LESSON 9

Read about this PROVISIONAL EDITION in the front matter to this book. Check the NFB website periodically for updates to this lesson.

- INTRODUCTION TO FRACTIONS
 - Simple Fractions
 - Mixed Numbers
 - Complex Fractions
- <u>More Fraction Rules</u>
- RADICAL EXPRESSIONS

FORMAT <u>LINKED EXPRESSIONS</u> <u>Division of a Long Linked Expression</u> <u>Special Linked Expression</u>

INTRODUCTION TO FRACTIONS

In a technical transcription, fractions are brailled in Nemeth Code. Two types of fractions are presented in this lesson: simple fractions (including mixed numbers) and complex fractions.

9.1 Recognition and Layout: A fraction is composed of three parts: a numerator, a denominator, and a fraction line.

numerator
$$\frac{3}{4} \leftarrow fraction line$$

Fractions are printed in a variety of ways. The numerator may be printed above the denominator or they may be printed on the same level. The fraction line may be horizontal or diagonal. Here are three examples of the fraction "three fourths" printed in different styles.

$$\frac{3}{4}$$
 $\frac{3}{4}$ $\frac{3}{4}$

The numerator and/or denominator may also consist of or contain words or abbreviations. Here are examples.

m/s	("meters per second")
ft./ _{sec.}	("feet per second")
rise run	("rise over run ")
3.5%/year	("3.5 percent per year")

Simple Fractions

9.2 Definition of Simple Fraction: A simple fraction is one in which neither the numerator nor the denominator is a fraction.

These are simple fractions:This is not a simple fraction:

$$\frac{1}{2}$$
 $\frac{a^2}{b^2}$ mi./hr. $\frac{1/3}{2/3}$

Note that the fraction line may be printed as a horizontal line or as a diagonal line.

A fraction is also considered to be a simple fraction when its numerator or denominator contains fractions at the superscript or subscript level only.

This is a simple fraction: $\frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

Regardless of print layout, unless otherwise stated the fraction is *transcribed linearly* so that the numerator, the fraction line, and the denominator are written horizontally across one braille line.

9.3 Simple Fraction Indicators and the Horizontal Simple Fraction Line: Simple fraction indicators are used to enclose a simple fraction whose numerator and denominator are separated by a *horizontal fraction line*.

Simple Fraction I	ndicators		
Opening	• • • • • •		
Closing	· • · •		

In a simple fraction, the horizontal fraction line is brailled as the symbol shown below.



Note that the horizontal simple fraction line consists of one braille cell.

$$\gg \frac{3}{4}$$

Example 9.3-1 Terry has 32 candy bars. She shares $\frac{3}{4}$ of them with her class.

$$\gg \frac{d}{t}$$

Example 9.3-2 Rate formula: rate $= \frac{\text{distance}}{\text{time}}$ or $r = \frac{d}{t}$.

Reminder: Words in Nemeth Code are brailled without contractions.

Example 9.3-3 Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{\Delta y}{\Delta x}$.

The numerator and denominator are unspaced from the fraction indicators and from the fraction line. Spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction.

Example 9.3-4 Multiplying fractions is easy!
$$\frac{3}{4} \cdot \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$$

There is no space before or after the operation signs (multiplication dots); there is a space before and after the comparison signs (equals signs).

Example 9.3-5 Use the reciprocal of the coefficient to solve for x in $\frac{3}{8}x = 72$.

The coefficient (fraction) is unspaced from the variable (x).

Example 9.3-6 Anderson sprinted $\frac{2}{3}$ of the $\frac{1}{4}$ -mile track.

The fraction in this hyphenated expression is unspaced from the hyphen.

Instructions: Treat the second heading as a connected title heading. (See *Braille Formats*, Section 4, for details.) Transcribe the first series of fractions as a paragraph, starting in cell 3. Begin a new line in the runover cell (cell 1) if the entire fraction or ellipsis will not fit on what remains of the current line. A blank line must precede the itemized portion. When you proofread, check that you closed each fraction, that you returned to the baseline after each superscript, that displayed expressions are placed in the proper cell, and that you terminated Nemeth Code where appropriate. Watch for end-of-sentence punctuation.

PRACTICE 9A

Horizontal Simple Fraction Line

Simple fractions:
$$\frac{1}{2} \dots \frac{15}{16} \dots \frac{x}{y} \dots \frac{a+b}{c+d} \dots \frac{\Delta y}{\Delta x} \dots \frac{(x+y)}{(x-y)} \dots$$

 $\frac{9}{12} \dots \left(\frac{3}{2}a + \frac{1}{2}b\right) \dots \frac{3x}{17y} \dots x - \frac{1}{4}(x-2x)$
1. $V = \frac{1}{3}\pi r^2 h$
2. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
3. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
4. Write an equation to show that $\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$.

- 5. $x^2 \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x^2 x 2}{(x+1)(y+1)}$
- 6. Solve this differential equation:

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = e^{x^2}$$

7. The number π is the ratio of the circumference of a circle to its diameter. That is,

$$\pi = \frac{\text{circumference}}{\text{diameter}}.$$

8. $\frac{35}{70} = \frac{x}{100}$ 9. $\frac{12}{33} = \frac{m}{11}$ 10. $\frac{x}{15} = \frac{12}{75}$ 11. $\frac{4}{32} = \frac{10.5}{x}$ 12. $\frac{1}{4} + \frac{3}{4} - \frac{1}{2} = \frac{1}{2}$ **9.4 The Diagonal Simple Fraction Line:** The type of fraction line used in the print copy (horizontal or diagonal) is replicated in the braille transcription. In a simple fraction, the diagonal fraction line is brailled as the symbol shown below.

Diagonal Simple Fraction Line /

Note that the diagonal simple fraction line consists of two braille cells.

When a diagonal fraction line is printed, it may not be clear where the fraction begins and where it ends. The transcriber must not attempt to analyze the math. Instead, application of the following rules will prevent misinterpretation of the expression.

9.4.1 Use of simple fraction indicators with the diagonal simple fraction line: When the numerator and denominator are *printed at different levels* of writing on either side of the *diagonal line*, the construction is clearly a fraction and so simple fraction indicators are used.

 $\gg \frac{3}{4}$

Do not confuse this type style with superscripts and subscripts. In this example, the numeral 3 is the numerator and the numeral 4 is the denominator.

Example 9.4-1 Terry has 32 bags of M&Ms. She shares $\frac{3}{4}$ of the bags with her class.

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If the numerator and denominator are printed at the same level of writing on either side of the diagonal line the transcriber must notice the <u>type size</u>. If the type is *in a different size* from that normally used for similar expressions throughout the text, identify this as a fraction by using simple fraction indicators. In the example below, note that the fraction is printed on the baseline of writing—it is *not* a subscript.

≥ 2 1/y

The numeral 1 is smaller than the numeral 2. The space between the coefficient and the fraction is not shown in braille.

Example 9.4-2 In the expression 21/y, 2 is the coefficient of the fraction.

9.4.2 Nonuse of simple fraction indicators with the diagonal simple fraction line: When the numerator and denominator are *printed at the same level of writing on either side of the diagonal line* and the *type size is normal* when compared to similar expressions, fraction indicators are <u>not</u> used. A switch to Nemeth Code is required. (Review **6.4.1** "Slash".)

≥ 3/4

Example 9.4-3 Jean has 16 bags of Skittles. She shares 3/4 of the bags with her class.

Example 9.4-2 is amended below. The first fraction is printed on the baseline but is in smaller type. Fraction indicators are used. However, the second fraction is printed in normal type size. This meets the requirements for <u>nonuse</u> of simple fraction indicators because the numerator and denominator are printed at the same level of writing on either side of the diagonal line.

>> 1/y

Example 9.4-4 In the expression 21/y, 2 is the coefficient of the fraction 1/y.

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The second occurrence of "1/y" is printed in normal print size, therefore fraction indicators are not used even though the text refers to it as a "fraction".

The next example meets the requirements for <u>nonuse</u> of simple fraction indicators because (a) the numerator and denominator are printed at the same level of writing on either side of the diagonal line; and (b) the type size is normal for similar expressions (subscripts). Simple fraction indicators are <u>not</u> used here, even though the text identifies the expression as a "fraction".

 $\gg 3_{x/y}$

Example 9.4-5 Notice the fraction in the subscript: $3_{x/y}$

The words next to the diagonal lines in the next two examples are mathematical. Consider the context – the words represent ratios and probabilities. Because the context is mathematical, a switch to Nemeth Code is required for each fraction. Fraction indicators are not used because (a) the numerator and denominator are printed at the same level of writing on either side of the diagonal line; and (b) the type size is normal for similar expressions.

Example 9.4-6 The input/output ratio is equal.

Example 9.4-7 The red/white probabilities are the same.

9.4.3 Reminder About the UEB Slash: When the slash is serving as a separator between words or numbers in literary context, it is not mathematical. If the slash does not represent a mathematical operation, the rules of UEB are followed.

and/or, c/o	(UEB)
true/false test	(UEB)
6/6/44 (date)	(UEB)

Example 9.4-8 On 11/15/59 U.S. Patent #14–5/2806ds-XRX was renamed *xerography* and Haloid/Xerox launched the world of photocopying into a new era with the Xerox 914/01.

Numerals printed with slashes such as dates, model numbers, etc. are not mathematical and therefore the rules of UEB are followed.

9.5 Additional Considerations

9.5.1 Fractions and Abbreviations: Abbreviations in the numerator or denominator of a fraction are unspaced from the fraction indicators or fraction lines. When the slash means *per, divided by,* or *over* the construction is considered to be a fraction and Nemeth Code is used.

Example 9.5-1 Express m/s in mph.

m/s means "meters <u>per</u> second"

Example 9.5-2 Express ^{ft.}/_{sec.} in mph.

ft./sec. means "feet per second"

An abbreviation is spaced from the value to which it applies (regardless of the spacing applied in print).

Example 9.5-3 Add 2/3 c. cornstarch—Oobleck!

A space is placed between 2/3 and its associated abbreviation, c.

Example 9.5-4 $1 \text{ ft/}_{sec} \approx 0.6818 \text{ mph}$

A space is between the numeral 1 and its associated abbreviation, ft/sec, as well as between the numeral 0.6816 and its associated abbreviation, mph.

9.5.2 Code Switch Reminders: Revisiting **Lesson 4**, as part of a math problem expressed in symbols and words, the words are included in the switch. Compare the next two examples.

Example 9.5-5 $\frac{1}{4}$ of 24 is 6

Only the fraction requires a switch to Nemeth Code.

Example 9.5-6 $\frac{1}{4}$ of 24 = 6

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The entire "math sentence" is a mathematical expression. The word "of" is part of the equation and is uncontracted in Nemeth Code.

Abbreviations are included inside the switches. Compare the next two examples.

Example 9.5-7 Convert feet to inches: change $\frac{1}{2}$ ft to 6 in. Add that to 3 in to get 9 in.

Remember, the abbreviation must fall on the same line as its value.

Example 9.5-8 ... Change 1/2 foot to 6 inches. Add that to 3 inches to get 9 inches.

The value need not be forced to fall on the same line as the related word.

Stay in Nemeth Code for a single word in UEB.

Example 9.5-9 Equivalent ratios of y/x (or x/y) can be seen in Table 4.1.

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Math grouping symbols are used for the parentheses here because we have not switched out of Nemeth Code.

Instructions: First determine whether the slash is used mathematically, that is, does it require a switch to Nemeth Code? If it does, then determine if fraction indicators are required.

PRACTICE 9B

Diagonal Simple Fraction Line

- A) How many $\frac{2}{3}$'s are there in $\frac{5}{6}$?
- B) Energy is absorbed at the rate of 880 J/s for each square meter of the surface.
- C) $y(0) = \pi/4$
- D) In y 1/5, y is the coefficient of the fraction 1/5.
- E) True/False: The rise/run ratio is 5 in graph A.
- F) $a/b \cdot c/d = ac/bd$
- G) A 5-year CD went from earning interest at the rate of 12.06%/year in 1984 to earning less than 0.87%/year in 2015.

Mixed Numbers

9.6 Definition of Mixed Number: A mixed number is an expression composed of a whole number followed by a simple fraction whose numerator and denominator are both numerals. Numerals in a mixed number may be represented by omission signs. An expression is <u>not</u> a mixed number if it contains any letters, even though the expression appears to be in the form of a mixed number. Here are some examples.

1 $\frac{2}{3}$ (Spoken "one and two-thirds") 99 $^{15}/_{16}$ (Spoken "ninety-nine and fifteen-sixteenths")

9.6.1 Use of Mixed Number Fraction Indicators: The opening and closing mixed number fraction indicators enclose the fractional part of a mixed number.

Mixed Number Fr	action Indicators
Opening	
Closing	· • · • • • • • • • •

The fractional part of the mixed number uses simple fraction lines, either horizontal or diagonal, according to the fraction line style used in print.

Horizontal Simple Fraction Line		• • • • • •
Diagonal Simple Fraction Line	/	

The examples shown above are brailled as follows.

$$\gg 1\frac{2}{3}$$

$$\gg 99^{15}/_{16}$$

$$\approx 10^{15} \times 10^{10} \times 1$$

Example 9.6-1 Russ is making a small flowerbed that is $3\frac{1}{2}$ feet by $1\frac{1}{2}$ feet.

9–10

9.6.2 Mixed Numbers and Omissions: If any part of a mixed number contains a sign of omission, the mixed number fraction indicators are used.

 $\gg \frac{7}{4} = 1\frac{?}{4}$ $\gg 1^{15}/_{25} = ?^{3}/_{5}$

9.6.3 Nonuse of Mixed Number Fraction Indicators: If the fractional part of the expression contains a letter, it no longer suits the definition of "mixed number." Appropriate fraction indicators are used (or are not used) according to the rules.

$$\gg \frac{7}{4} = 1 \frac{x}{4}$$

$$\gg 3 x/y$$

$$\Rightarrow 3 x/y$$

Instructions: Treat "FRACTION REVIEW" as a centered heading.

PRACTICE 9C

Mixed Numbers

1. Find the premium for a $1\frac{1}{2}$ -yr. policy at the yearly rate of 24¢ per \$100.

2.
$$2\frac{1}{2}$$
 ft + 8 in = ? inches

3. $\left(\frac{1}{2} \times 3\frac{1}{2}\right) + \left(3\frac{1}{2} \times 2\right)$

4.
$$13^{1}/_{2} + 2^{2}/_{3} = 16^{1}/_{6}$$

5.
$$7/4 = 1^{?}/_{4}$$

6. $\frac{9}{4} = 2\frac{x}{4}$

FRACTION REVIEW

Compute each unit rate (price/pound).

- a. \$1.50 for 2/3 pound of potatoes
- b. 4.20 for 1/2 pound of Edam cheese
- c. \$6.00 for $\frac{3}{4}$ pound of deli smoked turkey
- d. \$12.50 for $1\frac{1}{2}$ pounds of sliced ham

Complex Fractions

9.7 Definition of Complex Fraction: A complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers.

numerator
$$\frac{4\frac{3}{4}}{-5}$$
 \leftarrow complex fraction line denominator 5

Here are more examples of complex fractions. In this manual, the complex fraction line is usually longer or thicker than the fraction lines within the numerators and denominators.

$$\frac{\frac{4}{3}}{12} \qquad \frac{\frac{a}{b} - \frac{c}{d}}{\frac{a}{b} + \frac{c}{d}} \qquad \frac{1}{3/8} \qquad \frac{1}{2}/\frac{3}{4}$$

Reminder: A fraction is <u>not</u> a complex fraction if the only fractions it contains are at the superscript or subscript level. Such a fraction is a simple fraction.

This is a simple fraction, <u>not</u> a complex fraction:

 $\frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

9.7.1 Use of Complex Fraction Indicators and Complex Fraction Lines: The opening and closing complex fraction indicators are used to enclose a complex fraction.

Complex Fraction	Indicators	
Opening		
Closing		

The main complex fraction line is represented by its appropriate braille symbol—either horizontal or diagonal.

Horizontal Complex Fraction Line	· · · • · · · · · • •
Diagonal Complex Fraction Line	

The examples shown above are brailled as follows. Simple fraction indicators enclose each simple fraction when required; complex fraction indicators enclose the entire complex fraction. We suggest that you underline the complex fraction indicators and the complex fraction line in each example in order to analyze each transcription.

$$\gg \frac{4\frac{3}{4}}{5}$$

$$= \frac{\frac{4}{3}}{12}$$

$$= \frac{\frac{4}{3}}{\frac{1}{2}}$$

$$= \frac{\frac{1}{2}/\frac{3}{4}}{\frac{a}{5}} + \frac{c}{\frac{a}{6}}$$

$$= \frac{\frac{a}{b} - \frac{c}{d}}{\frac{a}{b} + \frac{c}{d}}$$

$$= \frac{1}{3/8}$$

$$= \frac{1}{3}$$

Instructions: Begin each complex fraction on a new braille line, not side-by-side as printed. Read left-to-right.

PRACTICE 9D Complex Fractions $\frac{\frac{1}{8} + \frac{3}{4}}{7} \qquad \frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{7}{9}} \qquad \frac{\frac{1/3 + 1/4}{4/5 - 1/2}}{\frac{1/3 + 1/4}{4/5 - 1/2}} \qquad \frac{\frac{\pi}{8}}{2\pi}$ $\frac{\frac{33\frac{1}{3}}{100}}{\frac{\frac{3}{5}}{6}} \qquad \frac{\frac{3}{5}}{\frac{5}{9}} / \frac{7}{9}$

More Fraction Rules

9.8 Fractions and the Baseline Indicator: When a fraction is on the baseline level, assure that the components of the fraction (the fraction indicators and the fraction line) are notated on the baseline of writing. Place the baseline indicator before the fraction line or the fraction indicator when there is a level in effect at the end of a numerator or a denominator. Correct placement of the baseline indicator assures accurate reading.

The baseline indicator precedes the fraction line following the superscript in the numerator.

The baseline indicator precedes the closing fraction indicator following the superscript in the denominator.

$$\gg \frac{\frac{x}{x-1} - 1^2}{\frac{x}{x+1} + 1^2}$$

These fractions are all on the baseline.

This fraction is in the superscript position.

The baseline indicator is not used after a numeric subscript that does not require a subscript indicator.

A return to the baseline after each subscript is understood.

9.9 Further Observations Regarding Spacing: As seen in previous examples, spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction.

Example 9.9-1 Explain why $\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$.

The plus sign is unspaced from the fractions before and after it; there is a space before and after the equals sign. There is no space between the components of a mixed number.

No space is left between a fraction and a letter, a numeral, a sign of grouping, a braille indicator, or another fraction when these items are part of the same expression.

Example 9.9-2 Multiply the fractions: $\left(\frac{5}{12}\right)\left(\frac{4}{12}\right) = _$

No space is left between factors even though one may appear in print.

Example 9.9-3 Differentiating the first two terms, $\frac{1}{2}x^{1/2} + \frac{5}{6}x^{-3/2}$.

No spaces are brailled in this long math expression.

9.10 Fractions and the Ellipsis and Long Dash: No space is left between an opening or closing *fraction indicator* and an ellipsis or long dash in the numerator or denominator of a fraction.

$$\frac{...\times 5}{2\times 10} = \frac{15}{20}$$

$$\therefore 3\times 5 = \frac{15}{20}$$

$$\frac{3\times 5}{2\times 2} = \frac{15}{20}$$

$$\therefore 3\times 5 = \frac{15}{20}$$

The space preceding this long dash is required. Review 2.13 in Lesson 2.

However, a space is left between a *fraction line* and an ellipsis or long dash.

$$\gg \frac{2+4+6+...}{1+3+5+...}$$

A space is left between a fraction and an ellipsis or long dash preceding or following the fraction.

$$\sum_{i=1}^{N} \frac{1}{10} \cdots \frac{10}{10}$$

$$Example 9.10-1$$
Fill in the blanks with the missing fractions: $\frac{1}{10} - \frac{4}{10}$

$$= \frac{4}{10}$$

$$= \frac{1}{10}$$

9.11 Enclosed Lists: Fractions and mixed numbers may be part of an enclosed list. (Review the rules for "enclosed list" in Lesson 5, section **5.15**.)

$$\gg \left\{0, \frac{1}{2}, 1, 1\frac{1}{2}\right\}$$

Instructions: Determine the formatting before beginning your transcription. Where does each paragraph begin? Which expressions are embedded and which are displayed? What is the proper cell placement for the displayed expressions?

PRACTICE 9E

These are simple fractions:

$$\frac{1}{2} \quad \frac{a^2}{b^2} \quad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$$

This is <u>not</u> a simple fraction: $\frac{1/3}{2/3}$

Review the rules in 8.11.5 regarding an ellipsis on the baseline of writing when it follows a superscript.

$$x^{\frac{1}{2}}$$
 ... $x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}}$... $\frac{x^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1}$

Plot the points $\left(-\frac{1}{2}, 4\right)$, $\left(3, 4\frac{1}{4}\right)$, and $\left(-9, \frac{3}{4}\right)$. Then express $\frac{a^{3/4}}{b^{5/4}}$ in radical form.

RADICAL EXPRESSIONS

9.12 Terminology: Here are the parts of a radical expression.

 $\sqrt{144}$ $\sqrt{}$ is the *radical sign*. 144 is the *radicand*. The horizontal bar above the radicand is the *vinculum*. The vinculum shows the extent to which the radical sign applies.

There may be a figure placed to the left and slightly above the radical sign, called the *index* of the radical. For example, this radical sign has an index of three: $\sqrt[3]{}$ When there is no index, the radical sign may be referred to as the "square root" sign.

9.13 The Termination Indicator: When a radical expression has a vinculum, the radical sign is placed before the radicand and the termination indicator is placed after the radicand.

Radical Sign	 •	
Termination Indicator	•• •• • •	

$$\gg \sqrt{x}$$

$$\gg \sqrt{64}$$

Reminders: An English letter indicator is not needed for an English letter (in regular type) which occurs in an unspaced sequence of mathematical symbols. A numeric indicator is not used when a numeral immediately follows an indicator.

When a vinculum is not shown in print, or when the radical sign occurs without a radicand, a termination indicator is <u>not</u> used.

 $\gg \sqrt{(x-1)}$

Example 9.13-1 The $\sqrt{}$ is called a "radical sign."

9.14 Spacing: The spacing before and after a radical expression is subject to the spacing rules for the signs preceding or following the radical expression.

No space is left between a radical expression and a letter, a numeral, a fraction, a sign of grouping, a braille indicator, or another radical expression when these items are unspaced in print and belong to the same expression.

$$\gg \sqrt{5}y$$

Reminder: In print, derivative notation dx, dy, etc. is often preceded and followed by a space within an expression, for clarity. In braille, the terms are not spaced unless a space is required with the item preceding or following them. (Review 5.13.1 in Lesson 5.)

Example 9.14-1 Simplify: $\frac{2-\sqrt{\frac{1}{4}}}{3-\sqrt{\frac{1}{2}}}$

9.15 Index of Radical: A small number or letter that may appear next to the radical sign is the *index*. This print example shows an index "3".

1

In braille, the index-of-radical indicator, as well as the index, precede the radical sign.

	Index-of-Radic	al Indicator	● · ● · · ●		
≫	$\sqrt[3]{27} = 9$				
۶	$\sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$				
Example	9.15-1 Simplify	y as follows: $\frac{\sqrt{9}}{\sqrt{2}}$	$\frac{1}{7} = \frac{3}{3} = 1$		
: : : : : : :					

PRACTICE 9F

Radical Expressions

1.	$\sqrt{\frac{b}{a} + \frac{a}{b}}$	2.√ <i>c/d</i>	3. $\frac{1}{4}\sqrt{\frac{1}{2}}$	$4. \sqrt{\frac{10}{16}} = \sqrt{10}/4$
5.	$(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})$	5)	6. $2\sqrt{2} + 7\sqrt{2} = (2+7)^{2}$	$\sqrt{2} = 9\sqrt{2}$
7.	$\frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{15}}{2}$:	8. $\frac{\sqrt{2} - \sqrt{\frac{1}{3}}}{\sqrt{3} - \sqrt{\frac{1}{2}}}$	9. $\sqrt{48x^3y}$
10.	$\sqrt{(y-1)} + \sqrt{(2y)}$	= 1	11. $\sqrt[n]{d}$	12. $\sqrt[a+b]{z-y}$
13.	$\sqrt[4]{729} + \sqrt[6]{27} = \sqrt[?]{?}$		14. $7\sqrt[3]{125} \cdot 7\sqrt[5]{2}$	15. $\sqrt[5]{m} \sqrt[5]{n} = \sqrt[5]{mn}$

9.16 Nested Radical Expressions: When radical expressions are nested one within the other, the appropriate number of order-of-radical indicators shows the depth of each inner radical expression.

Order-of-Radical Indicators	
First Inner Radical	. ● . ●
Second Inner Radical	• • • • • • • • •
Third Inner Radical	
Termination Indicator	

The appropriate order-of-radical indicator is placed before its radical sign and before its termination indicator.

When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

$$\gg \sqrt{x + \sqrt{y + \sqrt{z}}}$$

9.16.1 Nested Radical Expression with an Index: If an inner radical expression has an index, the appropriate order-of-radical indicator is placed before the index-of-radical indicator as well as before its termination indicator. When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

9.17 Radical Expressions and the Baseline Indicator: When a radical expression is on the baseline level, assure that the components of the construction (the radical sign or indicators) are notated on the baseline of writing. Place the baseline indicator before the component when it follows a superscript or a subscript.

The baseline indicator is not used after a numeric subscript that does not require a subscript indicator.

$$\gg \sqrt{x_1 + y_2}$$

9.18 Radical Expressions and the Ellipsis and Long Dash: These two omission symbols are generally preceded and followed by a space, but no space is left between a *braille indicator* and a related ellipsis or long dash. In the context of radical expressions this means no space is left between the ellipsis or long dash and the termination indicator or order-of-radical indicator.

A space is required between a radical expression and an ellipsis or long dash preceding or following the expression.

 $\gg \sqrt{4} \dots \sqrt{64}$

9.19 Radical Expressions and Abbreviations: When an abbreviation occurs within a radical, no space is left between the abbreviation and the termination or order-of-radical indicator following it.

 $\gg \sqrt{9 \text{ ft}}$

However, a space is required between a radical sign and an abbreviation.

 $\gg \sqrt{\mathrm{ft.}}$

A space is also required between a radical expression and an abbreviation preceding or following the expression.

9.20 Enclosed Lists with Radical Expressions: Radical expressions may be part of an enclosed list.

See 5.15 in Lesson 5 to review rules regarding enclosed lists.

PRACTICE 9G

Nested Radical Expressions

(12) $\sqrt{c+d+e+\cdots}$

- (1) $\sqrt{-\frac{1}{2} i\frac{\sqrt{3}}{2}}$ (2) $\sqrt{\sqrt{13} + \sqrt{15} + \sqrt{117}}$ (3) $\sqrt{1 - \sqrt{a - b}} \times \sqrt{1 + \sqrt{a - b}}$ (4) $\sqrt{b^3\sqrt{b\sqrt{b}}}$ (5) $\sqrt[a]{b\sqrt{\sqrt[b]{abc}}}$ (6) $\sqrt{a^2}\sqrt{b^4}\sqrt{c} = ab^2\sqrt{c}$ (7) $(s^2\sqrt[3]{s^4})^2$ (8) $\sqrt[3]{x^2\sqrt{64x^6}}$ (9) $\sqrt{\sqrt[3]{\sqrt[4]{5\sqrt{b^{48}}}}}$ (10) $\sqrt{x_1 + \sqrt{x_2}}$
- (11) $q^{\sqrt{r}} + s$

5/22/2019 revision

LINKED EXPRESSIONS

9.21 Definition of Linked Expression: A linked expression contains *at least one sign of comparison*. The part preceding the first sign of comparison is called the *anchor*. Each remaining part, beginning with a sign of comparison and ending before the next sign of comparison, is called a *link*.

Here are two examples of linked expressions.

 $12.5\% > \frac{1}{10}$ The anchor is 12.5% and the link is $> \frac{1}{10}$ $6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$ The anchor is 6×245 , followed by three links each beginning with an equals sign.

9.22 Division of Linked Expressions: Recall that a mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. If a *linked expression* is too long to fit on one line, the expression continues on the next line, beginning with a sign of comparison. If the expression contains more than one link, use as much of the line as possible before dividing the expression. It is not necessary to divide at every comparison sign. Begin the new line in the runover cell of the current format.

Example 9.22-1

The anchor starts in cell 5, displayed to itemized text. The first link fits on the same line. The second link starts on a new line in cell 7, the runover cell in this displayed format.

 Example 9.22-2
 How many hours? 3/8 of a day + 1/2 of a day = ____ hours.

 (Hint: A day is 24 hours.)

 \vdots \vdots

This is a narrative paragraph (3-1) with an embedded equation. The opening switch indicator will fit on the line with the anchor, in the runover position, cell 1. The link begins on the next line with its comparison sign.

9-22

9.22.1 Restrictions: In order to divide a long expression before a comparison sign, the comparison sign must be on the baseline of writing. Do not divide before a comparison sign that is part of an item enclosed in grouping symbols, between fraction indicators, or within radical signs. Details will be discussed in **Lesson 14**.

9.22.2 Other Considerations: A transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines.

$$\gg \left(\frac{a^2}{b}\right)^2 \div \left(\frac{b^2}{a}\right)^3 = \frac{a^7}{b^8}$$

A return to the baseline after the superscript "3" is triggered by the presence of the following comparison sign even though it is on the next line.

PRACTICE 9H

Linked Expressions

- 1. Is the following inequality true? $\frac{3}{5}\left(\frac{2}{3}x \frac{1}{2}\right) > \frac{2}{5}\left(\frac{1}{4}x + \frac{1}{3}\right)$
- 2. $33\frac{1}{3}\% + 40\% + 61\frac{2}{3}\% = 134\frac{3}{3}\% = 135\%$
- 3. In multiplying $5\frac{3}{4} \times 46$, recall that $5\frac{3}{4} = 5 + \frac{3}{4}$. Therefore, $46 \times (5 + \frac{3}{4}) = (46 \times 5) + (46 \times \frac{3}{4}) = 264\frac{1}{2}$.

9.23 SPECIAL CASE—Certain Displayed Linked Expressions: Lesson 8 introduced displayed math expressions—how to recognize them in the print copy and what margins to use in the braille transcription. A displayed linked expression is subject to special braille format rules if it appears in print in the following way:

- The expression is <u>displayed</u> (not embedded) in the text.
- Its signs of comparison are <u>vertically aligned</u>.
- Other than the anchor on the first line, no sign of comparison is preceded by any expression on its left. (There is one exception – see the fourth bullet, below)

The following linked expression meets the three "special" requirements.

To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

from factor.

$$ab + c^2 + ac + bc = (ab + ac) + (bc + c^2)$$

 $= a(b + c) + c(b + c)$
 $= (a + c)(b + c)$.

In print it is common for the last line of the expression to contain more than one link. As long as the other conditions are met, this layout is still considered to meet the "special" requirements.

We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way: $12\frac{1}{2}\% = 12.5\%$ = .125 $= \frac{125}{1000} = \frac{1}{8}$

Using the same example only with a different print layout, this <u>does not</u> meet the requirements for a linked expression requiring special margins because there is more than one link on the first line of the displayed expression. (Violation of the third bullet.)

We can reduce
$$12\frac{1}{2}\%$$
 to lowest terms in the following way:
 $12\frac{1}{2}\% = 12.5\% = .125$
 $= \frac{125}{1000} = \frac{1}{8}$

Using a similar example, the next layout <u>does not</u> meet the requirements for a linked expression requiring special margins because the expression is not displayed. (Violation of the first bullet.)

1.
$$12\frac{1}{2}\% = 12.5\%$$

= .125
= $\frac{125}{1000} = \frac{1}{8}$

If the print layout meets the requirements for a linked expression requiring special margins, one of the following braille formats is applied.

FORMAT

Margin Requirements for a Linked Expression Requiring Special Margins

9.23.1 In Narrative: When a linked expression requiring special margins as defined above occurs in unitemized explanatory portions of the text, the *anchor* begins in cell 3 and *each link* begins in cell 5. Any runovers to the anchor or to a link are indented two more cells, to begin in cell 7. (Runovers are discussed in a later lesson.)

Example 9.23-1

1 2

3

To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

$$ab + c^{2} + ac + bc = (ab + ac) + (bc + c^{2})$$

= $a(b + c) + c(b + c)$
= $(a + c)(b + c)$.

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Lines 1-3: Narrative paragraph (3-1)

**Line 4: Displayed math begins in cell 3.

(Note that the displayed anchor is indented two cells to the right of the runover cell of the preceding material.)

**Lines 5-7: Each link of this linked expression requiring special margins begins in cell 5. (Note that each link is indented two cells to the right of the anchor.) Even when the print copy shows more than one link on the last line, in braille *each link* begins on a new line.

Example 9.23-2

1

2 3

4

5 6 7 We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

Lines 1-2: Narrative paragraph (3-1)

**Line 3: Displayed math begins in cell 3.

**Lines 4-7: Each link of this linked expression requiring special margins begins in cell 5.

9.23.2 In Itemized Text Without Subdivisions: When a linked expression requiring special margins as defined above occurs in itemized text containing no subdivisions, the *anchor* begins in cell 5 and *each link* begins in cell 7. Any runovers to the anchor or to a link are indented two more cells, to begin in cell 9. (Runovers are discussed in a later lesson.)

Example 9.23-3

1 2

3

4

5

6 7 8 1. The example below shows a way of finding 6×245 .

$$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5)$$
$$= 1200 + 240 + 30$$
$$= 1470$$

Is there another way to find 6×245 ?

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Lines 1-2: Itemized text (1-3)

**Line 3: Displayed math begins in cell 5.

(Note that the displayed anchor is indented two cells to the right of the runover cell of the preceding material.)

**Lines 4-6: Each link of this linked expression requiring special margins begins in cell 7. (Note that each link is indented two cells to the right of the anchor.)
Lines 7-8: Text continues in the runover cell to itemized text, cell 3. **9.23.3 In Itemized Text With Subdivisions:** When a linked expression requiring special margins as defined above occurs in itemized text containing subdivisions, the *anchor* begins in cell 7 and *each link* begins in cell 9. Any runovers to the anchor or to a link are indented two more cells, to begin in cell 11. (Runovers are discussed in a later lesson.)

Example 9.23-4

1

2 3 4

5

6 7 8

- 2. Factor 2a(b-c) 3x(c-b).
 - (a) Factors b c and c b are divisible by (b c) since

c - b = (-1)(b - c).

(b) Thus

$$2a(b-c) - 3x(c-b) = 2a(b-c) + 3x(b-c)$$

= $(2a + 3x)(b-c)$.

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Line 1: Itemized text begins in cell 1.

Lines 2-3: Subitem text (3-5).

**Line 4: Displayed math begins in cell 7. This is not a linked expression requiring special margins so the link continues on the same line as the anchor.

Line 5: Next subitem begins in cell 3.

**Line 6: Displayed math begins in cell 7.

(Note that the displayed anchor is indented two cells to the right of the runover cell of the preceding material, even if the preceding material does not require a runover line.)

**Lines 7-8: Each link of this linked expression requiring special margins begins in cell 9. (Note that each link is indented two cells to the right of the anchor.)

Reminder: A line is not skipped above or below displayed mathematical material unless a blank line is required under other rules or guidelines.

Code Switch Decision: It would not be incorrect to terminate Nemeth Code at the end of line 1, braille the identifier (a) in UEB, and switch back to Nemeth Code after the word "Factors". The transcription shown minimizes code switching which is one of the considerations in the production of a fluent transcription.

Instructions: Treat "MULTIPLYING MIXED NUMBERS" as a centered heading.

PRACTICE 9I

Special Linked Expressions

To test the equation $R_t = \frac{R}{n}$, use four 1000- Ω resistors wired in series to

predict a total resistance of 250 $\Omega.$

$$R_t = \frac{R}{n} = \frac{1000 \ \Omega}{4}$$
$$\frac{1000 \ \Omega}{4} = 250 \ \Omega$$

Then, by using the resistance theory equation

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n},$$

plug the 150- Ω and 1000- Ω resistors into the equation as R_1 and R_2 .

$$\frac{1}{R_t} = \frac{1}{150 \,\Omega} + \frac{1}{1000 \,\Omega}$$
$$= 0.007 + 0.001$$
$$= 0.008$$
$$R_t = \frac{1}{0.008} = 125 \,\Omega$$

MULTIPLYING MIXED NUMBERS

A. Explain each step in the solution to this multiplication problem.

$$2\frac{1}{2} \cdot 1\frac{1}{4} = \left(2 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{4}\right)$$
$$= 2 + \frac{2}{4} + \frac{1}{2} + \frac{1}{8}$$
$$= 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8}$$
$$= 2 + 1 + \frac{1}{8} = 3\frac{1}{8}$$

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

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NOTES:

A) The apostrophe-s is included inside the switch and so a punctuation indicator is needed for the apostrophe. Review **3.7** in Lesson 3.

B) Because the slash means "per" ("Joules per second") a switch to Nemeth Code is required. The value ("880") is included in the switch. An English letter indicator is required for a single-letter abbreviation with no related period.

C) No fraction indicators are used because "pi" and "4" are of normal size and are printed on the baseline.

E) The first slash is in literary context. A switch to Nemeth Code is required for the second slash because it means "over" ("rise over run") in a ratio.

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EXERCISE 9

Exercise 9 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 10.