••••••••••••••••••••••••
	••••••••••••••••••••••••••••••••••••••	~	••••••••••••••••••••••••••••••••••••••	4	••••••••••••••••••••••••••••••••••••••
\rightarrow		\leftrightarrow			
	•				

An additional rule may apply to the first arrow shown above. See <u>Section 9.6</u>.

By comparison, the shafts in the arrows shown below are longer. A three-cell shaft is transcribed.

\rightarrow	••••••••••••• ••••••••••••	\Rightarrow	
\leftarrow		\Leftarrow	••••••••••••••••••••••••••••••••••••••

And, by comparison, the shafts in the arrows shown next are shorter. A one-cell shaft is transcribed.

 \rightarrow \vdots \vdots \vdots \leftarrow \vdots \vdots \vdots

Other shaft styles are illustrated below.

Dashee	l and dotted	Wavy											
>	•••••••••••••••••••••••••••••••••••••••	\sim											
~	••••	\sim	••••••••••••••••••••••••••••••••••••••										
>		⇜	••••••••••••••••••••••••••••••••••••••										

a. **Spacing.** Note the spacing before and after these arrows which are functioning as comparison signs.

 $X \rightarrow Y$ $X \rightarrow Y$ $X \rightarrow Y$ $X \Rightarrow Y$ $X \Rightarrow Y$

9–4

b. **Punctuation.** Mathematical punctuation is used with arrows.

A dot 6 comma is used in Nemeth.

Example 9-1

Consider the set $\mathcal{P} \leftarrow 2$.

The script letter P requires an English-letter indicator even though the letter precedes a sign of comparison. (A typeform indicator must always be followed by an alphabetic indicator.)

9.6 Special Case: The Contracted Form of the Right-Pointing Arrow [NC 22.1]

When the following criteria are all true, the right-pointing arrow is represented in its contracted form.

- the shaft is single and is of ordinary length*
- the barb is a full barb and is right pointing
- the arrow is printed in regular typeface

Contracted Arrow

 $\gg A \longrightarrow B$

Example 9-2

Is the following equation balanced: $2H_2O + O_2 \rightarrow 2H_2O$?

^{*&}quot;Ordinary length" is determined by comparing the arrow shafts printed in the book being transcribed.

9.6.1 **Nonuse of the Contracted Form of the Right-Pointing Arrow.** All other right-pointing arrows require the use of the appropriate shaft symbol.

≫	$X \dashrightarrow Y$	· · • • •	
	$X \sim Y$	· · • • • • • • • • • • • • • • • • • •	
\gg	$X \longrightarrow Y$	· · • •• · · • •	
≫	$X \Rightarrow Y$	· · • • • · • • •	

Instructions: Leave one space (one blank cell) between the arrows in A. and B. Assume shafts in items A, C, and D to be of regular length, and in item B to be long. In item D, divide the long expression before the arrow that comes between the double parentheses. Assume all to be in regular type even though they may appear darker.

PRACTICE 9A

Arrows with Barbed Ends

- $A. \quad \Leftrightarrow \quad \rightharpoonup \quad \leftrightarrow \quad \longleftarrow \quad \dashrightarrow \quad \dashrightarrow \quad \checkmark \quad \checkmark \quad \checkmark$
- $B_{\cdot} \longleftrightarrow \longleftrightarrow \Longleftrightarrow \Longleftrightarrow \Leftrightarrow$
- C. The principal operator of the left formula in Fig. 7.2 is \leftrightarrow , while the principal operator of the right formulas is \rightarrow .
- D. Construct a truth table for $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$.

9.7 Blunted, Straight, and Curved Arrowheads

Here are three other types of arrowheads identified in the Nemeth Code. Samples with a single shaft of ordinary length are shown. The longer horizontal line in each image represents the end of the arrow shaft.

BLUNTED ARROWHEADS Each blunted arrowhead can be left or right pointing.									
÷	Full		E						
		••••••	_]						
			[]						
	Lower only		Г						
		••••••••••••••••••••••••••••••••••••••	Г						
	Upper only		Ĺ						
		••••••••••••••••••••••••••••••••••••••	J						

Please disregard the differences in size and line weight in these images.

STRAIGHT ARROWHEADS									
Each straig	ht arrowhead can be lef	ft or right pointing.							
Full		\vdash							
	••••••••••••••••••••••••••••••••••••••	—							
Lower only									
]							
Upper only									
	••••••••••••••••••••••••••••••••••••••]							

CURVED ARROWHEADS								
Full, left pointing	••••••••••••••••••••••••••••••••••••••	\leftarrow						
Full, right pointing	••••••••••	\rightarrow						
		\leftrightarrow						
Lower only, left pointing	••••••••••••••••••••••••••••••••••••••							
🗄 🕄 Lower only, right pointing	••••••••••••••••••••••••••••••••••••••	\neg						
🗄 🕄 Upper only, left pointing	••••••••••••••••••••••••••••••••••••••	\frown						
🗄 🕄 Upper only, right pointing	••••••••••••••••••••••••••••••••••••••							

Example 9-3

Use $x \mapsto f(x)$ to denote that an element $x \in X$ is mapped to an element $f(x) \in Y$ by the mapping $f: X \to Y$.

1			. • • ·	•••			••		••		•••	••	••	•••	,	•••	••					•			• · · •		•••	•••	•••		•••				• •		
2		•			 	••		••		:• :::	•••	 	••		 	• . • • • •	 •:	. • • •		•••	•		•	•	•••			•••		• •			•••	• · • · • ·	•••	 	
3		•		••			:• :::	• . . •		· · · · •	••• ••		•••		•				••	•	:		•	•	::	::	::	::	::	::	::	::	::	::	::	::	::
4		•	•••	: • • •	••	 	••		•••	• . . •		 • • • •		•	• · • •	•••																					

Review Section 5.7 in Lesson 5 regarding transcription of the colon used in mapping notation.

9.8 Arrows with Dotted Ends

Arrows may be represented by a shaft preceded or followed by a solid or hollow dot.

Solid Dot • •••• Hollow Dot •

Dotted ends are most commonly encountered in geometry and will be illustrated in the lesson on modified expressions. *Note*: Arrow symbols used with graphic number lines are not represented by the braille symbols shown in the box above. Number lines are discussed in Lesson 18.

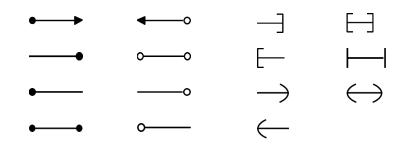
A sample list of arrows with dotted ends is shown on the next page.

o	••••••••••••••••••••••••••••••••••••••	∢ o	•• •• •• •• •• •• •• •• •• •• •• ••
~	60 · 0 0 · 0 0 • · · · · · 0 • · • • • · •	oo	•• ·• •• •• •• •• •• •• •• ••
oo	•• •• •• •• •• •• •• •• •• ••	o	•• •• •• •• •• ••
0—●	•• •• •• •• •• •• •• ••	o	•• ·• •• •• •• ••
••		••	
•>		•0	•• •· •· •• •• ••
∢ —●		•	•• • • • • • • • • • • • • • • • • • •
₀►	••••••••••••••••••••••••••••••••••••••	•	••••••••••••••••••••••••••••••••••••••

Instructions: Transcribe in four columns, as printed. Assume all shafts are of regular length. Arrowheads printed as a solid triangle are considered to be barbed arrowheads.

PRACTICE 9B

Other Types of Arrowheads



Vertical, Slanted, and Curved Arrow Shafts

The arrows shown so far point horizontally – to the left, to the right, or both left and right. When an arrow points in a vertical or slanted direction, arrow direction indicators are used.

9.9 Arrow Direction Indicators [NC 22.4]

An arrow's direction is assumed to be horizontal unless a direction indicator is used. There are two types of direction indicators – one set for vertical arrows and one set for slanted arrows.

9.9.1 Vertical Arrow Shaft. A vertical arrow requires either a "directly over" or a "directly under" indicator.

```
Pointing Up IndicatorPointing Down Indicator
```

The shape indicator is transcribed first, followed by the direction indicator, the arrow shaft, and the arrowhead.

a. **One-Headed Arrow.** When a vertical arrow has only one arrowhead, the symbol for the right arrowhead is used (dots 135).

```
↓ ↓ (up-pointing arrow)
↓ (down-pointing arrow)
```

b. **Two-Headed Arrow.** To show a two-headed vertical arrow (pointing up and down) the "pointing up" indicator is transcribed before a two-headed arrow.

(up-and-down-pointing arrow)

Example 9-4

 $x \downarrow a$ means x is approaching a "from above", much like $x \rightarrow a^+$.

Functioning as a sign of comparison, each arrow is spaced accordingly.

9.9.2 **Slanted Arrow Shaft.** Slanted arrows require either a superscript or a subscript indicator. The superscript indicator elevates the arrowhead by 45 degrees. The subscript indicator depresses the arrowhead by 45 degrees.

```
Superscript Indicator
Subscript Indicator
```

The shape indicator is transcribed first, followed by the appropriate "slant" indicator, and then the arrow.

a. **One-Headed Arrow.** When a slanted arrow has only one arrowhead, the appropriate left- or right-pointing arrowhead is used. The two arrows in the box below show a one-headed arrow pointing upward at a slant. The superscript indicator is used to show the upward slant of the arrowhead.

 Image: Image:

The two arrows in the box below show a one-headed arrow pointing downward at a slant. The subscript indicator is used to show the downward slant of the arrowhead.

4	(arrow points southwest)
7	(arrow points southeast)

Example 9-5

 $x \uparrow a = x \nearrow a = x \to a^-$

Each arrow is a sign of comparison and is spaced accordingly.

b. **Two-Headed Arrow.** To show a two-headed slanted arrow, the left-pointing arrowhead dictates the indicator used. The arrow in the next box shows the left arrowhead pointing upward, therefore the superscript indicator is used.

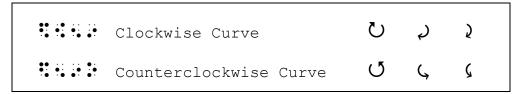
arrow points northwest to southeast (arrow points northwest to southeast)

The next arrow shows the left arrowhead pointing downward, therefore the subscript indicator is used.

 9.9.3 **Curved Arrow Shaft [NC 22.5.1]** Direction indicators are not used with curved arrows. Instead, the curved shaft symbol is used, and the type of arrowhead reveals the direction of curvature.

```
•••• Curved Shaft
```

Rotation direction is indicated as follows. A curved shaft preceded by a *left*-pointing arrowhead represents a clockwise arrow. A curved shaft followed by a *right*-pointing arrowhead represents a counterclockwise arrow.



PRACTICE 9C

Other Types of Arrow Shafts

Vertical	Slanted	Curved
€	7 7	\supset
↑	15	Ċ
\downarrow	1 Z	

Can you figure out how to construct this spear with a northwest, blunted arrowhead?



Boldface and Compounded Arrows

9.10 Boldface Arrow [NC 22.6]

When an arrow is printed in boldface type and the typeform is determined to be significant and therefore retained, the boldface typeform indicator (dots 456) is incorporated into the structure of the arrow. The typeform indicator is placed at the beginning of the arrow symbol, before the arrowhead or arrow shaft. When a direction indicator is required, the direction indicator is transcribed first, followed by the boldface typeform indicator.

••••••••••••••••••••••••••••••••••••••	Boldface Pointing Right	\rightarrow
	Boldface Pointing Left	←
	Boldface Pointing Left and Right	\leftrightarrow
•••••••••••••••••••••••••••••••••••••••	Boldface Pointing Up	1
00 00 00 00 00 00 00 01 00 00 00 00 00 00 00 00 00 00 00 00	Boldface Pointing Down	Ļ
••••••••••••••••••••••••••••••••••••••	Boldface Pointing Up and Down	\$

Reminder: A right-pointing arrow in nonregular type requires a shaft. The contracted form is not used.

9.11 Arrows Used as Signs of Comparison Compounded Vertically [NC 21.9]

Lesson 5 introduced signs of comparison compounded vertically—two or more simple signs of comparison arranged one under the other in print. When arrows are so arranged, the same principle applies. The combination of symbols becomes a single comparison sign "compounded vertically". The uppermost arrow symbol is transcribed first, immediately followed by and unspaced from the symbol for the lower arrow. Each arrow requires a shape indicator.

When the arrow is part of a sign of comparison compounded vertically, the uncontracted form of the right-pointing arrow is used. The contracted form of the right-pointing arrow is never used in the construction of arrows compounded vertically.



Arrows compounded vertically are most often encountered in the subject of chemistry. When the rules of *Chemical Notation Using the Nemeth Braille Code* are being followed, use the arrow constructions from that code, which differ from the Nemeth symbols.

When arrows compounded vertically are encountered in mathematics, use the constructions shown in the box on the next page. Mathematical arrows which are not shown in this list should be transcribed in accordance with the principle outlined in this section.

•••••	Pointing Right Over Pointing Left	$\stackrel{\rightarrow}{\leftarrow}$
•	Pointing Left Over Pointing Right	$\stackrel{\longleftarrow}{\rightarrow}$
: :: :: :: : : :: :: ::	Long Pointing Right Over Short Pointing Left	
	Short Pointing Right Over Long Pointing Left	$\stackrel{\longrightarrow}{\longleftarrow}$
	Pointing Right, Upper Barb Only Over Pointing Left, Lower Barb Only	$\stackrel{\sim}{\leftarrow}$
	Pointing Right Over Boldface Pointing Left	$\overrightarrow{\leftarrow}$
	Boldface Pointing Left Over Pointing Right	$\stackrel{\clubsuit}{\rightarrow}$
	Boldface Pointing Right Over Boldface Pointing Left	↓

9.12 Arrows Used as Signs of Comparison Compounded Horizontally [NC 21.11]

Lesson 5 also introduced the topic of signs of comparison compounded horizontally. When arrows are arranged side by side, the same principle applies. The combination of symbols becomes a single comparison sign "compounded horizontally". A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that the symbols are printed side by side. Arrows not shown in this list should be transcribed in accordance with this principle.

!: !: !: !: ! : ! :	Pointing Up Followed by Pointing Down	$\uparrow\downarrow$
\$: : •• •• : •• •• :	Pointing Down Followed by Pointing Up	$\downarrow\uparrow$
	Pointing Up Followed by Boldface Pointing Down	↑↓
	Boldface Pointing Down Followed by Boldface Pointing Up	↓ ↑

Arrows compounded horizontally are most often encountered in the subject of chemistry. When the rules of *Chemical Notation Using the Nemeth Braille Code* are being followed, use the arrow constructions from that code, which differ from the Nemeth symbols.

9.13 Nonmathematical Arrows

Nemeth arrows should not be used for nonmathematical purposes. Rules and guidelines in BANA publications examine flowchart arrows, arrows in graphic organizer diagrams, lead lines in tactile graphics, etc. Refer to *Guidelines and Standards for Tactile Graphics* for techniques regarding arrows in specific applications such as box-and-whisker plots, clock faces, graphs, line plots, measurement tools, number lines, Cartesian graphs, and spinners.

Here are some samples. (Transcriptions are not shown.)

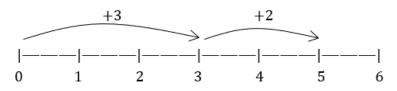
Example 9-6

$$F = 0 \qquad I \qquad L$$

$$(2x + 3)(x - 5) = 2x^2 - 10x + 3x - 15$$

These "pointing" arrows should be drawn as a raised-line tactile graphic.

Example 9-7



Methods for depicting arrows above number lines are given in Guidelines and Standards for Tactile Graphics.

Example 9-8

$$\frac{3}{4} \leftarrow fraction line$$

The lead line in this diagram could be omitted without a loss of information.

INTRODUCTION TO SPATIAL ARRANGEMENTS

9.14 Background

Up to this point we have looked at mathematical material that is read linearly, from left to right, whether it appears embedded within the narrative or set apart from the text as displayed material. When material is arranged on more than one line in print and there is essential vertical alignment of place values or characters, a spatial arrangement is required in braille. ("Alignment" refers to vertical location in the same column or cell.)

- 9.14.1 **Numeric Indicator.** The numeric indicator is generally not used in arrangements which are aligned for computation. Use of the numeric indicator in certain situations abides by rules particular to that specific focus. This distinction will be clarified as each topic is presented.
- 9.14.2 Format. A blank line is required above and below a spatial arrangement. However, transition to a new braille page before beginning or after ending a spatial arrangement takes the place of the required blank line. When no running head is used, a spatial arrangement can begin on line 1 unless a code switch indicator is required at the page turn, or unless the print page number interferes with the alignment. Similarly, a spatial arrangement can end on line 25 unless a Nemeth Code terminator is required or unless the braille page number interferes with the alignment. These exceptions are illustrated later in this lesson.

This lesson teaches rules for the transcription of spatially arranged addition and subtraction problems.

Spatial Arrangements with Addition and Subtraction

The Preliminary Lesson introduced linear problems using the symbol + for plus, - for minus, and = for equals.

 $\gg 2+3=5$ $\Rightarrow 7-6=1$

When addition and subtraction problems are printed in a vertical arrangement, Nemeth rules for spatial arrangements apply. We will use standard terminology to refer to the parts.

2	addend	7	minuend
+3	addend	-6	subtrahend
	separation line		separation line
5	sum	1	difference

9.15 Separation Line

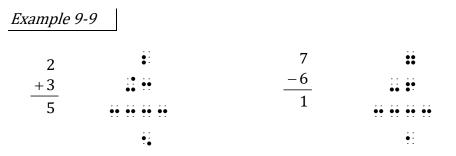
The line printed between the addends and the sum or between the subtrahend and the difference (signifying "equals") is called the "separation line" and is transcribed as a line of dots 25. In braille, the separation line must be made one cell longer at either end than the overall width of the arrangement.

👯 👯 👯 👯 👯 🗰 Separation Line (varying in length)

9.16 Alignment with Addition and Subtraction

In spatial arrangements for addition and subtraction, the corresponding digits, commas, and decimal points are vertically aligned one below the other. In other words, digits under digits, commas under commas, decimal points under decimal points.

The Examples Placement of code switch indicators with spatial arrangements is discussed later in this lesson. Assume Nemeth context in these examples. Switch indicators and the blank line before and after each example are not shown.



Numeric indicators are not used. Digits are aligned by place value, as printed. The separation line extends one cell to the left and one cell to the right of the overall arrangement.

Example 9-	10		
2,017	••••••••••••••••••••••••••••••••••••••	7.5	··· · • • · · · •• · · • • •
+1,963	· • · · · · · · · · · · · · · · · · · ·	-6.7	· · · · · · · · · · · · · · · · · · ·
3,980	•• •• •• •• •• •• ••	0.8	•• •• •• •• ••
	•••••••••••••••••••••••••••••••••••••		· · · • • · · · · · · · · · · · · · · ·

Commas and decimal points are aligned as printed. The separation line extends one cell to the left and one cell to the right of the overall arrangement.

9.16.1 **Intentional Misalignment.** If items have been intentionally misaligned as an exercise for the student, the misalignment is preserved in the transcription. In <u>Example 9-11</u> the student has been instructed to arrange the digits for proper place value.

Example 9-11								
25.92			•	••••	• • •		•	
10.4				•	 		•••	
+3.796		:• ••	÷	. • . •		 	•••	
	••	÷÷	÷÷	÷÷	••	÷÷	÷÷	ėė

The decimal points are intentionally misaligned. The braille transcription matches the print arrangement.

9.16.2 **Operation Sign is Absent.** The plus sign is not always present in addition problems. If there is no operation sign, examine the surrounding text to determine that this is indeed an addition problem. Then apply alignment rules for addition.

Example 9-12	,					
.36			· • · ·	÷÷	•••	
7.02			•		•	
3.04			:• :•		-	
	••	••	••	••	••	••

9.17 Placement of Symbols

9.17.1 **Operation Symbols.** The plus or minus symbol indicating addition or subtraction is placed one column of cells to the left of the leftmost numeric symbol in the part of the arrangement which lies *above* the separation line, regardless of print layout.

Example	9-13		
97 +1	• • • • • • • • • • • • • • • • • • •	97	··· ··· •• ••
+1	• • • • • • • • • • • • • • • • • • •	+1 - 98	· • · · · · · · · · · · · · · · · · · ·
	••••••		•• •• •• ••
			••••

The plus sign is printed within the column reserved for the tens place. In braille, the plus sign is placed one column to the left of the leftmost numeric symbol that appears above the separation line.

Example 9-14

877		
-16	••••••••••	
861	••••••••••	
	• • • • • • •	

The minus sign is printed under the "8" in the minuend. In braille, the minus sign is placed one column to the left of the leftmost numeric symbol that appears above the separation line.

a. **Considerations Below the Separation Line.** Placement of the operation symbol is in relation to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may lie beneath symbols which appear above the separation line.

Example 9-15				
$65 \\ +56 \\ 121$	•••	· • • • •	· · · • • • • • • • • • • • •	•••

The "ones" and "tens" place values in the sum are aligned with those place values in the addends. Because the plus sign is placed one cell to the left of the "tens" column, the "hundreds" place value in the sum (below the separation line) lies in the same column as the plus sign.

9.17.2 **Monetary Symbols.** Monetary symbols which appear above the separation line must also be similarly placed—that is, one column of cells to the left of the leftmost numeric symbol in the part of the arrangement which lies *above* the separation line. A plus or minus symbol (one cell) aligned below a monetary symbol (two cells) is right adjusted in the column.

Example 9-16							
\$.36	:• :::			:• :•	÷÷		
7.02				:• :•	 	•	
+3.04		:• :: ••		• • • • • •	 	•••	
	••	÷÷	÷÷	÷÷	÷÷	••	••

a. **Considerations Below the Separation Line.** Placement of the monetary symbol applies to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may be shown beneath the symbols which appear above it.

Example 9-17									
\$.36 7.02			••	. • • • • •		· • · ·	••	 ••• • •	
+3.04						•		•••	
\$10.42				••	••	•••••••••••••••••••••••••••••••••••••••	•••	•••	
	ėė	÷÷	÷÷	÷÷	÷÷	••	÷÷	••	••
		:• ::	•	•		:• :•	•••	•	

The decimal point and place values in the sum align with the decimal points and place values in the addends. The dollar signs above and below the separation line are not aligned in print and so need not be aligned in braille.

b. Further Considerations. The alignment rules for symbols above the separation line apply regardless of print layout, as illustrated in the <u>Eexamples 9-18</u> and <u>9-19</u>. Below the separation line, normal spacing rules for monetary symbols apply.

Example 9-18

\$1.45		•	•			• •	
- 1.05			•:		· •	••	
	••	••		••	••	••	••

The minus sign is printed to the left of the dollar sign. This alignment is not duplicated in braille.

Example 9-19

\$1.45	
-1.05	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
\$.40	•• •• •• •• •• •• ••

Below the separation line, the dollar sign in print is spaced away from the decimal in order to align with the dollar sign above the separation line. In braille, no space comes between the dollar sign and the decimal.

9.18 Side-by-Side Layout

When there are two or more spatial problems, it is preferred that they be transcribed side by side across the page. The leftmost symbol, which is often the first symbol of a separation line, is placed in the appropriate cell following the rules for itemized, unitemized, or displayed material, which will be demonstrated in the examples in this section.

In side-by-side layout, care must be taken to leave at least three blank cells between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement. "Any symbol" does not refer to a separation line. In a typical arrangement of similar problems, this will mean that one blank space will be left between the end of one separation line and the beginning of the next.

Exampl	e 9-20			
35	975	12,618	17.17	
<u>-24</u>	+25	-1,414	+.17	
••••	• • • •	• • • • • • • • •	· · · · · · · · · · · · · · · · · · ·	
· · · · · · · · · · · · · · · · · · ·		··· ··· · •• ··· ·	· · · · · · · · · · · · · · · · · · ·	
•• •• •• ••		•	• • • • • • • • •	• •• •• •• •• •• ••

Each separation line extends one cell to the left and one cell to the right of its overall arrangement. One clear column of blank space is between the ends of the separation lines.

9.18.1 **Top Alignment.** The minuend or the first addend in each problem is transcribed on the same line, regardless of the print arrangement. When the separation lines are not on the same braille line, at least one clear column of blank space (one cell's width) must be maintained between the ends of the separation lines. This assures that at least three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring spatial arrangement (separation lines excluded).

Example	9-21			
		\$.56	\$6.26	
\$3.00	\$7.39	.37	5.83	
99	+ .25	.93	.27	
	· • · · · · · · · · · · · · · · · · · ·			
· · · · • •	· • · · · · · · · · · · · · · · · · · ·	· • · · · · · · · · · · · · · · · · · ·		• • • • • • • •
•• •• •• ••	•• •• •• ••	• •• •• •• •• •• ••		
				••••••••••••••

In print, the problems are arranged so that all separation lines are across the same row. In braille, the minuend and addends are top adjusted. One clear column of blank space is between the ends of the separation lines, even those that do not occur on the same braille line.

Instructions: Placement of code switches and blank lines has not yet been discussed. Please follow these directives in your transcription of this PRACTICE. Transcribe the opening Nemeth Code indicator on a line by itself, in cell 1. The next line should be blank. Begin the problems on the line following the blank line. Use side-by-side layout, with the first cell of the leftmost separation line in cell 1. Leave only one blank line between the two sets of problems. After completion of the last problem, leave one line blank and transcribe a Nemeth Code terminator in cell 1.

PRACTICE 9D

$900 \\ + 100 \\ 1000$	$4391 \\ + 81 \\ \overline{4472}$	$25.763 \\ - 4.239 \\ \hline 21.524$	$718 \\ -437 \\ 281$	$5,025 \\ + 3,999 \\ \hline 9,024$
\$.27 6.13 2.15	\$3,854 602 + 918	\$14.99 - 6.37 \$ 8.62		

9.19 Omissions in Work Arranged Spatially for Computation

Recall that, in a linear problem, the transcriber indicates the omission of an answer by transcribing a general omission symbol. (See Lesson 1.)

On the other hand, when nothing is printed below the separation line in a spatially arranged problem, the transcriber follows print. Do not insert a general omission symbol to represent blank space below a spatially arranged problem.

An omission may be printed any number of ways – as a question mark, an underscore, a box, etc. In braille, only the general omission symbol is used to show an omission within a spatial arrangement. The one-cell symbol allows the transcription to maintain alignment of the place values.

		General Omission Symbol
Exa	mple 9-22	2
94 +? 100		

The omission is printed as a questions mark.

9.19.1 The number of general omission symbols transcribed reflects the number of omission signs used in print.

Example 9-23					
946		 	•••		
+??	:•		::	::	
1002	••	••	••	••	••
	•		 	 • . • .	

In print, each omission is denoted as a questions mark.

9.19.2 When only one sign of omission is used in print, only one general omission symbol is transcribed. Follow print for placement of the symbol. When the print sign spans more than one column in the arrangement, align the general omission symbol beneath the rightmost character of the omission.

Example 9-24		
\$7.18 <u>- ?</u> \$5.20	\$7.18 -1.98 \$5	\$ - 1.98 \$ 5.20
•• ••	• • • • • • • • • • • •	• • • • • • • • • • • • • • • •

The first omission is printed as a question mark aligned with the decimal point. The second omission is printed as an underscore beneath the decimal digits. (A multipurpose indicator is required between the decimal and the general omission symbol. This will be covered in detail in Lesson 13.) The third omission is printed as a rectangle above the subtrahend.

9.20 Spatially Arranged Polynomials

In polynomials arranged spatially for addition or subtraction, vertical alignment is important.

9.20.1 Alignment. Terms, symbols, and indicators are aligned.

Example 9-25														
5r + 16s + 17t			•••		:• :: ••	•	•••		:• :: ••	•				
-4r - 3s		:: ••	•••		:: ••		••							
r + 13s + 17t		••	••	••	÷	÷	÷÷	÷÷	••	÷	••	••	••	
					· • · ·	•	÷÷		·• ••	•	:::			
Example 9-26														
$3x^2 + 4xy + 2$		••	••		•••	:• :•	 	 ••• •••	••	•• ••			 • · • ·	
$x^2 + 2xy - 15$			••	•	•••	•	 	•••	••	•• ••	••	•	•••	
$4x^2 + 6xy - 13$	••	••	••	••	••	••	••	••	••	••	••	••		••
		•••	••	:•	•	: :•	· • · · ·	•••	••	•• •• ••	••	•	••	

Level indicators align, as well as coefficients and variables.

9.20.2 **Placement of the Baseline Indicator.** A space used for alignment does not cancel the need for a baseline indicator. When the baseline indicator is required, it is placed in the first (leftmost) possible position consistent with the alignment. Any spacing required for alignment is placed following the baseline indicator.

Example 9-27		
3x ² + 2	•• • • • • • • •	••••
$x^2 + 4xy - 5$	••• ••• <td></td>	
$4x^2 + 4xy - 3$	•• •• •• •• •• •• •• ••	

9.20.3 **Misalignment in Print.** The braille transcription must show proper alignment, even if items are not aligned in print. *Exception*: If the terms are intentionally misaligned in an exercise or illustration, follow print alignment in the braille transcription.

Example 9-28

4x + 20y + 6z		•••	••	•••	 • . • .		• • • •	•••	•••	• . • •	
18x - 9z	•	•••	••					:: ••	•	• · · •	
5y + 4z						• •	• • • •			· •	
			••								

Terms are not aligned in this print example. The braille transcription aligns terms and symbols.

Example 9-29

Rearrange the second polynomial in order to add like terms in columns.

						3	y ⁵	+	y ⁴	+	2	y ³	_	27	y																					
							2	$2y^5$	+	<u>3</u> y	/3	+ '	7 y	+	2																					
1		 		•••		• · • •	• ·	•••		•••				. • • · • ·	•••	••	• · · • • ·		••		•••	• . . •	• · • · • ·	•••	•••	• · · •	•• • •	 •:	•••	• · • · • ·	 	• . . •	•••	••		
2			•••	•••	•••		•••				••			•••		••	• · • • • •		• · • •	••	• • • •	 • . • .	•••		 	••										
3	· · · · · · · · · · · · · · · · · · ·		• •																												 	 	• •		 ::	: : : :
4				•• ••	:•	 • •.•	•	· • · ·	•• ••	:•	•••	•	 	•	•• ••	:•	••	•	· · · • •	•	•••	•• ••														
5							•		:•	•••	•	:• :••	••	•••		••	:	· • · · ·	::		:• :::	•														
6	· · · · · · · · · · · · · · · · · · ·	••	••	••	••	••	••	••	••	••	••	••	••	••	••		••		•••	••	••	••	••													
7		::	::	::	::	::	::	::	::	::																					 	 	::			:::
8																																				

Both polynomials are right justified in print. The student is instructed to align the terms and so the print layout is duplicated in braille. (This example illustrates the proper placement of switch indicators and margins for displayed spatial material. Both topics are discussed later in this lesson.)

9.21 Abbreviations

Abbreviations which occur in a spatially arranged addition or subtraction problem are vertically aligned and follow the rules for abbreviations outlined in Lessons 3 and 4.

Example 9-30														
4 ft. 7 in.					•••							•••		
+ 3 ft. 5 in.		:•	••		••		•••		•:		•••	•••	•••	
7 ft. 12 in.	••	•••	••	••	••	•••	••	÷÷	•••	÷÷	••	•••	••	•••
					•••			•	 • •		:• • :			

9.22 Fractions

In a spatial addition or subtraction arrangement containing fractions, the fraction lines and fraction indicators are vertically aligned throughout the problem. Also, each numerator and denominator is in contact with its fraction line, regardless of the number of digits. (Digits in numerators and denominators do not align by place value.) Indicators and numbers must not appear in the same column.

Spaces may need to be inserted to achieve alignment.

Example 9-31	
1/4	
+ 2/4	· • · · · · • · • · · · · · · · · · · ·
	•••••••••••••••

Recall from Section 8.4.2 in Lesson 8 that fraction indicators are not transcribed when a fraction is printed in this manner.

Example 9-32

13/		• • : •	•	÷÷		•• ••	•	•••	· • · •	
$+\frac{5}{16}$	• •			•••			•	•••		
/ /16	•• •	••	••	÷÷	••	÷÷	÷÷	••	••	••

The opening and closing fraction indicators are aligned. The fraction lines are aligned. A space is inserted before the 1-digit numerator in order to have it touch the fraction line.

9.22.1 Alignment with Mixed Numbers. The whole number part of a mixed number is vertically aligned by place value.

Example 9-33														
$2\frac{15}{1}$						••							. •	
16											•			
$+10\frac{5}{2}$		••	•	••				•	•	••			••	
+ 10	••	••	••	••	••	••	••	••	••	••	••	••	••	••

Whole numbers 2 and 10 align by place value.

9.22.2 Alignment of Different Fraction Indicators. When different kinds of fraction indicators occur in the same problem, align the (1456) and (3456) symbols. A space is inserted in order to achieve alignment.

Example 9-34									
$\frac{3}{8}$	$5\frac{1}{2}$								
$+9\frac{1}{8}$	$+\frac{11}{12}$								
				•••	•		: • : • : •	: • : •	
	· • · •	· • · · ·				•			
** ** ** ** ** ** ** **	•••	••	•••	••••	••	••	••	••	••

Notice the blank cell above or below the (456) part of the mixed number indicators to assure that indicators and numbers do not appear in the same column. Reminder: Each numerator and denominator is in contact with its fraction line.

9.22.3 Alignment with the Answer

The fraction indicators and fraction lines in the answer below the separation line must also align with those above the separation line. Blank cells are inserted in order to achieve alignment.

Example 9-35		
12	15	9
16	16	4
$-\frac{3}{16}$	$+\frac{5}{8}$	$+ 6 \frac{1}{4}$
9	?	$15\frac{3}{4}$
16	16	4
•••••••••••••••••••••••••••••	••••••••••••••••••••••	•• •• •• •• •• •• •• •• ••

Fraction indicators and fraction lines are aligned. Each numerator and denominator is in contact with its fraction line. Take particular notice of the the single-digit numerators and denominators as well as the general omission symbol in the second problem. Recall from <u>Section 9.17.1.a</u> that the plus sign in the third problem can be in the same column as the tens place of the 2-digit whole number in the answer.

9.22.4 A Linear Portion of a Spatial Arrangement. An equals sign printed within a spatial arrangement follows linear spacing rules for signs of comparison.

A numeric indicator is required for the final answer because it is not part of the vertical computation.

Example 9-37 $4\frac{4}{8} = 3\frac{12}{8}$ $-2\frac{5}{8} = -2\frac{5}{8}$ $1\frac{7}{8}$ •• •• •• •• •• •• •• •• •• •• •• •• •• ••

The equals signs show a relationship between the two side-by-side spatial arrangements.

9.23 Placement of Identifiers

The numeric or alphabetic identifier associated with a spatially arranged addition or subtraction problem is positioned at the top line of the problem—that is, with the first addend or with the subtrahend. One clear column of blank space (one cell's width) must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines.

Example 9-38

1. 3 <u>- 1</u>	-	= 1	b	u.	-	ok	•											
						•••				:•		••	•••	•••	•••			 •••

9.23.1 **Side-by-Side Layout.** Spatially arranged itemized material may be arranged side by side. The identifiers are located on the same braille line across the width of the page, in sequential order. Problems are spaced so that no less than three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded) including the neighboring identifier.

Note that this is a special rule pertaining to spatially arranged itemized material. As taught in Lesson 1, each main item in *nonspatial* itemized material must start in cell 1, even if the print copy arranges the items side by side across the page.

		E	ixa	m	ple	9	9-3	9																												
		1)	1! +3				2)	1				3)		15 13			4	1 _1	15 13		5)		1 +1	15 13			6		15 13				
10	::		:::							• •											 			 								 ::		::	: : : :	::
11	· • · •	•		::			•	 • •.•	::	::	::	 	•		::			•	• •		 ::	-	••	:::			•	•••								
12				::		·• ••		••							::		••		••					::			•	••								
13				::	••	••	••	••							::	••	••	••	••					::	••	••	••	••								
14			::																		 			 								 	 ::			::
15		•••					•	•••	::			· • · •	 • •.•		::			•	••		 							•	•••	 						
16				::		:: ••	•								::		 	••	••						::		:: ••	•	••	 						
17				::	••	••	••	••	••						::	••	••	••	••	••					::		••	••	••	••	••					
18			::						::	::											 		::	 									 ::	::	· · · · ·	::

Lines 11 and 15: Three blank cells come between the rightmost symbol in each problem (the "ones" column) and the numeric indicator of the next identifier.

Lines 11-13 and 15-17: One blank cell comes between the right parenthesis of each identifier and the leftmost cell of the separation line on line 3.

Line 14: A blank line precedes the second group of spatially arranged problems.

Line 15: No attempt is made to align identifiers with those on line 11.

Instructions: Transcribe the opening Nemeth Code indicator on a line by itself, in cell 1. The next line should be blank. Begin the problems on the line following the blank line. Use side-by-side layout. After completion of the last problem, leave one line blank and transcribe a Nemeth Code terminator in cell 1.

PRACTICE 9E

1)	621 +??? 1096	2)	$ \frac{17x - 8y - z}{-2x + 17y + 6z} \\ \frac{15x + 9y + }{-2x + 9y + } \\ $	3)	$3x^{2}-5x+4$ $-5x^{2}+12x-12$ $-2x^{2}+7x-8$
4)	$ \frac{1\frac{2}{3} \text{ yr}}{+5\frac{5}{12} \text{ yr}} \\ \frac{6\frac{13}{12} \text{ yr}}{6\frac{13}{12} \text{ yr}} = $	$7\frac{1}{12}$	yr		

9.24 Regrouping Numbers in Addition Problems

Small numbers printed above or below digits in a problem are referred to as "regrouping numbers". (Older texts may call them "carried numbers".) The transcription places the regrouping numbers in the same location as printed, inserting the appropriate regrouping indicator between the regrouping numbers and their associated digits. A regrouping line usually does not appear in print, since the smaller font is enough to distinguish them.

The first cell of the regrouping indicator tells the reader to read the numbers above the line or below the line.

```
Regrouping Indicator for Numbers Above
the Arrangement (varying in length)
Regrouping Indicator for Numbers Below
the Arrangement (varying in length)
```

The regrouping indicator will be one cell longer on the left than the separation line. It is important to place the regrouping numbers in the same columnar positions as in print.

In <u>Example 9-40</u>, the regrouping numbers are printed above the first addend. In <u>Example 9-41</u>, the regrouping numbers are printed above the separation line.

Example 9-40

1 1			•	•		
213	• • • • • • • • • • • • • • • • • • •	:::				::
+ 87			•••	•	••	
300		· • • •		•••	•••	
	••	••	••	••	••	••
			••		 	

In the print copy, the regrouping numbers are in a smaller typeface and a regrouping line is not present.

 Example 9-41
 Image: Second symple 1

 7
 9
 Image: Second symple 1

 2
 1
 4
 Image: Second symple 1

 +
 1
 8
 Image: Second symple 1

 3
 1
 1
 Image: Second symple 1

 Image: Second symple 1
 Image: Second symple 1
 Image: Second symple 1

 Image: Second symple 1
 Image: Second symple 1
 Image: Second symple 1

 Image: Second symple 1
 Image: Second symple 1
 Image: Second symple 1

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 Image: Second symple 1
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 Image: Second symple 1

 Image: Second symple 1
 Image: Second symple 1
 Image: Second symple 1

 Image: Second symple 1
 Image: Second symple 1
 Image: Second symple 1

 Image: Second symple 1
 Image: Second symple 1
 Image: Second symple 1

 Image: Second symple 1
 Image: Second

In the print copy, the regrouping numbers are in a smaller typeface and a regrouping line is not present.

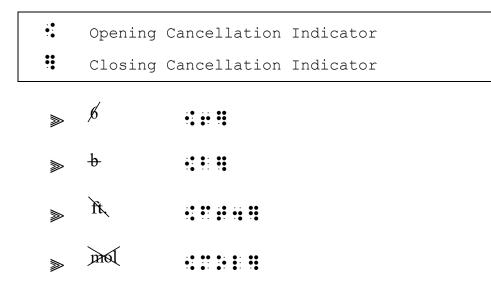
9.24.1 With Identifiers. If an identifier is present, it is placed on the first line of the addition problem (the first addend) regardless of the presence of regrouping numbers. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines and regrouping indicators.

l	Example 9-42									
						 • . • .	•			
			::							•••
1.	² ¹ 25.9		::			•	•••	: • : •	•	
	49.4					•••	•	. • . •	•••	
	87.2					 • . • •		. • . •	•••	
	162.5			••	••	••	••	••	••	••
					•	•••	•••	: • : •	•••	

Introduction to Cancellation

[NC Rule 12]

A spatial arrangement is required when numbers, letters, abbreviations, or words in a mathematical expression are canceled in print by any type of stroke through them, and replacement values are shown. The opening and closing cancellation indicators enclose the material being canceled.



9.25 Cancellation in Subtraction Problems

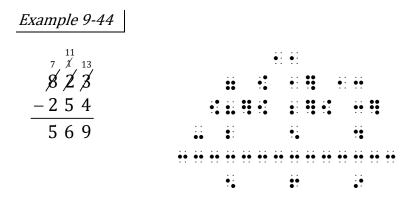
When cancellation is shown in subtraction, a regrouping indicator is not needed. Each regrouping number is aligned with the corresponding canceled digit in the problem.

Digits, decimal points, or other symbols lying above and below the line with cancellation must be carefully aligned for computation. Spaces are inserted where necessary to achieve alignment. Numerals or other symbols lying above *and* below the separation line must not appear in the same columns as cancellation indicators.

Example 9-43

4 10							•	· •	
$1 \not {\overset{4}{\not {D}}} \not {\overset{10}{\not {D}}}$ - 3 9		••	••	•••				 	
$\frac{-39}{111}$	•••			••				 •	
1 1 1	•• ••	••		••	••	••	••	••	 ••

Place values align throughout the arrangement, including the regrouping numbers. To assure a clear column above the cancellation indicators, a space is inserted before the two-digit regrouping number on line 1.



Cancellation indicators align with cancellation indicators. The minus sign on line 4 is placed one column to the left of the opening cancellation indicator on line 3 to assure a clear column below that indicator.

9.25.1 With Identifiers. If an identifier is present, it is placed on the first line of the subtraction problem (the minuend) regardless of the presence of canceled items. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

	Example 9-45	-																				
	7 12 4 16											•	 • : • :			 • •			•	 •••		
A.	× ZZZ	:::::::::::::::::::::::::::::::::::::::	•••	•••	::			••	•••		. • • · • •		••		••	•••				•••		
	-2347				::		: : ••		•••				••			•••				•••		
	5909				::	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••
									•••				 			 				 • .		

PRACTICE 9F

Regrouping Numbers and Cancellation

.

1.	${\overset{1}{9}}{\overset{1}{4}}{\overset{1}{8}}$	2. $\$ 2.52^{12}$	3.	$\overset{4}{\not{5}}\overset{16}{\not{6}}\overset{1}{\not{2}}\overset{13}{\not{3}}$
	+ 75	65.89		- 804
	1023	91.99		4819
		32.03		
		\$192.43		

Arrangement on the Page

A spatial arrangement should be confined to one braille page. Strategies for larger arrangements are beyond the scope of this course.

9.26 **Blank Lines**

A blank line is required above and below a spatial arrangement in most situations.

9.26.1 Blank Lines and the Page Change Indicator. The presence of a page change indicator does not affect the blank line requirement. There may be times when a blank line is needed both before and after the page change indicator, for example, when spatial arrangements occur both before and after the page change.

	Example 9-46	
	15 +13	15 -13
	150 +130	new print page 123
9		
10	• • • • • • • • • • • • • • • • • • •	•. •. •. •.
11		••••
12	•• •• •• •• ••	** ** **
13		
14	•• •• •• •• •• •• •• ••	
15		
16	• • • • • •	
17	•••••••••	
18	•• •• •• •• •• ••	•• •• ••
19		

Each set of spatial problems in this example requires a blank line before and after it.

9.26.2 Blank Lines and Box Lines. A blank line is left both above and below a spatial arrangement even if the arrangement directly precedes or follows a box line. (No examples are given.)

- 9.26.3 **Pagination and Blank Lines.** Transition to a new braille page before beginning or after ending the transcription of a spatial arrangement takes the place of the required blank line. The discussion in this section assumes that code switch indicators are not needed, imagining that Nemeth material precedes and follows each example.
 - a. Starting a Braille Page with a Spatial Arrangement. When no running head is in use, a spatial arrangement may begin on line 1. There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the print page number. If a running head is present, line 2 must be blank and the spatial arrangement will begin on line 3.

	Example 9-47					
	1) 15 +3	2) 155 +3	3) 15 +13	55 33		
1		· · · · · · · · · · · · · · · · · · ·	••••••••••••••••••••••••••••••••••••••	· · · • • • • • • • • • • • • • • • • •	• • • • • · · · ·	
2	··· ·• •• ···	::	• • • • • • • • • • • • • • • • • • •			· · · · · · · · · · · · · · · · · · ·
3	·· •• •• •• ••		•• •• •• ••	· · · • •	•••••••	· · · · · · · · · · · · · · · · · · ·
4						: .: .: .: .: .:
	This exam	ple has no running	head. The prob	olems begin on l	line 1.	
	<i>Example 9-48</i> 1) 1 +1	2) 2 3 +2	3) 13 +3	4) 14 +4		

1		
2		
3		• •• ••
4		
5	EE ++ ++ ++ ++ EE ++ ++ ++ ++ ++ EE ++ ++	··· •• •• •• •• ••
6		

This example has a running head. Line 2 is blank.

b. Ending a Braille Page with a Spatial Arrangement. A spatial arrangement may be placed on the last line of the braille page. There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the braille page number.

	-	Exa	m	ole	9 9	-4	9																															
		1)	15 + 3						2)		45 -3(3)	+	95 1	9 3																	
22		· · ·	• •					• •	 						• •				• •	• •	• •	• •					• •		• •	• •			• •	• •				•••
23			:::			••	• •		 :::	• ě	••	••				••	• •	••	::			- Ö	••	••				· •	 • •.•	· •		• •	::					
24			::		:• ::: ••		••						::			••	 	••							::		:• :: ••		•			• •	::					
25			••	••	••	••	••	••						••	••	••	••	••	••							••	••	••	••	••	••				- · ě	•••	۰.	

Line 25: There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the braille page number.

9.27 Wide Arrangements [NC 26.6.1]

If any portion of a spatial arrangement occurs on lines 1, 2, 24, or 25 of the braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement and the first symbol of the page number. This restriction includes the rightmost symbol of a separation line. If this spacing cannot be achieved, the arrangement must begin on line 3 or end on line 23, respectively.

Example 9-50 illustrates a wide arrangement that begins in cell 3, assuming it is displayed to narrative.

Example 9-50

 $S = 1 + 3 + 5 + \dots + 45 + 47 + 49$ $S = 49 + 47 + 45 + \dots + 5 + 3 + 1$ $2S = 50 + 50 + 50 + \dots + 50 + 50 + 50$

Example 9-50-a (Arranged near the top of a braille page, with no running head.)

1																																		•••	::	Ξ.	- · ě	ě.	. •
2	· · · · ·			• •	• •					• •		• •				• •				• •			•••	• •				•••				• •		•••		• •	• •		••
3				· · · · ·	. • • : • :		••• •••	• · • ·			•	· • · · ·		••	· • · · ·		 • •.•	· • · ·		•	•	•		· • · · ·	•••	 • •.•	· • · · ·	•••	•••	 	•••	 		• •	::				
4				· · · · ·	. • • ·		● 	• · · · • ·		•••	 	· • · · ·	•••	::	· • · · ·	•••	 • •.•	 		•	•	•		· • · · ·		 • •.•	· • · · ·		••	: • : : • •		•		••	:::				
5	· · · · ·		••	••	••	••	••	••	••	••	••		••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	•••					
6		:		 			••• •••	• · • ·		•••	 	:• :: ••	•••		· • · · ·	 • •.•	 	·• ••		• •	•			• •	 • •.•	· •	• •	••	· •		••	· •							

Lines 1-2: The separation line (line 5) invades the three-cell area before the print page number on line 1, so this problem is not allowed to occupy lines 1 or 2.

Example 9-50-b	(Arranged near the bottom of a braille page.)
	(

19	··· ·· ·· ·· ··	• •	• •					• •	• •			• •	• •		• •		• •				• •	• •			•••	• •									•••	•••	
20		 	•		• • • • • •				•				· • • •		••	·• ••		• •	•				•••	۰.		••	••		••	· •							
21		:. :. ••	•		·• ••	•••		••	•		••	••		••	••			• •	•					•••			••			•							
22	· · · · • • •	••	••	••	••	••	••	••	••	••	••	••			••	••	••			••	••	••	••	••	••	••	••	••	••	••		••	••				
23	• • • •		•		· • · ·	•••		••	 		••	· •		••	· •			• •	•				•••	· •		• •	· •		••	· •		••	::				
24	··· ·· ·· ··	•••	•••	• •	••	• •	• •	•••	•••	• •	• •	••	• •	••	••	• •	••	• •	••	• •	•••	•••	•••	••	••	•••	•••	•••	• •	••	••	••	••	• •	••	••	••
25																																	::	- ē	Ξ.	Ξ.	ě .

Lines 24-25: The separation line (line 22) invades the three-cell area before the braille page number on line 25, so this problem is not allowed to occupy lines 24 or 25.

9.28 Itemized Spatial Problems with Subdivisions

9.28.1 Without Narrative. When a set of spatial arrangements contains both main divisions and subdivisions, the following must be observed. If there is no material between the main division identifier and the first subdivision identifier, place the main division at the margin in cell 1, and place the first subdivision on the same braille line after one blank cell. As many subdivisions as possible may be placed across the line. The identifiers are placed as outlined in Sections 9.23, 9.24.1, and 9.25.1, and must be located on the same braille line across the width of the page. If additional subdivisions remain after the width of the page has been met, insert a blank line below the last line of the arrangements. Then begin the next subdivision identifier in cell 3.

 Example 9-51

 1. a. 15 b. 15

 +3 -3

• •	- ÷ •	••	 - · •	•	•	- ē	••				••	••	 •	•	 · •	۰	••	• •				•	
								::				••						::		•••		•••	
								••	• •	••	••	÷	 •					• •	••			••	

	Examp	ole 9-52						
	1. a.	15 +3	. .	15 с. -3	15 +13	d. 15 -13		
1				• • • • • • •		ě ee	• • • • • • • • • • • • • • • • • • •	• • •
2				· • · · · · · · · · · · · · · · · · · ·		· · · ·		· · · · · · · · · · · · · · · · · · ·
3				•••••••			• • • •	: •• •• •• •• ••
4								 : :: :: :: :: :: ::
5			•					
6			•					
7		· · · · ·		••				

9.28.2 With Narrative. If there is narrative associated with the main division number, the subdivisions begin on the next available line (following the required blank line that precedes spatial material) starting in cell 3.

	Example 9-53
	1. Solve: a. 15 b. 15 c. 15 d. 15 +3 -3 $+13$ -13
1	
2	
3	
4	
5	E
6	
7	
8	
9	
10	
11	

Placement of Code Switch Indicators

Each code switch indicator should appear on the same braille page as the mathematical material to which it applies. To keep the switches from interfering with the layout of the problem, they are placed outside of the arrangement. A code switch indicator does not take the place of a blank line. Position the code switch indicators *before* and *after* the requisite blank lines.

9.29 Code Switching with Unitemized Spatial Arrangements

- i. If the problem is not preceded by text, the opening Nemeth Code indicator is placed in <u>cell 1</u> on a line by itself. If there is text before the problem, the opening Nemeth Code indicator is placed on the same line with the end of the text, but if there is not room on that line for the switch indicator, a special rule for spatial arrangements places it in <u>cell 1</u> on the next line.
- ii. The required blank line comes next.
- iii. When unitemized, undisplayed spatial problems are transcribed side by side, the leftmost symbol of the first problem in each row is placed in cell 1.

When the problems are displayed, the leftmost symbol of the first arrangement is placed in the appropriate cell for displayed material. If all problems will not fit across the page, the next problem is placed in the same display cell, not the runover cell.

- iv. When Nemeth terminates after a spatial problem, the required blank line comes first, followed by the Nemeth Code terminator in <u>cell 1</u> on a line by itself.
- v. Resume UEB text on the line following the terminator, unless a format change that requires a leading blank line follows, such as a heading or Nemeth instructions.

For spatial material following a heading, see Section 9.31.

	Example 9-54
	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
1	
2	
3	
4	
5	** ** ** ** ** ** ** ** ** ** ** ** **
6	
7	

Add or subtract, as indicated by the signs.

	15 +3	115 -30	155 +13	515 -33	133 +55	333 - 55	113 + 5	
1	· · • · ••					• • • • • • • • • • •		
2								
3								
4	• • • • • • • • •	•••••	• • • •	• • • • •	• • • • •	•	• • • •	•• •• ••
5	· • · · · · · · · · · · · · · · · · · ·	••• ••		• •	••••••••••••••••••••••••••••••••••••••	• •	• • • • • • • • • • • • • • • • • • •	··· ··· ··· ··· •· •·
6	•• •• •• ••	•• •• •• ••	•••••	• •• •• ••	•• •• •• ••	• • •	•••••••	•• •• •• •• ••
7								
8	$\begin{array}{cccc} \cdot & \cdot & \cdot & \cdot \\ \bullet & \cdot & \bullet & \cdot \\ \cdot & \cdot & \cdot & \bullet \end{array}$							
9	· • · · · · · · · · · · · · · · · · · ·							
10	•• •• •• ••	••						
11								
12								
			ds like "instr			not followe	d by itemized	l material, it is

- formatted as a 3-1 narrative paragraph. Line 2: The opening switch fits at the end of the line of text.
- *Line 3: A blank line precedes the spatial material.*
- Line 6: The problems are printed side by side and begin at the left margin. They are not interpreted as being "displayed." The leftmost symbol of the first problem is placed in cell 1.
- *Line 7: A blank line follows the spatial material.*
- *Lines* 8-10: *The last problem does not fit on lines* 4-6. *The leftmost symbol is placed in cell* 1.
- *Line 12: The Nemeth Code terminator is placed in cell 1 outside of the arrangement, after the required blank line (line 11).*

1. Add or subtract. Circle your answer.

15	115	155	515	133	333
+3	-30	+13	-33	+55	-55

2. Solve the following.

1 2 3 4 • 5 6 7 8 9 10 11 12 13

Line 1: Main item identifier is in cell 1.

- Line 2: The opening Nemeth Code indicator does not fit at the end of line 1. Its position on line 2 is determined by what follows. Placement of the opening switch before an unitemized spatial problem is cell 1. See <u>Section 9.29.i</u>.
- *Line 3: The code switch indicator on line 2 does not take the place of the required blank line before the spatial problems which follow.*
- Line 6: The leftmost symbol of the side by side displayed arrangements is placed in cell 5 according to the rules for displayed material following 1-3 itemized text. Five problems will fit in this row.

Line 10: The leftmost symbol of the continuation of the displayed spatial material is in cell 5. Line 11:A blank line follows the spatial material.

Line 8: The Nemeth Code terminator is placed in cell 1 outside of the arrangement, after the required blank line.

9.30 Code Switching with Itemized Spatial Arrangements

- i. An opening Nemeth Code indicator which precedes itemized spatial material follows the same directives as with any itemized material. (See Lesson 2.) That is, if there is text, the opening Nemeth Code indicator is placed on the same line with the end of the text preceding the itemized problem unless there is no room, in which case the opening Nemeth Code indicator is placed in the runover cell.
- ii. The required blank line comes next.
- iii. The layout follows the directives given in Sections 9.23 and 9.28.
- iv. When Nemeth terminates after a spatial problem, the required blank line comes first, followed by the Nemeth Code terminator in <u>cell 1</u> on a line by itself.
- v. Resume UEB text on the line following the terminator, unless a format change that requires a leading blank line follows, such as a heading.

Example 9-57

1. Add or subtract as indicated by the signs.

			ä	a.	1 + _	15 -3			b.	-	15 -3			c.		- + -	15 13																				
1	 • •				•••				• · · •	•••		 • . • .	• •				•••	••	 		• · · •		 • •	•••	 ••	• ·		•••		• · • ·							
2				•••	•••	::		 •: •:	•••			••																									
3	 										• •						• •								 										 ::	 	
4	 	· •		· •	•••				••	••	::				••	· •					• •	•••			 · •		- · ě					۰	•••				
5						::		 		••								::		 ••		••							::			•	••				
6						• •	••	••	••	••	••								••	••	••	••	••						::	••	••		••	••			
7	 										• •						• •								 										 ::	 	
8	• . : •																																				

Lines 1-2: Margins for itemized material with subdivisions is 1-5. Line 2: The opening switch fits at the end of the line of text. Line 8: The Nemeth Code terminator is placed in cell 1 outside of the arrangement, after the required blank line.

1. Solve these problems using counting cubes.

a. 15 b. 15 c. 15 +3 -3 +13

2. Solve these problems with your pencil.

1		•	•••							•••							•••	•			•••	••	•				· • · · ·					· • · · •											
2						•																																					
3	•	•	•••	•••	•••	•	•	• •						 • •		• •		•		•	•••	••		• •		• •	::				• •	• •	• •	• •	•		•	•••	•••	••	• •	•	•
4	•	•	٠.	• •	· • • •		•	• •			••	••		 	 •• ••	••	• •	•	• •	•			••	••		• •	::	- · •	• •	· •		• •			٠	÷.	•						
5										:• :::		••								:		:: ••		••								::		• •	٠	•	•						
6								• •	••	••	••	••	••						•	•	••	••	••	••	••							• •		••			• •	••					
7	•	•	•••	•••	• •	•	•	• •						 • •		• •		•		•	•••	••		• •		• •	::				• •	• •	• •	• •	•		•	•••	•••	••	• •	•	•
8	:	•																																									
9		•	•••			٠	•			•••	•••		- · ě		•	•••	• •				•••						• • • •			•		••			÷	•							

Line 2: The opening Nemeth Code indicator does not fit at the end of the itemized text on line 1. Its position on line 2 is determined by what follows. Before itemized material, the opening switch is placed in the runover position of the text from which it comes (cell 5).
Line 3: The code switch indicator on line 2 does not take the place of the required blank line

before the spatial problems which follow.

Line 8: The Nemeth Code terminator is placed outside of the arrangement in cell 1.

Example 9-59

Solve these problems using counting cubes.

1.	15	2. 15	3. 15
	+3	-3	+13

1	• •	• •								•••		•••			•			•		•••	•••			•••		:• ••		••	:• :•		:••		••		•			•	;	
2			· · •	•••																																				
3		• •		• •	• •			• •	• •	• •	::		• •				• •				• •			• •	• •	• •		• •	• •	• •		• •			•				• •	
4		•						• •	••		::		- · ě	• •	- · ě					• •	••				- · ě		- · ě	•••				•	 • •.•							
5					: :		· • · ·		••								::		:: ••		••								::		:• :: ••	۰	••							
6					::	••	••		••								• •	••		••	••								::	••			••							
7	::	::	: :	• •	• •			• •	• •	• •	::		• •				• •				• •			• •	• •	• •		• •	• •	• •		• •			•				• •	
8		• . . •																																						
		Ι	in	e 2	2: 2	The	е о	ре	nir	ıg.	Ne	тe	eth	Ca	ode	e ir	ndi	ica	to	r d	0e.	s n	ot.	fit	at	th	e e	nd	of	^c th	ie i	ins	trı	ict	io	ns	on	liı	ne	1.

- Its position on line 2 is determined by what follows. Before itemized material, the opening switch is placed in the runover position of the text from which it comes (cell 3).
- *Line 3: The code switch indicator on line 2 does not take the place of the required blank line before the spatial problems which follow.*
- *Line 8: The Nemeth Code terminator is placed outside of the arrangement in cell 1, after the required blank line.*

9.31 Code Switching with Cell-5 and Cell-7 Headings

Placement of the opening Nemeth Code indicator before a spatial arrangement supersede those taught in Lesson 4 regarding a cell-5 or a cell-7 heading. If the opening Nemeth Code indicator does not fit on the line with the heading, it is placed in <u>cell 1</u> on the next line.

Example 9-60

Addition Problems

10	10	10	10	10
+10	+20	+30	+40	+50

1		•	•••			÷ •	- ÷ é	- ÷ é	•	· •	- ÷ •					• · • •	•••	• · • · • ·	•••	••			 	••														
2	:: : :: :										•••	• •	• •	•••	• •				•••		•••			• •	• •			• •		•••	• •	• •		 • •		• •		
3			•	· •					•	 					•	· •					•	· •						 										
4	:	•	•	 				· • · · ·	•••	 				:• ::: ••	••	 				· • · · ·	••	 				· • · · ·	 • •.•	-										
5	••••	•	••	••	••		••	••	••	••	••		••	••	••	••	••		••	••	••	••	••			••	••	••	••									
6	:: : :: :	•	••	• •	• •	•••				• •	• •		• •	• •	• •	• •		• •	• •	• •	• •	• •	• •	• •	• •	• •		• •	• •	• •		• •	• •	 • •	• •	• •	• • •	 • •
7		•																																				

Line 1: The opening switch is placed after the last word of the cell-5 heading. Line 2: A blank line is required before spatial material.

More Addition Problems for Tomorrow's Class

	10 +10	10 +20	10 +30	10 + 40	10 + 50	
1	· · · · · · · · · · · · · · · · · · ·	··· •· •• •• ••			
2						
3						
4	• •	• • • • • • • •	• • • • • • • •	• • • • • • • • •	•	
5	· • · · · · · · · · · · · · · · · · · ·	· • · · · · · · · · · · · · · · · · · ·	· • · · · · · · · · · · · · · · · · · ·	· • · · · · · · · · · · · · · · · · · ·	· • · · · · · · · · · · · · · · · · · ·	
6	•• •• •• •• •• •	• •• •• ••	•• •• •• ••	•• •• •• ••	** ** ** ** **	
7						
8						

Line 1: The opening switch does not fit on the line with the cell-5 heading. Line 2: The opening switch is placed in cell 1 because spatial material follows. Line 3: A blank line is required before spatial material.

9.32 Code Switching and the Page Change Indicator

Recall from <u>Section 9.26.1</u> that a blank line is required before a page change indicator when the print page ends with a spatial arrangement. <u>Example 9-62</u> illustrates blank lines necessary when a heading begins on the new print page and code switching is necessary. Blank lines are required before and after each centered heading, and before and after each problem set. Each code switch indicator is placed in cell 1 on a line by itself.

Example	9-62					
		ADDITION	PROBLEMS			
10 +10	10 +20	10 +30	10 + 40	10 + 50		
=======	========	=========	========	========	== new print page 4	ł5
	5	SUBTRACTI	ON PROBLE	MS		
10	20	30	40	50		
-10	-10	-10	-10	-10		

3		
4		
5		
6		
7		
8		
9		
10	** ** ** ** ** ** ** ** ** ** ** ** **	
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21	** ** ** ** ** ** ** ** ** ** ** ** **	
22		
23		
	Line 5. A blank line is required following the contered heading on line A	

Line 5: A blank line is required following the centered heading on line 4. Line 6: The opening Nemeth Code indicator is placed in cell 1 on a line by itself. Line 7: A blank line precedes the spatial problem set. Line 11: A blank line follows the spatial problems. Line 12: The Nemeth Code terminator is placed in cell 1 on a line by itself. Line 14: A blank line precedes the centered heading on line 15. Lines 16-23: The same rules apply as to lines 5-12.

For further practice, see Addendum 1—Reading Practice.

Submit Exercise 9 to your instructor.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 9A

1										: : : •								: : : •			•	•••		: : : •	•••	•••											
2																																					
3																																					
4	•		•			. • • : • •	::	::	• . . • • ·			••	••	•	•••				••	••	• . . • • :				••	••	· · · · •	•••			•	•	•	• . . • • ·			
5	•	••	•	•	• · • •		•••		•••	••	•••	• . • •		•••	••		••	••	· · · · •	•••																	
6	•		•			. • • •	••		••			•		••	••	••	• . • •		•••	· · · · •			••	••	• . • •		•••		•••	::		• · • • • •			• . . • . •		
7	•••	•	:				•••		•	••	. • • ·		• •	• · • · • ·		• · • •			• •		• · • • • •								•••	•••			::	••	•••	• •	
8	•	•		•••	••• •••		•••				•••	•			•		· • · •	••		•••			••	•••			•••	•		•••			• · • • • •				
9			•	•••		• • • •	• •	• · • · • ·		• · • •			• ·		• . . •							•			::	••	•••		• •								
10		•••	•		• · • •		· • • •	• · • •	•••																												
11	•	•••	1		••	•		•••	••			• ·				•••	• • • •			• •	•		• · · • · •		::												
12		•••	•				•••	• · • • • •						• . • •																							
13			,					•••	• · • • • •					•••	• . . • • •			•• • ·		•••	• · · •							•••	•••								

Line 5: The dashed shaft must not be divided between lines.

Lines 12-13: This long expression will not fit on one line. The arrows are comparison signs, making this is a "linked expression". In order to organize this expression into mathematical units, division is made before the main sign of comparison – the third arrow in this case (line 13).

PRACTICE 9B

PRACTICE 9C

1	
2	
3	
4	∃• •• •• •• •• •• •• •• •• ∃∃ ∃∃ ∃• •• •
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13	

Note: It is acceptable to have two blank cells between items in the second column.

PRACTICE 9D

1	
2	
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4	
5	•• •• •• •• •• •• •• •• •• •• •• •• ••
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13	

PRACTICE 9E

1																																					
2	··· ·· ··	::	:::	::	:::	::	:::	::	::	::	::	::	::	::	:::	::	::	::	::	::	::	:::	::	::	::	::	::	:::	::	::	::	::	::	::	::		:
3						•	•	::	::	::		•		::			•	::	••	· · · • •		•••	• • • •	· · • •		•••											
4		::		••	::	::	::							::		:: ••		•••	••	· • · · ·	•	•••	• • • •	 	•••	• . • •											
5		::	••	••	••	••	••	••						::	••	••	••	••	••	••	••	••	••	••	••	••	••										
6				•		•	•										•	•••	••	· • · · ·		 •	• • • •	 		::											
7	··· ·· ·· ··	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::		:
8		 			••	••		 • . • .	 ••	••		•••	••	. • 		•••																					
9		::		••	•••	••		 • . • .	 ••	·• ••	•	•••	••	••	•	•																					
10		::	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••																				
11				••	•	••		•	•	:• ::: ••			••	:: ••		•••																					
12	··· ·· ·· ·· ··· ·· ··	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::		:
13		 			•	: • : •			•••	• • • •	••			 																							
14		::		· • · ·	 • •.•	: • : •			 • •.•	• • • •	•	•		 																							
15		::	••	••	••	÷	••	••	••	••	••	••	••	••	••	••	••	••																			
16					•••		• • • •	•	••	•	•	•	 	 		• • • • • •			· • · · •	• · · · • ·		 			• • • • • •	•	 	•	•				•• ••				
17	··· ·· ··	::	:::	::	::	::	:::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::	::		:
18																																					

PRACTICE 9F

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3						
4		· · · · · · · · ·				
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6						** ** ** ** **
7		•		• • • • • • • • • • • • • • • • • • •		
8		••	•		* •	· · · · · · · · · · · · · · · · · · ·
9		•• •• •• •	• •• ••		•	· · · · · · · · · · · · · · · · · · ·
10			•		••	
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16	· · · · · · · · · · · · · · · · · · ·	· · · · ••	• •	••		
17	· · ••	••••••	••••••	•• •• •• ••	•• ••	
18		••	• • • •	••••••		
19						
20						